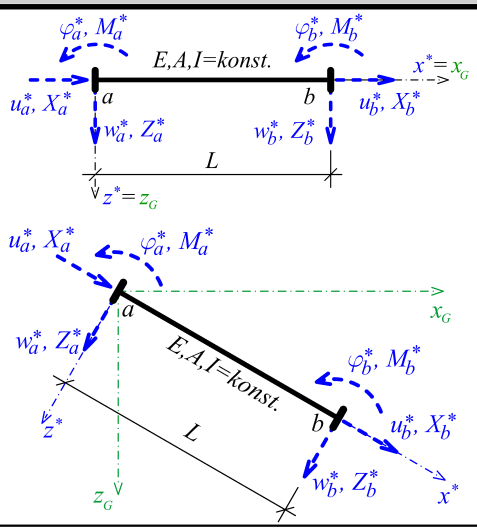
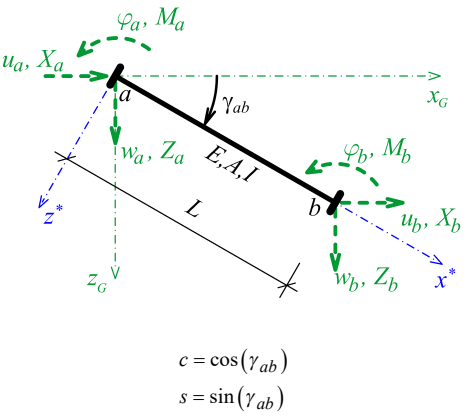
Vetknutí - Vetknutí |——————|



Lokální matice tuhosti

$$\left[K^*_{a,b}\right]=\begin{array}{c}\\ \widehat{X}^*_a\\ \widehat{Z}^*_a\\ \widehat{M}^*_a\\ \widehat{X}^*_b\\ \widehat{Z}^*_b\\ \widehat{M}^*_b\end{array}\begin{array}{c}\begin{array}{cccccc} u^*_a & w^*_a & \varphi^*_a & u^*_b & w^*_b & \varphi^*_b\end{array}\\ \left[\begin{array}{cccccc} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0\\ 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2}\\ 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} & 0 & \frac{6EI}{L^2} & \frac{2EI}{L}\\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0\\ 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2}\\ 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} & 0 & \frac{6EI}{L^2} & \frac{4EI}{L}\end{array}\right]\end{array}$$

