

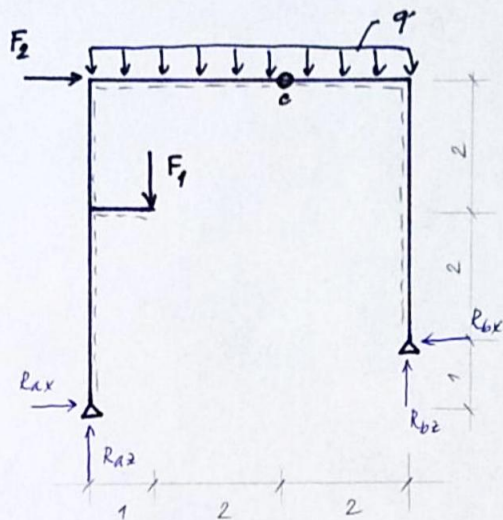
* TROJKLOUBOVÝ LOMENÝ NOVNÍK
s TÁHLEM A BEZ TÁHLA

POZN.: VSTAT. VROTILOST
 $2b + 3d = a_1 + 2a_2 + 3a_3 + 2\sum_{n=3}^{m-1} k_n$ (8) 1
 k_n - vnitřní kloub aplikován $n > 2$ dle

Vnitřní kloub - stěží 2 stupně volnosti; $H_0 = 0$

Pr. 34: Všechny rovinné novníkové prvky - 3kloubový lomový novník bez táhla

$F_1 = 15 \text{ kN}$; $F_2 = 10 \text{ kN}$
 $q = 2 \text{ kN} \cdot \text{m}^{-1}$



POZN.: $b = 0$; $d = 2$

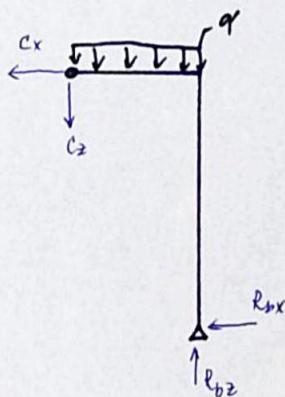
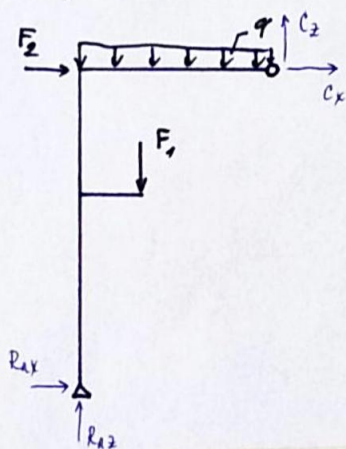
$a_1 = 0$; $a_2 = 3$; $a_3 = 0$; $k_n = 0$

$3 \cdot 2 = 2 \cdot 3 \rightarrow 6 = 6 \rightarrow$ stat. určitý kó

R... reakce vazeb

C... interakce vazeb

• Řešení: a) obecné řešení:



$\sum F_{ix} = 0$: $R_{ax} + F_2 + C_x = 0$ (1a)

$\sum F_{ix} = 0$: $-R_{bx} - C_x = 0$ (4a)

$\sum F_{iz} = 0$: $R_{ax2} - F_1 - q \cdot 3 + C_z = 0$ (2a)

$\sum F_{iz} = 0$: $-C_z - q \cdot 2 + R_{bz} = 0$ (5a)

$\sum M_{ia} = 0$: $-C_x \cdot 5 - F_1 \cdot 1 - F_2 \cdot 5 - q \cdot 3^2 \cdot \frac{1}{2} + C_z \cdot 3 = 0$ (3a) $\sum M_{ib} = 0$: $C_x \cdot 4 + C_z \cdot 2 + q \cdot 2^2 \cdot \frac{1}{2} = 0$ (6a)

6 lineárních rovnic pro 6 neznámých
 $\rightarrow$ každá z nich má obecně 2 neznámé

Řešení např.: z (6a) $\rightarrow C_z = \frac{-2q - 4C_x}{2} = -q - 2C_x$

dokladíme do (3a): $-C_x \cdot 5 - F_1 \cdot 1 - F_2 \cdot 5 - q \cdot 3^2 \cdot \frac{1}{2} + (-q - 2C_x) \cdot 3 = 0 \Rightarrow -11C_x = 80$
 $C_x = -7,27 \text{ kN}$

$\rightarrow C_z = 12,54 \text{ kN}$; $R_{ax} = -2,73 \text{ kN}$; $R_{ax2} = 8,46 \text{ kN}$; $R_{bx} = 7,27 \text{ kN}$; $R_{bz} = 16,54 \text{ kN}$

b) 4 momente' potinily pro slozky reakci' vnitřní uzel

$$\sum M_{ib} = 0 : -R_{A2} \cdot 5 + R_{Ax} \cdot 1 + F_1 \cdot 4 - F_2 \cdot 4 + q \cdot 5^2 \cdot \frac{1}{2} = 0 \quad (1b)$$

$$\sum M_{ia} = 0 : -F_1 \cdot 1 - F_2 \cdot 5 - q \cdot 5^2 \cdot \frac{1}{2} + R_{bx} \cdot 1 + R_{b2} \cdot 5 = 0 \quad (2b)$$

