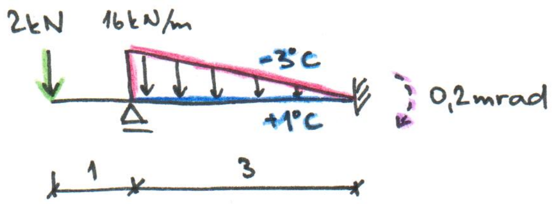
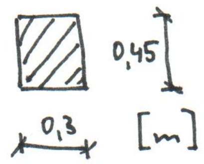
Deformační metoda - konstrukce zatížená teplotou, deformací, převislý konec

Určete průběh vnitřních sil na kci



$E = 27\ GPa$

$\alpha = 1\cdot 10^{-5}\ K^{-1}$



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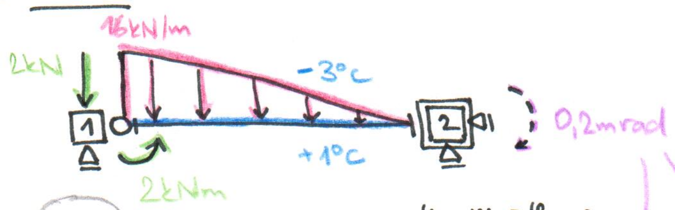
① model

$n_p = 1$



$u_1 = ?$

$w_1 = \varphi_1 = 0$

$u_2 = w_2 = \varphi_2 = 0$

② $\{r\}, \{s\}$

$\{r\} = \{u_1\} \qquad \{s\} = \{0\}$

③ $[k_{12}],\ \{\bar{R}^*_{12}\} \rightarrow \{\bar{R}_{12}\},\ \{\tilde{R}_{12}\}$

$$[k_{12}] = 10^6 \cdot \begin{array}{c|c} \begin{array}{cccccc} u_1 & w_1 & \varphi_1 & u_2 & w_2 & \varphi_2 \end{array} & \\ \begin{bmatrix} 1215 & & & & & 0 \\ 0 & & & & & -20{,}502 \\ 0 & & & & & 0 \\ -1215 & & & & & 0 \\ 0 & & & & & 20{,}502 \\ 0 & & & & & 61{,}506 \end{bmatrix} & \begin{array}{c} u_1 \\ w_1 \\ \varphi_1 \\ u_2 \\ w_2 \\ \varphi_2 \end{array} \end{array}$$

11.3(c)

$l = 3m$

$E = 27\cdot 10^9 Pa$

$A = 0{,}135 m^2$

$I = 2{,}278\cdot 10^{-3} m^4$

$$\{\bar{R}^*_{12}\} = \{\bar{R}^*_{12,M}\} + \{\bar{R}^*_{12,q}\} + \{\bar{R}^*_{12,t}\}$$

↳ teplota ... tab. 11.5

! prut 1-2 : ○— X v tabulkách prim. konc. sil : —○ !

→ možná řešení : a) použít jiné tabulky (další sada tabulek pro pruty ○—)

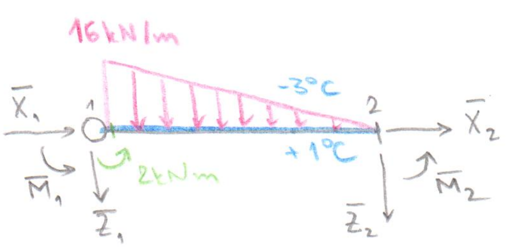
b) počítat jako prut 2-1 ... —○, $\gamma = 180°$ ⇒ nutno transformovat, dolní vlákna budou nahoře !

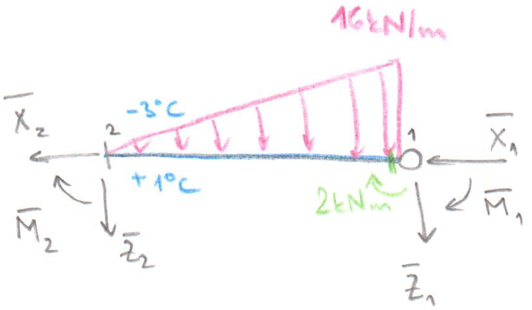
c) pomocný prut (zrcadlení)