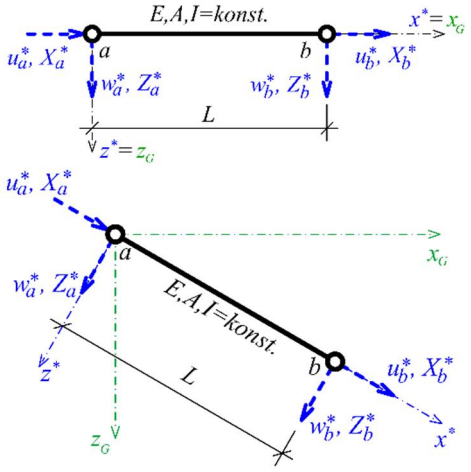


$$c=\cos(\gamma_{ab})$$
$$s=\sin(\gamma_{ab})$$

Globální matice tuhosti

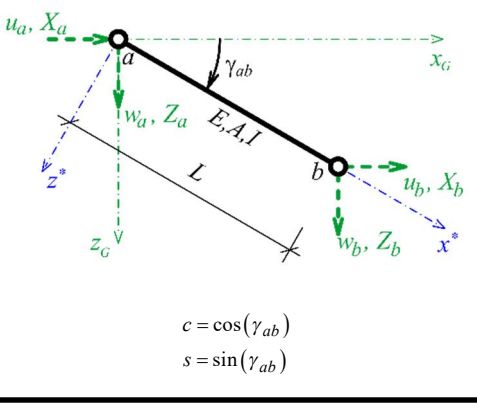
$$\left[K_{a,b}\right]=\begin{array}{c}\\ \widehat{X}_a\\ \widehat{Z}_a\\ \widehat{M}_a\\ \widehat{X}_b\\ \widehat{Z}_b\\ \widehat{M}_b\end{array}\begin{array}{c}\begin{array}{cccccc} u_a & w_a & \varphi_a & u_b & w_b & \varphi_b\end{array}\\ \left[\begin{array}{cccccc} \underbrace{\left(\frac{EA}{L}c^2+\frac{12EI}{L^3}s^2\right)}_{D11} & \underbrace{\left(\frac{EA}{L}-\frac{12EI}{L^3}\right)cs}_{D12} & \frac{6EI}{L^2}s & -D11 & -D12 & \frac{6EI}{L^2}s\\ \underbrace{\left(\frac{EA}{L}-\frac{12EI}{L^3}\right)cs}_{D12} & \underbrace{\left(\frac{EA}{L}s^2+\frac{12EI}{L^3}c^2\right)}_{D22} & -\frac{6EI}{L^2}c & -D12 & -D22 & -\frac{6EI}{L^2}c\\ \frac{6EI}{L^2}s & -\frac{6EI}{L^2}c & \frac{4EI}{L} & -\frac{6EI}{L^2}s & \frac{6EI}{L^2}c & \frac{2EI}{L}\\ -D11 & -D12 & -\frac{6EI}{L^2}s & D11 & D12 & -\frac{6EI}{L^2}s\\ -D12 & -D22 & \frac{6EI}{L^2}c & D12 & D22 & \frac{6EI}{L^2}c\\ \frac{6EI}{L^2}s & -\frac{6EI}{L^2}c & \frac{2EI}{L} & -\frac{6EI}{L^2}s & \frac{6EI}{L^2}c & \frac{4EI}{L}\end{array}\right]\end{array}$$

Kloub - Kloub ○————○



Lokální matice tuhosti

$$\left[K^*_{a,b}\right]=\begin{array}{c}\\ \widehat{X}^*_a\\ \widehat{Z}^*_a\\ \widehat{M}^*_a\\ \widehat{X}^*_b\\ \widehat{Z}^*_b\\ \widehat{M}^*_b\end{array}\begin{array}{c}\begin{array}{cccccc} u^*_a & w^*_a & \varphi^*_a & u^*_b & w^*_b & \varphi^*_b\end{array}\\ \left[\begin{array}{cccccc} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\end{array}\right]\end{array}$$