! platí $\sum M_c = \sum M_d = 0$ $\left\{ \begin{array}{l} \sum M_{ic} = 0 : -R_{A2} \cdot 3 + F_1 \cdot 2 + q \cdot 3^2 \cdot \frac{1}{2} + R_{Ax} \cdot 5 = 0 \quad (3b) \\ \sum M_{id} = 0 : -q \cdot 2^2 \cdot \frac{1}{2} - R_{bx} \cdot 4 + R_{b2} \cdot 2 = 0 \quad (4b) \end{array} \right.$

$\Rightarrow$ z (1b) a (3b) $\rightarrow R_{Ax} ; R_{A2}$

z (2b) a (4b) $\rightarrow R_{bx} ; R_{b2}$

$\rightarrow$ z (1b) $R_{A2} = \frac{1}{5} (R_{Ax} \cdot 1 + F_1 \cdot 4 - F_2 \cdot 4 + q \cdot 5^2 \cdot \frac{1}{2})$

do (3b) $-\frac{1}{5} (R_{Ax} \cdot 1 + F_1 \cdot 4 - F_2 \cdot 4 + q \cdot 5^2 \cdot \frac{1}{2}) \cdot 3 + F_1 \cdot 2 + q \cdot 3^2 \cdot \frac{1}{2} + R_{Ax} \cdot 5 = 0$

$$\Rightarrow (-\frac{3}{5} + 5) R_{Ax} = (\frac{4}{5} F_1 - \frac{4}{5} F_2 + q \cdot 5 \cdot \frac{1}{2}) \cdot 3 - F_1 \cdot 2 - q \cdot 3^2 \cdot \frac{1}{2}$$

$$\underline{R_{Ax} = -2,73 \text{ kN}}$$

$$\Rightarrow \underline{R_{A2} = \frac{1}{5} (-2,73 \cdot 1 + 15 \cdot 4 - 10 \cdot 4 + 25 \cdot \frac{1}{2}) = 8,45 \text{ kN}}$$

$\rightarrow$ z (4b)

$$R_{bx} = \frac{1}{4} (-q \cdot 2^2 \cdot \frac{1}{2} + R_{b2} \cdot 2)$$

do (2b)

$$-F_1 \cdot 1 - F_2 \cdot 5 - q \cdot 5^2 \cdot \frac{1}{2} + \frac{1}{4} (-q \cdot 2^2 \cdot \frac{1}{2} + R_{b2} \cdot 2) \cdot 1 + R_{b2} \cdot 5 = 0$$

$$\Rightarrow (\frac{2}{4} + 5) R_{b2} = +F_1 \cdot 1 + F_2 \cdot 5 + q \cdot 5^2 \cdot \frac{1}{2} + \frac{1}{4} q \cdot 2^2 \cdot \frac{1}{2}$$

$$\underline{R_{b2} = 16,55 \text{ kN}}$$

$$\Rightarrow \underline{R_{bx} = \frac{1}{4} (-2 \cdot 2^2 \cdot \frac{1}{2} + 16,55 \cdot 2) = 7,27 \text{ kN}}$$

kontrola: $\sum F_{ix} = 0 : R_{Ax} + F_2 - R_{bx} = 0$

$$-2,73 + 10 - 7,27 = 0 \quad \checkmark$$

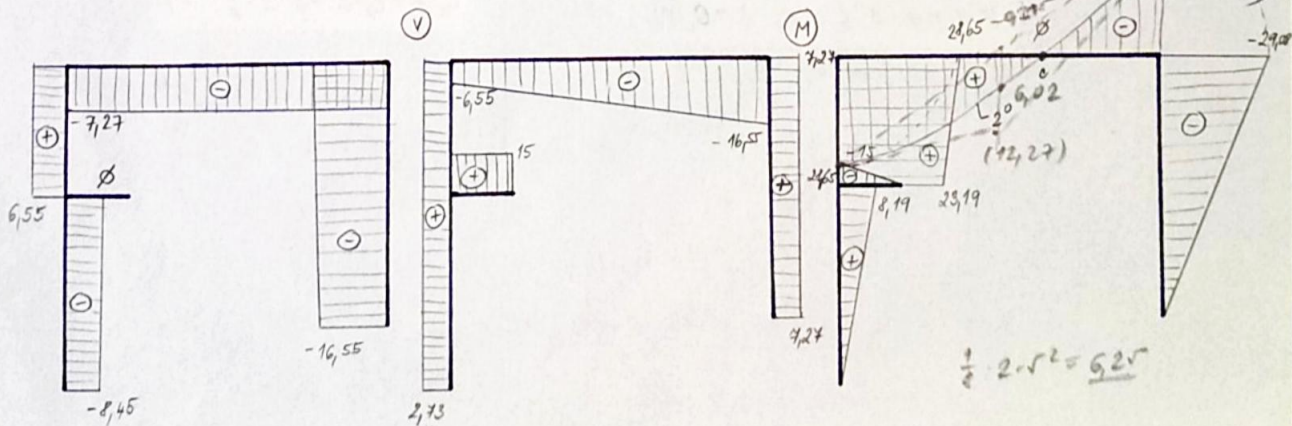
$$\sum F_{iz} = 0 : R_{A2} - F_1 - q \cdot 5 + R_{b2} = 0$$