↳ tento postup bude použit v tomto příkladu

řešený prut
(vč. prim. konc. sil)

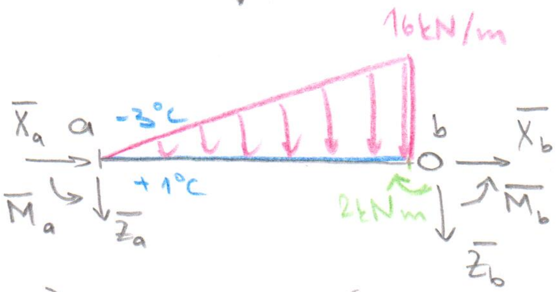
ozrcadlený prut





$\overline{X}_1 = -\overline{X}_b$
$\overline{Z}_1 = \overline{Z}_b$
$\overline{M}_1 = -\overline{M}_b$
$\overline{X}_2 = -\overline{X}_a$
$\overline{Z}_2 = \overline{Z}_a$
$\overline{M}_2 = -\overline{M}_a$

pomocný prut
(prim. konc. síly dle tabulek)



$$\{\overline{R}^*_{ab,M}\} = \begin{Bmatrix} 0 \\ 1000 \\ -1000 \\ \hline 0 \\ -1000 \\ 0 \end{Bmatrix} \qquad \{\overline{R}^*_{ab,q}\} = \begin{Bmatrix} 0 \\ -10800 \\ 8400 \\ \hline 0 \\ -13200 \\ 0 \end{Bmatrix} \qquad \{\overline{R}^*_{ab,t}\} = \begin{Bmatrix} -36450 \\ -2734 \\ 8201 \\ \hline 36450 \\ 2734 \\ 0 \end{Bmatrix}$$

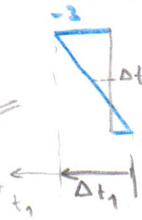
$M = 2000\,Nm$
$b = 0\,m$
$l = 3\,m$

$n = 0$
$q_1 = 0$
$q_2 = 16000\,N/m$
$l = 3\,m$

-3
+1
$E = 27\cdot10^9\,Pa$
$A = 0{,}135\,m^2$
$I = 2{,}278\cdot10^{-3}\,m^4$
$\alpha_t = 1\cdot10^{-5}\,K^{-1}$
$h = 0{,}45\,m$
$l = 3\,m$
$\Delta t_0 = -1°C$
$\Delta t_1 = 4°C$

Δt_0 – změna teploty v těžišti

$$\Delta t_0 = \frac{\Delta t_d + \Delta t_h}{2} = \frac{1+(-3)}{2} = -1°C$$



↳ rozdíl teplot mezi krajními vlákny
$\Delta t_1 = \Delta t_d - \Delta t_h = 1-(-3) = 4°C$
(pokud je dole vyšší teplota → víc se natahují dolní vlákna → kladný moment od teploty → Δt_1 … kladné)

$$\{\overline{R}^*_{ab}\} = \{\overline{R}^*_{ab,M}\} + \{\overline{R}^*_{ab,q}\} + \{\overline{R}^*_{ab,t}\}$$

$$\{\overline{R}^*_{ab}\} = \begin{Bmatrix} 0 \\ 1000 \\ -1000 \\ \hline 0 \\ -1000 \\ 0 \end{Bmatrix} + \begin{Bmatrix} 0 \\ -10800 \\ 8400 \\ \hline 0 \\ -13200 \\ 0 \end{Bmatrix} + \begin{Bmatrix} -36450 \\ -2734 \\ 8201 \\ \hline 36450 \\ 2734 \\ 0 \end{Bmatrix} = \begin{Bmatrix} -36450 \\ -12534 \\ 15601 \\ \hline 36450 \\ -11466 \\ 0 \end{Bmatrix} \begin{matrix} \overline{X}_a \\ \overline{Z}_a \\ \overline{M}_a \\ \overline{X}_b \\ \overline{Z}_b \\ \overline{M}_b \end{matrix}$$

$$\{\overline{R}^*_{12}\} = \begin{Bmatrix} -36450 \\ -11466 \\ 0 \\ \hline 36450 \\ -12534 \\ 15601 \end{Bmatrix} = \{\overline{R}_{12}\}$$