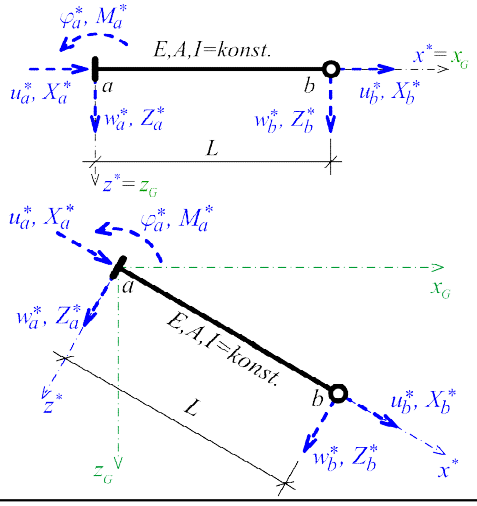


$$c=\cos(\gamma_{ab})$$
$$s=\sin(\gamma_{ab})$$

Globální matice tuhosti

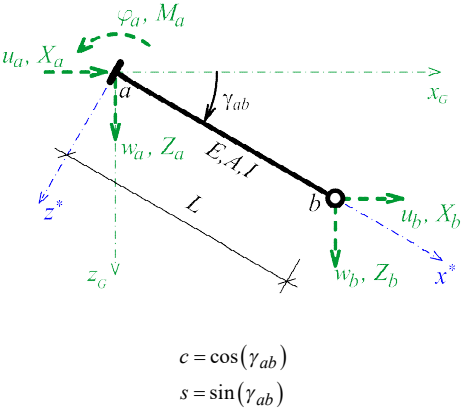
$$\left[K_{a,b}\right]=\begin{array}{c}\\ \widehat{X}_a\\ \widehat{Z}_a\\ \widehat{M}_a\\ \widehat{X}_b\\ \widehat{Z}_b\\ \widehat{M}_b\end{array}\begin{array}{c}\begin{array}{cccccc} u_a & w_a & \varphi_a & u_b & w_b & \varphi_b\end{array}\\ \left[\begin{array}{cccccc} \frac{EA}{L}c^2 & \frac{EA}{L}cs & 0 & -\frac{EA}{L}c^2 & -\frac{EA}{L}cs & 0\\ \frac{EA}{L}cs & \frac{EA}{L}s^2 & 0 & -\frac{EA}{L}cs & -\frac{EA}{L}s^2 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ -\frac{EA}{L}c^2 & -\frac{EA}{L}cs & 0 & \frac{EA}{L}c^2 & \frac{EA}{L}cs & 0\\ -\frac{EA}{L}cs & -\frac{EA}{L}s^2 & 0 & \frac{EA}{L}cs & \frac{EA}{L}s^2 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\end{array}\right]\end{array}$$

Vetknutí - Kloub



Lokální matice tuhosti

$$\left[K^*_{a,b}\right]=\begin{array}{c|ccc|ccc|c}
 & u^*_a & w^*_a & \varphi^*_a & u^*_b & w^*_b & \varphi^*_b & \\
\hline
 & \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 & \widehat{X}^*_a \\
 & 0 & \frac{3EI}{L^3} & -\frac{3EI}{L^2} & 0 & -\frac{3EI}{L^3} & 0 & \widehat{Z}^*_a \\
 & 0 & -\frac{3EI}{L^2} & \frac{3EI}{L} & 0 & \frac{3EI}{L^2} & 0 & \widehat{M}^*_a \\
\hline
 & -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 & \widehat{X}^*_b \\
 & 0 & -\frac{3EI}{L^3} & \frac{3EI}{L^2} & 0 & \frac{3EI}{L^3} & 0 & \widehat{Z}^*_b \\
 & 0 & 0 & 0 & 0 & 0 & 0 & \widehat{M}^*_b
\end{array}$$



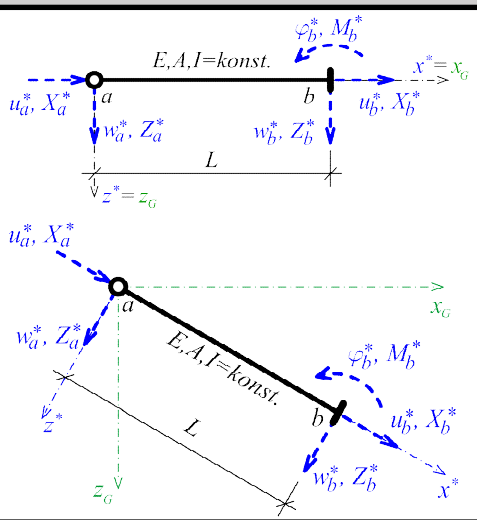
$c=\cos(\gamma_{ab})$

$s=\sin(\gamma_{ab})$

Globální matice tuhosti

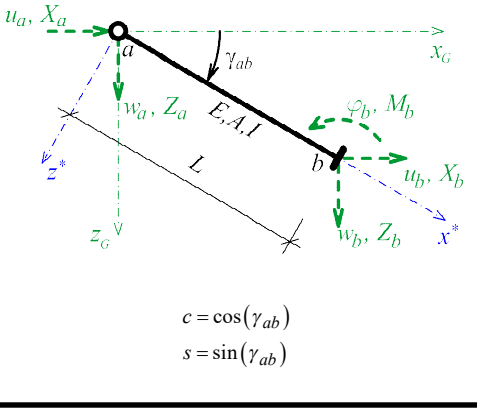
$$\left[K_{a,b}\right]=\begin{array}{c|ccc|ccc|c}
 & u_a & w_a & \varphi_a & u_b & w_b & \varphi_b & \\
\hline
 & \underbrace{\left(\frac{EA}{L}c^2+\frac{3EI}{L^3}s^2\right)}_{D11} & \underbrace{\left(\frac{EA}{L}-\frac{3EI}{L^3}\right)cs}_{D12} & \frac{3EI}{L^2}s & -D11 & -D12 & 0 & \widehat{X}_a \\
 & \underbrace{\left(\frac{EA}{L}-\frac{3EI}{L^3}\right)cs}_{D12} & \underbrace{\left(\frac{EA}{L}s^2+\frac{3EI}{L^3}c^2\right)}_{D22} & -\frac{3EI}{L^2}c & -D12 & -D22 & 0 & \widehat{Z}_a \\
 & \frac{3EI}{L^2}s & -\frac{3EI}{L^2}c & \frac{3EI}{L} & -\frac{3EI}{L^2}s & \frac{3EI}{L^2}c & 0 & \widehat{M}_a \\
\hline
 & -D11 & -D12 & -\frac{3EI}{L^2}s & D11 & D12 & 0 & \widehat{X}_b \\
 & -D12 & -D22 & \frac{3EI}{L^2}c & D12 & D22 & 0 & \widehat{Z}_b \\
 & 0 & 0 & 0 & 0 & 0 & 0 & \widehat{M}_b
\end{array}$$