$$8,45 - 15 - 2 \cdot 5 + 16,55 = 0 \quad \checkmark$$

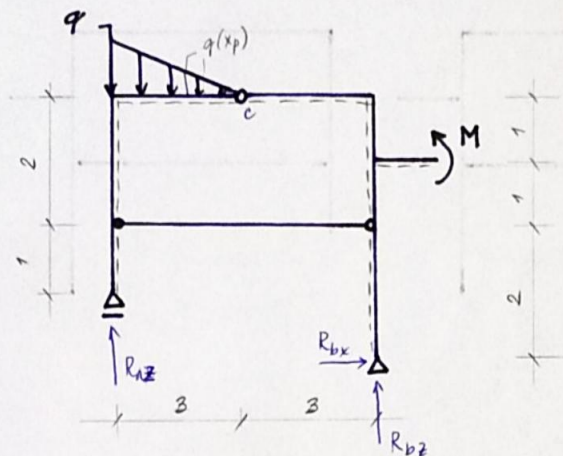
slozky interakci' R_{Ax} a R_{A2} vnitřní uzel c

$$\sum F_{ix}^{(N)} = 0 : R_{Ax} + F_2 + C_x = 0 \Rightarrow C_x = -R_{Ax} - F_2 = -(-2,73) - 10 = -7,27 \text{ kN} \quad (5b)$$

$$\sum F_{iz}^{(N)} = 0 : R_{A2} - F_1 - q \cdot 3 + C_2 = 0 \Rightarrow C_2 = -R_{A2} + F_1 + q \cdot 3 = -8,45 + 15 + 2 \cdot 3 = 12,55 \text{ kN} \quad (6b)$$



Pr. 35: Uložení rovinná novníková soustava - 3 km body' novník u tálém 8 2



$$M = 2 \text{ kNm}$$

$$q = 2 \text{ kN} \cdot \text{m}^{-1}$$

$$POZN.: b=0; d=2$$

$$a_1 = 2; a_2 = 2; a_3 = 0; l_n = 0$$

$$3 \cdot 2 = 2 + 2 \cdot 2 \rightarrow 6 = 6 \rightarrow \text{stat. urdění ke}$$

• Rěšení: vložit reakce vzhledem k vazbám

$$\sum F_{Ax} = 0 : R_{Bx} = 0$$

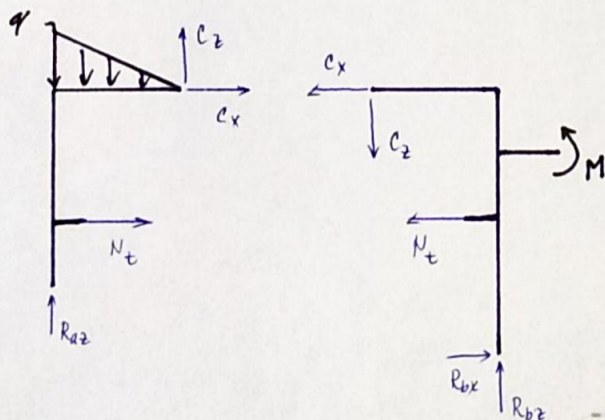
$$\sum H_{Aa} = 0 : -\frac{1}{2} q \cdot 3 \cdot \frac{1}{3} \cdot 3 + M + R_{Bz} \cdot 6 + R_{Bx} \cdot 1 = 0 \Rightarrow R_{Bz} = \frac{1}{6} = 0,16 \text{ kN}$$

$$\sum M_{Aa} = 0 : -R_{Az} \cdot 6 + \frac{1}{2} q \cdot 3 \cdot (3 + \frac{2}{3} \cdot 3) + M = 0 \Rightarrow R_{Az} = \frac{1}{6} = 2,83 \text{ kN}$$

$$\text{kontrola } \sum F_{Az} = 0 : 2,83 - \frac{1}{2} \cdot 2 \cdot 3 + 0,16 = 0 \checkmark$$

vložit interakce vazeb

→ 2 rovnice sil působících na uvolněné levé nebo pravé části



$$\sum H_{Aa} = 0 :$$

$$-R_{Az} \cdot 3 + N_t \cdot 2 + \frac{1}{2} \cdot q \cdot 3 \cdot \frac{2}{3} \cdot 3 = 0$$

$$N_t = \frac{1}{2} \left(2,83 \cdot 3 - \frac{1}{2} \cdot 2 \cdot 3 \cdot \frac{2}{3} \cdot 3 \right) = 1,25 \text{ kN}$$