(LSS ≈ GSS ⇒ není třeba transformovat)

prim. konc. síly způsobené popuštěním podpor:

$$\{\tilde{R}_{12}\} = [K_{12}]\cdot\{\tilde{r}_{12}\} = 10^6 \cdot \begin{array}{c} \begin{matrix} u_1 & w_1 & \varphi_1 & u_2 & w_2 & \varphi_2 \end{matrix} \\ \begin{bmatrix} & & & & & 0 \\ & & & & & -20{,}502 \\ & & & & & 0 \\ \hline & & & & & 0 \\ & & & & & 20{,}502 \\ & & & & & 61{,}506 \end{bmatrix} \end{array} \cdot \begin{Bmatrix} 0 \\ 0 \\ 0 \\ \hline 0 \\ 0 \\ -2\cdot10^{-4} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 4100 \\ 0 \\ \hline 0 \\ -4100 \\ -12301 \end{Bmatrix}$$

x
z
⊕
$\tilde{\varphi}_2 = -0{,}2\,mrad = -2\cdot10^{-4}\,rad$

4 $[K], \{\bar{R}\}, \{\tilde{R}\}$ (pouze jedna neznámá deformace ... u_1)

$[K] = 10^6 \cdot [1215] \qquad \{\bar{R}\} = \{-36450\} \qquad \{\tilde{R}\} = \{0\}$

5 $\{F\}, \{r\}$

$\{F\} = \{S\} - \{\bar{R}\} - \{\tilde{R}\} = \{0\} - \{-36450\} - \{0\} = \{36450\}$

$[K] \cdot \{r\} = \{F\}$

$[1215 \cdot 10^6] \cdot \{u_1\} = \{36450\} \quad \Rightarrow u_1 = 3 \cdot 10^{-5}\,m$

6 $\{r_{12}\}$

$$\{r_{12}\} = \begin{Bmatrix} 3 \cdot 10^{-5} \\ 0 \\ 0 \\ \hline 0 \\ 0 \\ 0 \end{Bmatrix}$$

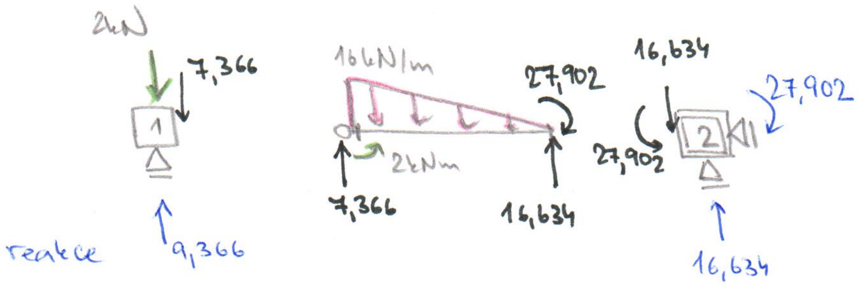
7 $\{\hat{R}_{12}\}, \{R_{12}\} \rightarrow$ kontrola $\rightarrow \{R^*_{12}\}$

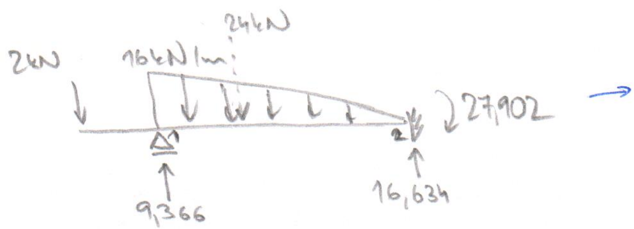
$$\{\hat{R}_{12}\} = [K_{12}] \cdot \{r_{12}\} = 10^6 \cdot \begin{array}{c} \begin{matrix} u_1 & w_1 & \varphi_1 & u_2 & w_2 & \varphi_2 \end{matrix} \\ \begin{bmatrix} 1215 & & & & & \\ 0 & & & & & \\ 0 & & & & & \\ \hline -1215 & & & & & \\ 0 & & & & & \\ 0 & & & & & \end{bmatrix} \end{array} \cdot \begin{Bmatrix} 3 \cdot 10^{-5} \\ 0 \\ 0 \\ \hline 0 \\ 0 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 36450 \\ 0 \\ 0 \\ \hline -36450 \\ 0 \\ 0 \end{Bmatrix}$$