Kloub - Vetknutí



Lokální matice tuhosti

$$\left[K^*_{a,b}\right]=\begin{array}{c|ccc|ccc|c}
 & u^*_a & w^*_a & \varphi^*_a & u^*_b & w^*_b & \varphi^*_b & \\
\hline
 & \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 & \widehat{X}^*_a \\
 & 0 & \frac{3EI}{L^3} & 0 & 0 & -\frac{3EI}{L^3} & -\frac{3EI}{L^2} & \widehat{Z}^*_a \\
 & 0 & 0 & 0 & 0 & 0 & 0 & \widehat{M}^*_a \\
\hline
 & -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 & \widehat{X}^*_b \\
 & 0 & -\frac{3EI}{L^3} & 0 & 0 & \frac{3EI}{L^3} & \frac{3EI}{L^2} & \widehat{Z}^*_b \\
 & 0 & -\frac{3EI}{L^2} & 0 & 0 & \frac{3EI}{L^2} & \frac{3EI}{L} & \widehat{M}^*_b
\end{array}$$



$c=\cos(\gamma_{ab})$

$s=\sin(\gamma_{ab})$

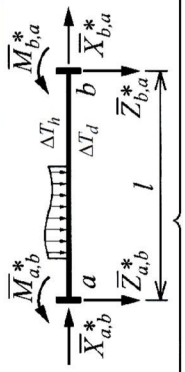
Globální matice tuhosti

$$\left[K_{a,b}\right]=\begin{array}{c|ccc|ccc|c}
 & u_a & w_a & \varphi_a & u_b & w_b & \varphi_b & \\
\hline
 & \underbrace{\left(\frac{EA}{L}c^2+\frac{3EI}{L^3}s^2\right)}_{D11} & \underbrace{\left(\frac{EA}{L}-\frac{3EI}{L^3}\right)cs}_{D12} & 0 & -D11 & -D12 & \frac{3EI}{L^2}s & \widehat{X}_a \\
 & \underbrace{\left(\frac{EA}{L}-\frac{3EI}{L^3}\right)cs}_{D12} & \underbrace{\left(\frac{EA}{L}s^2+\frac{3EI}{L^3}c^2\right)}_{D22} & 0 & -D12 & -D22 & -\frac{3EI}{L^2}c & \widehat{Z}_a \\
 & 0 & 0 & 0 & 0 & 0 & 0 & \widehat{M}_a \\
\hline
 & -D11 & -D21 & 0 & D11 & D12 & -\frac{3EI}{L^2}s & \widehat{X}_b \\
 & -D12 & -D22 & 0 & D12 & D22 & \frac{3EI}{L^2}c & \widehat{Z}_b \\
 & \frac{3EI}{L^2}s & -\frac{3EI}{L^2}c & 0 & -\frac{3EI}{L^2}s & \frac{3EI}{L^2}c & \frac{3EI}{L} & \widehat{M}_b
\end{array}$$