$$\sum F_{Ax} = 0 :$$

$$N_t + C_x = 0 \Rightarrow C_x = -N_t = -1,25 \text{ kN}$$

$$\sum F_{Az} = 0 :$$

$$R_{Az} - \frac{1}{2} \cdot q \cdot 3 + C_2 = 0$$

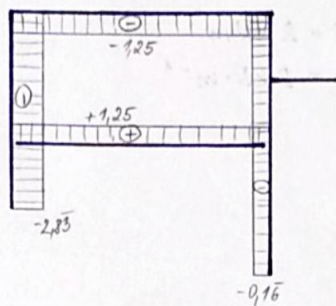
$$C_2 = \frac{1}{2} \cdot 2 \cdot 3 - 2,83 = 0,16 \text{ kN}$$

$$\text{kontrola: } \sum H_{Aa} = M - N_t \cdot 2 + R_{Bx} \cdot 4 + R_{Bz} \cdot 3 = 2 - 1,25 \cdot 2 + 0,16 \cdot 3 + 0 = 0 \checkmark$$

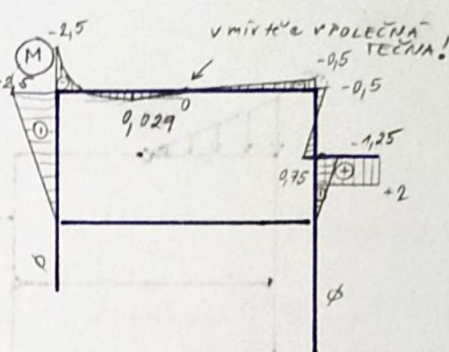
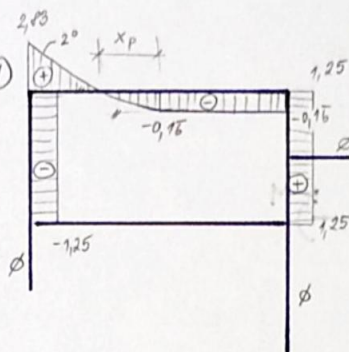
$$\sum F_{Ax} = R_{Bx} - N_t - C_x = 0 - 1,25 - (-1,25) = 0 \checkmark$$

$$\sum F_{Az} = -C_2 + R_{Bz} = -0,16 + 0,16 = 0 \checkmark$$

(N)



(V)



$$! M_c^L = M_c^P = 0 !$$

$$\frac{q}{3} = \frac{q(x_p)}{x_p}$$

$$x_p: -0.16 + \frac{1}{2} \cdot q(x_p) \cdot x_p = 0$$

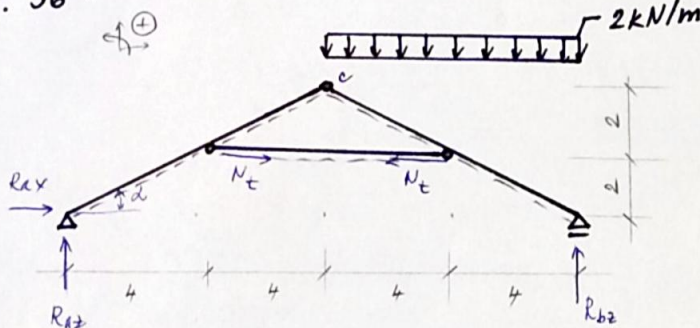
$$-0.16 + \frac{1}{2} \cdot \frac{2}{3} \cdot x_p \cdot x_p = 0$$

$$x_p = \pm \sqrt{0.16 \cdot 3}$$

$$x_p = 0.707 \text{ m}$$

$$M_{max} = R_{bb}(3+x_p) + R_{bx} \cdot 4 - N_t \cdot 2 + M - \frac{1}{2} \cdot \frac{2}{3} \cdot x_p \cdot x_p \cdot \frac{2}{3} x_p = 0.16 \cdot 3.707 + 0 - 1.25 \cdot 2 + 0 - \frac{1}{2} \cdot \frac{2}{3} \cdot 0.707^3 \cdot \frac{1}{3} = 0.079 \text{ kNm}$$

pr. 36:



$$\sum F_{ix} = 0 \Rightarrow R_{ax} = 0$$

$$\sum M_{ia} = 0:$$

$$-2 \cdot 8 \cdot 12 + R_{bz} \cdot 16 = 0 \Rightarrow R_{bz} = 12 \text{ kN}$$

$$\sum M_{ib} = 0:$$

$$-R_{az} \cdot 16 + 2 \cdot 8 \cdot 4 = 0 \Rightarrow R_{az} = 4 \text{ kN}$$

$$\sum M_{ic} = 0:$$

$$-R_{az} \cdot 8 + R_{ax} \cdot 4 + N_t \cdot 2 = 0$$

$$2N_t = R_{az} \cdot 8 - R_{ax} \cdot 4 = 4 \cdot 8 - 0 = 32$$

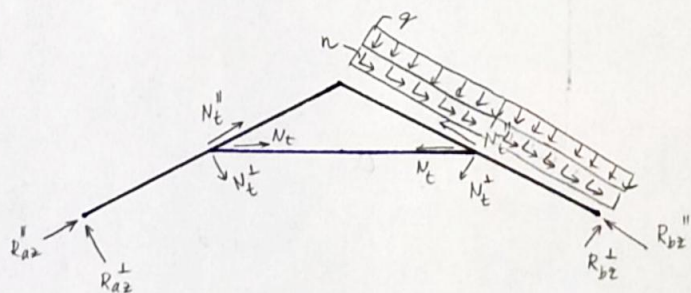
$$N_t = \frac{32}{2} = 16 \text{ kN}$$

$$\sin \alpha = \frac{4}{\sqrt{4^2 + 8^2}} = 0.447$$

$$\cos \alpha = \frac{8}{\sqrt{4^2 + 8^2}} = 0.894$$

$$q = 2 \cdot \cos^2 \alpha = 2 \cdot 0.894^2 = 1.6 \text{ kN/m}$$

$$n = 2 \cdot \cos \alpha \cdot \sin \alpha = 2 \cdot 0.894 \cdot 0.447 = 0.8 \text{ kN/m}$$



$$R_{az}'' = R_{az} \cdot \sin \alpha = 1.79 \text{ kN}$$

$$R_{az}^\perp = R_{az} \cdot \cos \alpha = 3.58 \text{ kN}$$

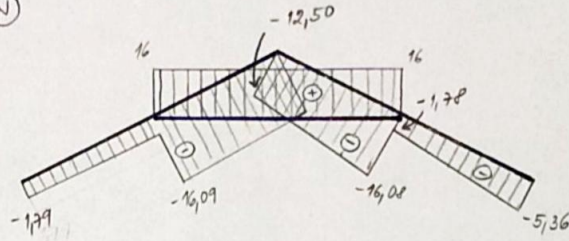
$$R_{bz}'' = R_{bz} \cdot \sin \alpha = 5.36 \text{ kN}$$

$$R_{bz}^\perp = R_{bz} \cdot \cos \alpha = 10.73 \text{ kN}$$

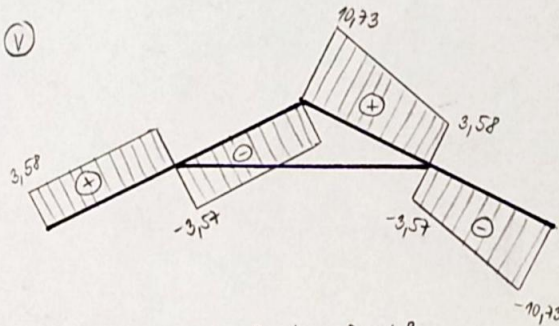
$$N_t'' = N_t \cdot \cos \alpha = 14.30 \text{ kN}$$

$$N_t^\perp = N_t \cdot \sin \alpha = 7.15 \text{ kN}$$

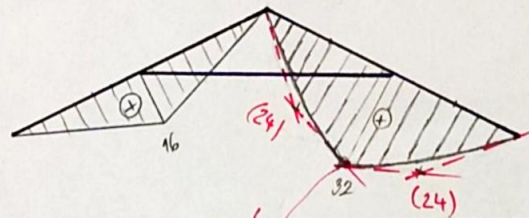
(N)



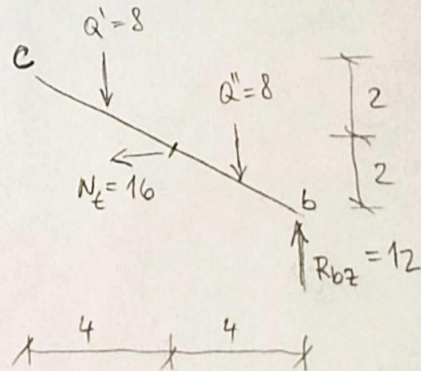
(V)



$$M_L^L = M_L^P = 0$$



není
vodorovná tečna
(ani vrchol paraboly)