$$\{R_{12}\} = \underbrace{\{\bar{R}_{12}\} + \{\tilde{R}_{12}\}}_{\text{primární}} + \underbrace{\{\hat{R}_{12}\}}_{\text{sekundární}}$$

$$\{R_{12}\} = \begin{Bmatrix} -36450 \\ -11466 \\ 0 \\ \hline 36450 \\ -12534 \\ -15601 \end{Bmatrix} + \begin{Bmatrix} 0 \\ 4100 \\ 0 \\ \hline 0 \\ -4100 \\ -12301 \end{Bmatrix} + \begin{Bmatrix} 36450 \\ 0 \\ 0 \\ \hline -36450 \\ 0 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 0 \\ -7366 \\ 0 \\ \hline 0 \\ -16634 \\ -27902 \end{Bmatrix}$$

kontrola





$\rightarrow$

$\Sigma F_x = 0$ ✓

$\Sigma F_z = 0 : \; 2 - 9{,}366 + 24 - 16{,}634 = 0$ ✓

$\Sigma M_y = 0$ – např. M_{y1}:

$2 \cdot 1 - 24 \cdot 1 - 27{,}907 + 16{,}634 \cdot 3 = -0{,}005 \doteq 0$ ✓

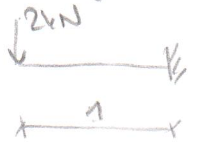
– např. M_{y2}:

$2 \cdot 4 - 9{,}366 \cdot 3 + 24 \cdot 2 - 27{,}907 = -0{,}005 \doteq 0$ ✓

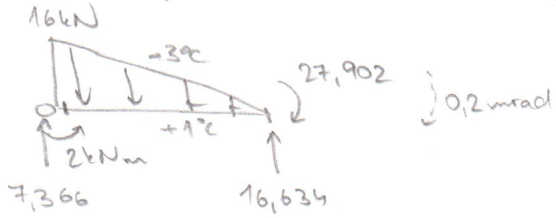
$\{R_{12}^*\} = [T_{12}] \cdot \{R_{12}\}$ (zde GSS ≈ LSS $\Rightarrow \{R_{12}^*\} = \{R_{12}\}$

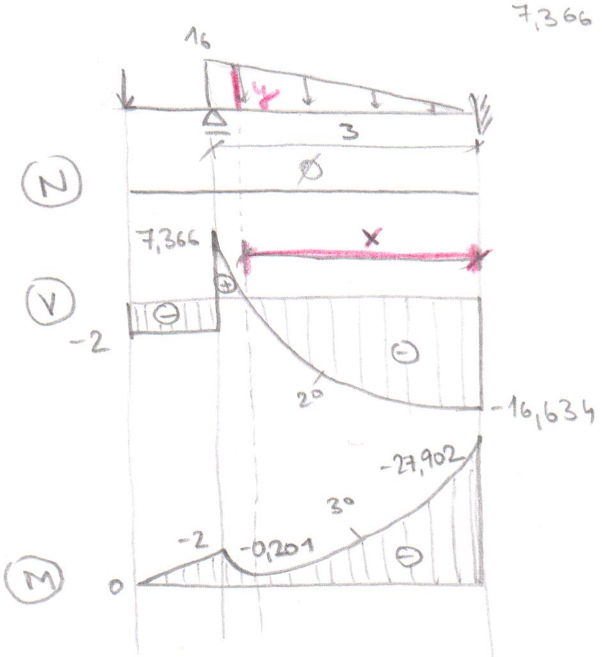
$$\{R_{12}^*\} = \begin{Bmatrix} 0 \\ -7366 \\ 0 \\ \hline 0 \\ -16\,634 \\ -27\,902 \end{Bmatrix}$$

převislý konec



prut 1-2 ... zakreslit všechna zatížení, se kterými jsme počítali, plus koncové síly





$\frac{x}{y} = \frac{3}{16} \Rightarrow y = \frac{16x}{3}$

$\frac{x \cdot y}{2} = 16{,}634$

$\frac{x \cdot 16x}{6} = 16{,}634 \Rightarrow x = 2{,}498\ m$

$y = 13{,}32\ kN/m$

náhr. břemeno: 16,634 kN