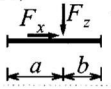
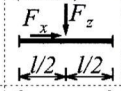
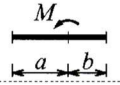
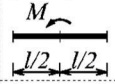
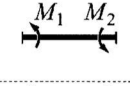
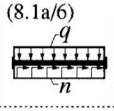
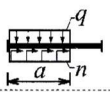
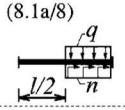
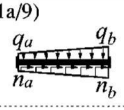
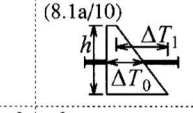
Tabulka 8.1 Primární lokální vektory koncových sil prutu $\{R^*_{a,b}\}$

Oprava: Ing. Zbyněk Vlk, Ph.D. - 2025

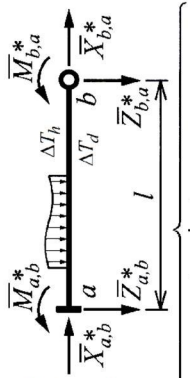
(a)



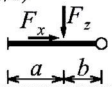
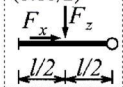
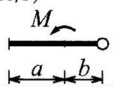
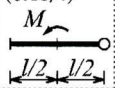
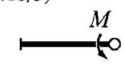
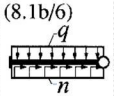
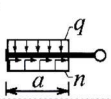
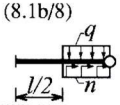
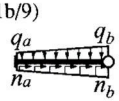
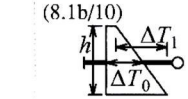
Obdélník.průřez:
$\Delta T_0=(\Delta T_d+\Delta T_h)/2$
$\Delta T_1=\Delta T_d-\Delta T_h$

	(8.1a/1)	(8.1a/2)	(8.1a/3)	(8.1a/4)	(8.1a/5)	(8.1a/6)	(8.1a/7)	(8.1a/8)	(8.1a/9)	(8.1a/10)
										
$\overline{X}^*_{a,b}$	$-\frac{b}{l}F_x$	$-\frac{1}{2}F_x$	0	0	0	$-\frac{1}{2}nl$	$-\frac{2al-a^2}{2l}n$	$-\frac{1}{8}nl$	$-\frac{2n_a+n_b}{6}l$	$EA\alpha_T\Delta T_0$
$\overline{Z}^*_{a,b}$	$-b^2\frac{3l-2b}{l^3}F_z$	$-\frac{1}{2}F_z$	$-6\frac{ab}{l^3}M$	$-\frac{3}{2}\frac{M}{l}$	0	$-\frac{1}{2}ql$	$-\frac{2al^3-2a^3l+a^4}{2l^3}q$	$-\frac{3}{32}ql$	$-\frac{7q_a+3q_b}{20}l$	0
$\overline{M}^*_{a,b}$	$\frac{ab^2}{l^2}F_z$	$\frac{1}{8}F_zl$	$b\frac{2l-3b}{l^2}M$	$\frac{1}{4}M$	$-M_1$	$\frac{1}{12}ql^2$	$\frac{6a^2l^2-8a^3l+3a^4}{12l^2}q$	$\frac{5}{192}ql^2$	$\frac{3q_a+2q_b}{60}l^2$	$\frac{EI}{h}\alpha_T\Delta T_1$
$\overline{X}^*_{b,a}$	$-\frac{a}{l}F_x$	$-\frac{1}{2}F_x$	0	0	0	$-\frac{1}{2}nl$	$-\frac{a^2}{2l}n$	$-\frac{3}{8}nl$	$-\frac{n_a+2n_b}{6}l$	$-EA\alpha_T\Delta T_0$
$\overline{Z}^*_{b,a}$	$-a^2\frac{3l-2a}{l^3}F_z$	$-\frac{1}{2}F_z$	$6\frac{ab}{l^3}M$	$\frac{3}{2}\frac{M}{l}$	0	$-\frac{1}{2}ql$	$-\frac{2a^3l-a^4}{2l^3}q$	$-\frac{13}{32}ql$	$-\frac{3q_a+7q_b}{20}l$	0
$\overline{M}^*_{b,a}$	$-\frac{a^2b}{l^2}F_z$	$-\frac{1}{8}F_zl$	$a\frac{2l-3a}{l^2}M$	$\frac{1}{4}M$	$-M_2$	$-\frac{1}{12}ql^2$	$-\frac{4a^3l-3a^4}{12l^2}q$	$-\frac{11}{192}ql^2$	$-\frac{2q_a+3q_b}{60}l^2$	$-\frac{EI}{h}\alpha_T\Delta T_1$