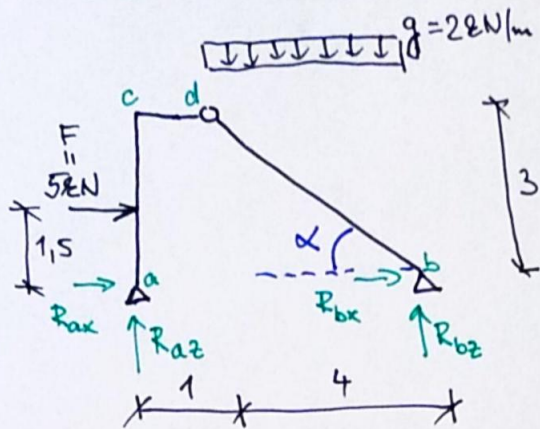


$$M_{Q''} = R_{b2} \cdot 2 = 24 \text{ kNm}$$

$$M_{N_L} = R_{b2} \cdot 4 - Q'' \cdot 2 = 32 \text{ kNm} \checkmark$$

$$M_{Q'} = R_{b2} \cdot 6 - Q'' \cdot 4 - N_L \cdot 1 = 24 \text{ kNm}$$

TROJKLOUBOVÝ NOSNÍK BEZ TAHLA



$$\textcircled{1} \sum M_{bi} = 0$$

$$F \cdot 1,5 + g \cdot 4 \cdot 3 - R_{bz} \cdot 5 = 0$$

$$R_{bz} = 6,38 \text{ kN}$$

$$\textcircled{2} \sum M_{bi} = 0$$

$$F \cdot 1,5 - g \cdot 4 \cdot 2 + R_{az} \cdot 5 = 0$$

$$R_{az} = 1,4 \text{ kN}$$

$$\textcircled{3} \sum M_{di}^P = 0$$

$$g \cdot 4 \cdot 2 - R_{bz} \cdot 4 - R_{bx} \cdot 3 = 0$$

$$R_{bx} = -3,06 \text{ kN}$$

$$\textcircled{4} \sum M_{di}^L = 0$$

$$-F \cdot 1,5 - R_{ax} \cdot 3 + R_{az} \cdot 1 = 0$$

$$R_{ax} = -1,93 \text{ kN}$$

KONTROLA:

$$\textcircled{5} \sum F_{xi} = 0$$

$$R_{ax} + R_{bx} + F = 0$$

$$0 = 0 \checkmark$$

$$\textcircled{6} \sum F_{zi} = 0$$

$$R_{az} + R_{bz} - g \cdot 4 = 0$$

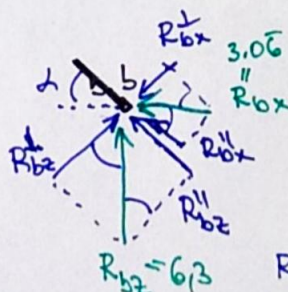
$$0 = 0 \checkmark \quad (6 \text{ min})$$

$$\cos \alpha = \frac{4}{5} \quad \sin \alpha = \frac{3}{5}$$

$$g' = g \cos \alpha = 2 \cdot \frac{4}{5} = 1,6 \text{ kN/m}$$

$$q = g \cdot \cos^2 \alpha = 2 \left(\frac{4}{5} \right)^2 = 1,28 \text{ kN/m}$$

$$n = g \cdot \cos \alpha \sin \alpha = 2 \cdot \frac{4}{5} \cdot \frac{3}{5} = 0,96 \text{ kN/m}$$

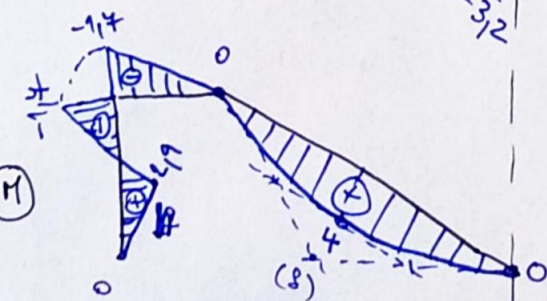
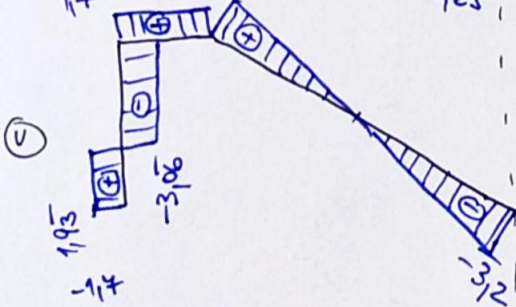
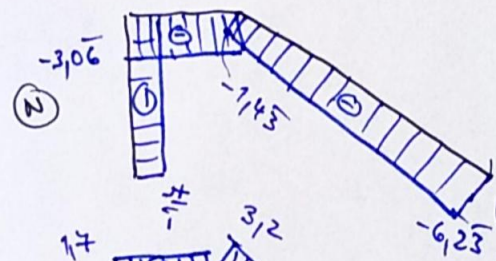
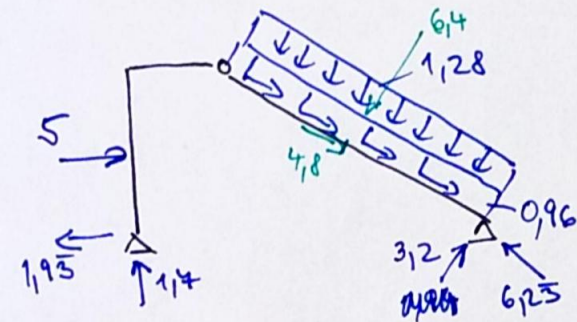


$$R_{bx}^{\perp} = R_{bx} \sin \alpha = 3,06 \cdot \frac{3}{5} = 1,84 \text{ kN}$$

$$R_{bx}^{\parallel} = R_{bx} \cos \alpha = 3,06 \cdot \frac{4}{5} = 2,45 \text{ kN}$$

$$R_{bz}^{\perp} = R_{bz} \cos \alpha = 6,38 \cdot \frac{4}{5} = 5,04 \text{ kN}$$

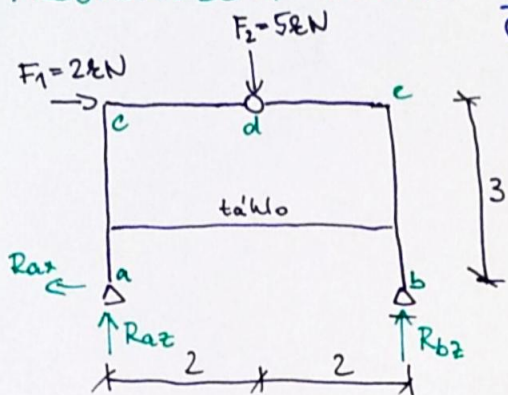
$$R_{bz}^{\parallel} = R_{bz} \sin \alpha = 6,38 \cdot \frac{3}{5} = 3,78 \text{ kN}$$



KONTROLA sil ve styčnicku

d
c

TROJKLOUBOVÝ NOSNÍK S TAHLÉM



$$\sum F_{xi} = 0$$

$$F_1 - R_{ax} = 0 \Rightarrow R_{ax} = 2 \text{ kN}$$

$$\sum M_{ai} = 0$$

$$F_1 \cdot 3 + F_2 \cdot 2 - R_{bz} \cdot 4 = 0$$

$$R_{bz} = 4 \text{ kN}$$

$$\sum M_{bi} = 0$$

$$R_{az} \cdot 4 + F_1 \cdot 3 - F_2 \cdot 2 = 0$$

$$R_{az} = 1 \text{ kN}$$

$$\sum F_{zi} = 0$$

$$R_{az} + R_{bz} - 5 = 0$$

$$0 = 0 \checkmark$$

$$\sum M_{di}^L = 0$$

$$R_{az} \cdot 2 + R_{ax} \cdot 3 + F_t \cdot 2 = 0$$

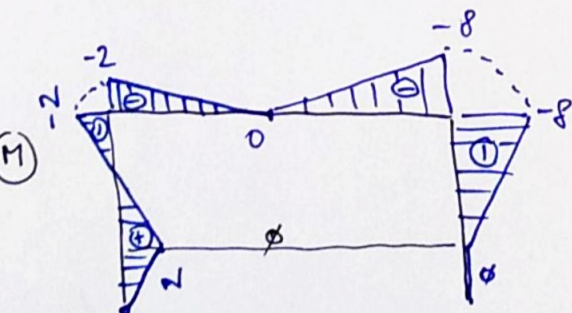
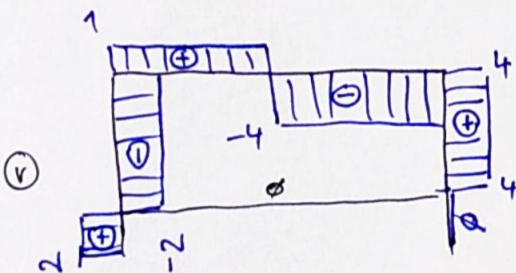
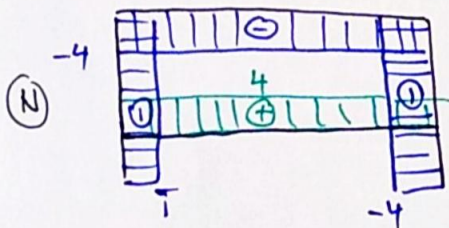
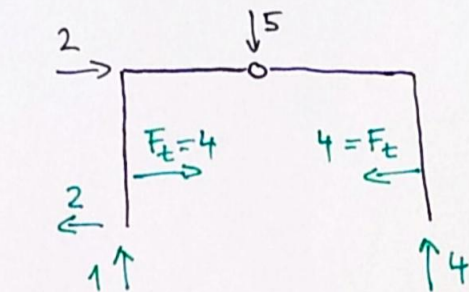
$$F_t = 4 \text{ kN}$$