(b)



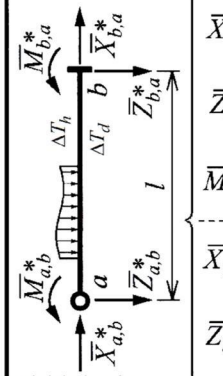
Obdélník.průřez:
$\Delta T_0=(\Delta T_d+\Delta T_h)/2$
$\Delta T_1=\Delta T_d-\Delta T_h$

	(8.1b/1)	(8.1b/2)	(8.1b/3)	(8.1b/4)	(8.1b/5)	(8.1b/6)	(8.1b/7)	(8.1b/8)	(8.1b/9)	(8.1b/10)
										
$\overline{X}^*_{a,b}$	$-\frac{b}{l}F_x$	$-\frac{1}{2}F_x$	0	0	0	$-\frac{1}{2}nl$	$-\frac{2al-a^2}{2l}n$	$-\frac{1}{8}nl$	$-\frac{2n_a+n_b}{6}l$	$EA\alpha_T\Delta T_0$
$\overline{Z}^*_{a,b}$	$-b\frac{3l^2-b^2}{2l^3}F_z$	$-\frac{11}{16}F_z$	$-3\frac{l^2-b^2}{2l^3}M$	$-\frac{9}{8}\frac{M}{l}$	$-\frac{3}{2}\frac{M}{l}$	$-\frac{5}{8}ql$	$-\frac{8al^3-4a^3l+a^4}{8l^3}q$	$-\frac{23}{128}ql$	$-\frac{16q_a+9q_b}{40}l$	$-\frac{3}{2}\frac{EI}{hl}\alpha_T\Delta T_1$
$\overline{M}^*_{a,b}$	$ab\frac{l+b}{2l^2}F_z$	$\frac{3}{16}F_zl$	$\frac{l^2-3b^2}{2l^2}M$	$\frac{1}{8}M$	$\frac{1}{2}M$	$\frac{1}{8}ql^2$	$\frac{(2al-a^2)^2}{8l^2}q$	$\frac{7}{128}ql^2$	$\frac{8q_a+7q_b}{120}l^2$	$\frac{3}{2}\frac{EI}{h}\alpha_T\Delta T_1$
$\overline{X}^*_{b,a}$	$-\frac{a}{l}F_x$	$-\frac{1}{2}F_x$	0	0	0	$-\frac{1}{2}nl$	$-\frac{a^2}{2l}n$	$-\frac{3}{8}nl$	$-\frac{n_a+2n_b}{6}l$	$-EA\alpha_T\Delta T_0$
$\overline{Z}^*_{b,a}$	$-a^2\frac{3l-a}{2l^3}F_z$	$-\frac{5}{16}F_z$	$3\frac{l^2-b^2}{2l^3}M$	$\frac{9}{8}\frac{M}{l}$	$\frac{3}{2}\frac{M}{l}$	$-\frac{3}{8}ql$	$-\frac{4a^3l-a^4}{8l^3}q$	$-\frac{41}{128}ql$	$-\frac{4q_a+11q_b}{40}l$	$\frac{3}{2}\frac{EI}{hl}\alpha_T\Delta T_1$
$\overline{M}^*_{b,a}$	0	0	0	0	0	0	0	0	0	0

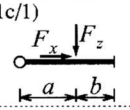
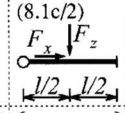
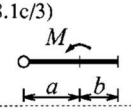
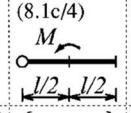
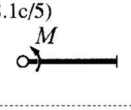
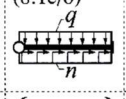
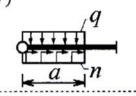
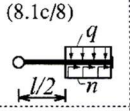
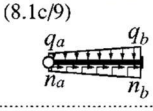
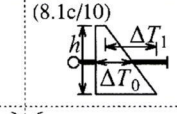
Tabulka 8.1 Primární lokální vektory koncových sil prutu $\{R^*_{a,b}\}$ (pokračování)

Oprava: Ing. Zbyněk Vlk, Ph.D. - 2025

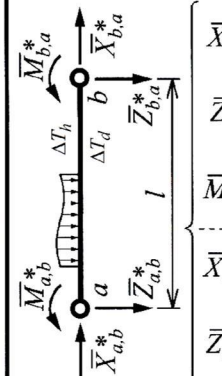
(c)



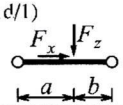
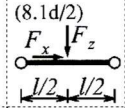
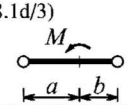
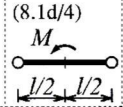
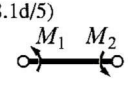
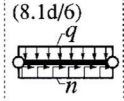
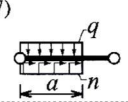
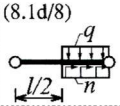
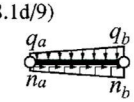
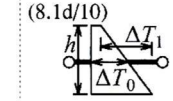
Obdélník.průřez:
$\Delta T_0=(\Delta T_d+\Delta T_h)/2$
$\Delta T_1=\Delta T_d-\Delta T_h$

	(8.1c/1)	(8.1c/2)	(8.1c/3)	(8.1c/4)	(8.1c/5)	(8.1c/6)	(8.1c/7)	(8.1c/8)	(8.1c/9)	(8.1c/10)
										
$\overline{X}^*_{a,b}$	$-\frac{b}{l}F_x$	$-\frac{1}{2}F_x$	0	0	0	$-\frac{1}{2}nl$	$-\frac{2al-a^2}{2l}n$	$-\frac{1}{8}nl$	$-\frac{2n_a+n_b}{6}l$	$EA\alpha_T\Delta T_0$
$\overline{Z}^*_{a,b}$	$-b^2\frac{3l-b}{2l^3}F_z$	$-\frac{5}{16}F_z$	$-3\frac{l^2-a^2}{2l^3}M$	$-\frac{9}{8}\frac{M}{l}$	$-\frac{3}{2}\frac{M}{l}$	$-\frac{3}{8}ql$	$-\frac{8al^3-6a^2l^2+a^4}{8l^3}q$	$-\frac{7}{128}ql$	$-\frac{11q_a+4q_b}{40}l$	$\frac{3}{2}\frac{EI}{hl}\alpha_T\Delta T_1$
$\overline{M}^*_{a,b}$	0	0	0	0	0	0	0	0	0	0
$\overline{X}^*_{b,a}$	$-\frac{a}{l}F_x$	$-\frac{1}{2}F_x$	0	0	0	$-\frac{1}{2}nl$	$-\frac{a^2}{2l}n$	$-\frac{3}{8}nl$	$-\frac{n_a+2n_b}{6}l$	$-EA\alpha_T\Delta T_0$
$\overline{Z}^*_{b,a}$	$-a\frac{3l^2-a^2}{2l^3}F_z$	$-\frac{11}{16}F_z$	$3\frac{l^2-a^2}{2l^3}M$	$\frac{9}{8}\frac{M}{l}$	$\frac{3}{2}\frac{M}{l}$	$-\frac{5}{8}ql$	$-\frac{6a^2l^2-a^4}{8l^3}q$	$-\frac{57}{128}ql$	$-\frac{9q_a+16q_b}{40}l$	$-\frac{3}{2}\frac{EI}{hl}\alpha_T\Delta T_1$
$\overline{M}^*_{b,a}$	$-ab\frac{l+a}{2l^2}F_z$	$-\frac{3}{16}F_zl$	$\frac{l^2-3a^2}{2l^2}M$	$\frac{1}{8}M$	$\frac{1}{2}M$	$-\frac{1}{8}ql^2$	$-\frac{2a^2l^2-a^4}{8l^2}q$	$-\frac{9}{128}ql^2$	$-\frac{7q_a+8q_b}{120}l^2$	$-\frac{3}{2}\frac{EI}{h}\alpha_T\Delta T_1$