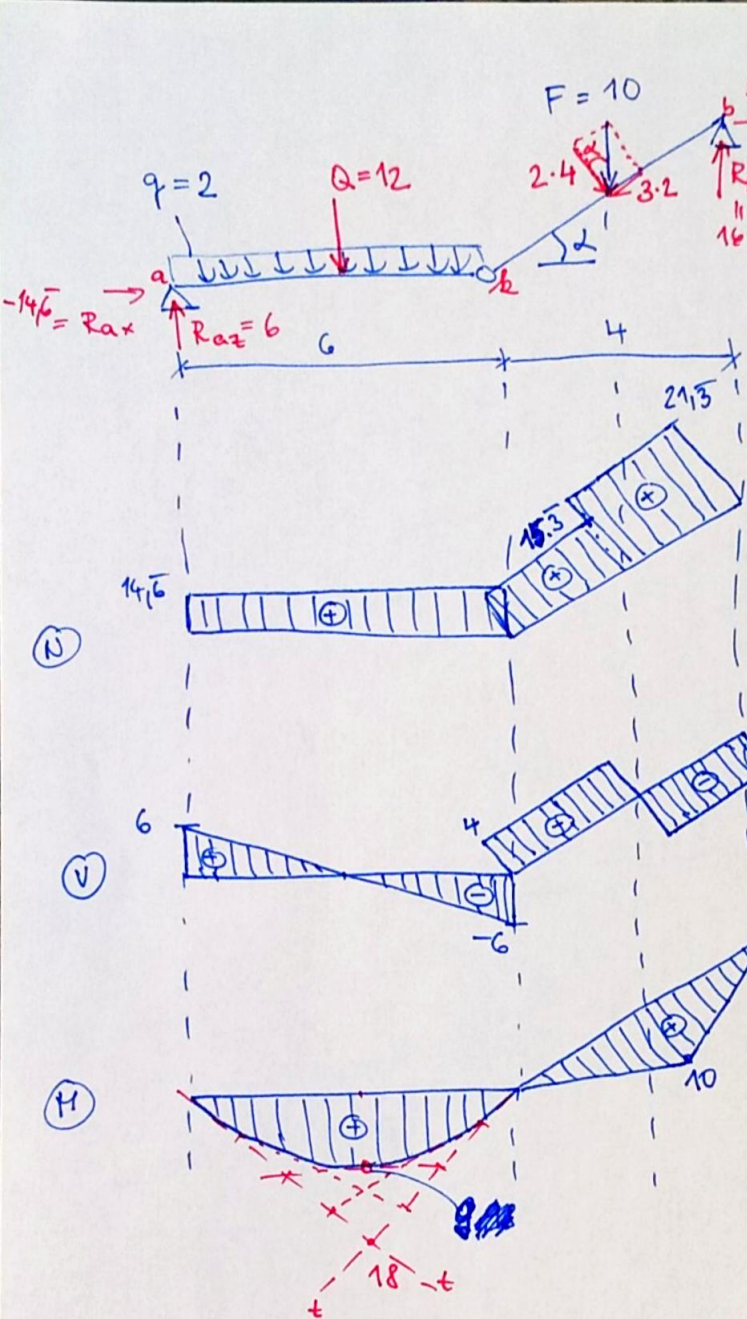
$$\sum M_{di}^P = 0$$

$$R_{bz} \cdot 2 - F_t \cdot 2 = 0$$

$$F_t = 4 \text{ kN}$$

✓





$$\sum M_{Ai}^P = 0$$

$$-R_{Az} \cdot 6 + Q \cdot 3 = 0 \Rightarrow R_{Az} = \frac{Q}{2} = \underline{\underline{6 \text{ kN}}}$$

$$\sum M_{Bi} = 0$$

$$-R_{Az} \cdot 10 + Q \cdot 7 + 10 \cdot 2 + R_{Ax} \cdot 3 = 0$$

$$3R_{Ax} = + \frac{Q}{2} \cdot 10 - 7Q - 20$$

$$R_{Ax} = \underline{\underline{-14,6 \text{ kN}}}$$

$$\sum M_{Ai}^P = 0$$

$$-10 \cdot 2 + R_{Bz} \cdot 4 - R_{Bx} \cdot 3 = 0$$

$$\sum M_{Bi} = 0$$

$$-Q \cdot 3 - F \cdot 8 + R_{Bz} \cdot 10 - R_{Bx} \cdot 3 = 0$$

$$-20 + 4R_{Bz} + 3Q + 8F - R_{Bz} \cdot 10 = 0$$

$$R_{Bz} = \underline{\underline{16 \text{ kN}}}$$

$$R_{Bx} = \underline{\underline{14,6 \text{ kN}}}$$

$$\sum F_{ix} = 0$$

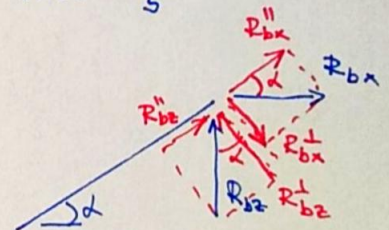
$$-14,6 + 14,6 = 0 \quad \checkmark$$

$$\sum F_{iz} = 0$$

$$6 - 12 - 10 + 16 = 0 \quad \checkmark$$

$$\alpha: \cos \alpha = \frac{4}{5}$$

$$\sin \alpha = \frac{3}{5}$$



$$R_{Bx}^{\parallel} = R_{Bx} \cdot \cos \alpha = 14,6 \cdot \frac{4}{5} = 11,73 \text{ kN}$$

$$R_{Bx}^{\perp} = R_{Bx} \cdot \sin \alpha = 14,6 \cdot \frac{3}{5} = 8,76 \text{ kN}$$

$$R_{Bz}^{\parallel} = R_{Bz} \cdot \sin \alpha = 16 \cdot \frac{3}{5} = 9,6 \text{ kN}$$

$$R_{Bz}^{\perp} = R_{Bz} \cdot \cos \alpha = 16 \cdot \frac{4}{5} = 12,8 \text{ kN}$$

KONTROLA:

rozklad R_{Ax} , R_{Az} , Q