(d)



Obdélník.průřez:
$\Delta T_0=(\Delta T_d+\Delta T_h)/2$
$\Delta T_1=\Delta T_d-\Delta T_h$

	(8.1d/1)	(8.1d/2)	(8.1d/3)	(8.1d/4)	(8.1d/5)	(8.1d/6)	(8.1d/7)	(8.1d/8)	(8.1d/9)	(8.1d/10)
										
$\overline{X}^*_{a,b}$	$-\frac{b}{l}F_x$	$-\frac{1}{2}F_x$	0	0	0	$-\frac{1}{2}nl$	$-\frac{2al-a^2}{2l}n$	$-\frac{1}{8}nl$	$-\frac{2n_a+n_b}{6}l$	$EA\alpha_T\Delta T_0$
$\overline{Z}^*_{a,b}$	$-\frac{b}{l}F_z$	$-\frac{1}{2}F_z$	$-\frac{M}{l}$	$-\frac{M}{l}$	$-\frac{M_1+M_2}{l}$	$-\frac{1}{2}ql$	$-\frac{2al-a^2}{2l}q$	$-\frac{1}{8}ql$	$-\frac{2q_a+q_b}{6}l$	0
$\overline{M}^*_{a,b}$	0	0	0	0	0	0	0	0	0	0
$\overline{X}^*_{b,a}$	$-\frac{a}{l}F_x$	$-\frac{1}{2}F_x$	0	0	0	$-\frac{1}{2}nl$	$-\frac{a^2}{2l}n$	$-\frac{3}{8}nl$	$-\frac{n_a+2n_b}{6}l$	$-EA\alpha_T\Delta T_0$
$\overline{Z}^*_{b,a}$	$-\frac{a}{l}F_z$	$-\frac{1}{2}F_z$	$\frac{M}{l}$	$\frac{M}{l}$	$\frac{M_1+M_2}{l}$	$-\frac{1}{2}ql$	$-\frac{a^2}{2l}q$	$-\frac{3}{8}ql$	$-\frac{q_a+2q_b}{6}l$	0
$\overline{M}^*_{b,a}$	0	0	0	0	0	0	0	0	0	0

Transformace vektorů

z lokálních do globálních souřadnic

$$\left\{\bar{\mathbf{R}}^{*}_{a,b}\right\} \;\rightarrow\; \left\{\bar{\mathbf{R}}_{a,b}\right\}$$

$$\left\{\bar{\mathbf{R}}_{a,b}\right\} = \left\{\begin{array}{c} \bar{X}^{*}_{a}\cos\gamma_{ab} - \bar{Z}^{*}_{a}\sin\gamma_{ab} \\ \bar{X}^{*}_{a}\sin\gamma_{ab} + \bar{Z}^{*}_{a}\cos\gamma_{ab} \\ \bar{M}^{*}_{a} \\ \hline \bar{X}^{*}_{b}\cos\gamma_{ab} - \bar{Z}^{*}_{b}\sin\gamma_{ab} \\ \bar{X}^{*}_{b}\sin\gamma_{ab} + \bar{Z}^{*}_{b}\cos\gamma_{ab} \\ \bar{M}^{*}_{b} \end{array}\right\}$$

z globálních do lokálních souřadnic

$$\left\{\mathbf{r}_{a,b}\right\} \;\rightarrow\; \left\{\mathbf{r}^{*}_{a,b}\right\}$$

$$\left\{\mathbf{r}^{*}_{a,b}\right\} = \left\{\begin{array}{c} u_{a}\cos\gamma_{ab} + w_{a}\sin\gamma_{ab} \\ -u_{a}\sin\gamma_{ab} + w_{a}\cos\gamma_{ab} \\ \varphi_{a} \\ \hline u_{b}\cos\gamma_{ab} + w_{b}\sin\gamma_{ab} \\ -u_{b}\sin\gamma_{ab} + w_{b}\cos\gamma_{ab} \\ \varphi_{b} \end{array}\right\}$$