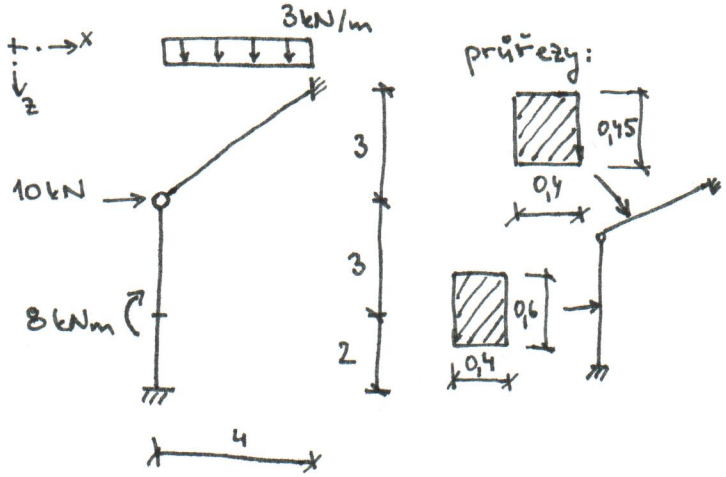
Deformační metoda - rám

Určete průběh vnitřních sil na kci

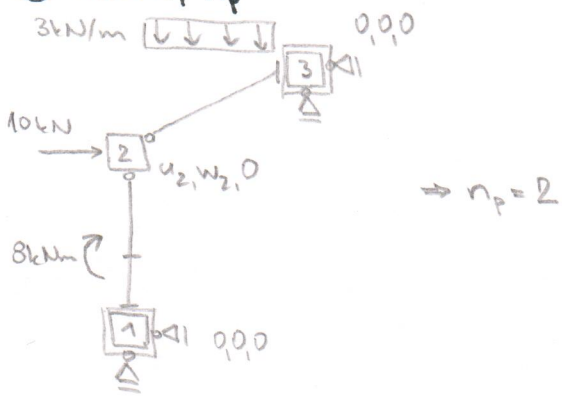


$E = 27\ GPa$

V průběhu výpočtu zaokrouhlujte na 3 platná místa.

---

① model, n_p



$\Rightarrow n_p = 2$

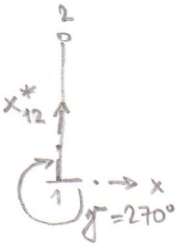
② $\{r\}, \{S\}$

$$\{r\} = \begin{Bmatrix} u_2 \\ w_2 \end{Bmatrix}$$

$$\{S\} = \begin{Bmatrix} 10\,000 \\ 0 \end{Bmatrix}$$

③ $[K_{ij}]$, $\{\bar{R}^*_{ij}\} \rightarrow \{\bar{R}_{ij}\}$

$[K_{12}]$ ... tab. 11.4 (6)



$c = \cos\gamma = 0$

$s = \sin\gamma = -1$

$A_{12} = 0{,}4 \cdot 0{,}6 = 0{,}24\ m^2$

$I_{12} = \frac{1}{12} \cdot 0{,}4 \cdot 0{,}6^3 = 7{,}2 \cdot 10^{-3}\ m^4$

$\frac{EA}{l} = \frac{27 \cdot 10^9 \cdot 0{,}24}{5} = 1300 \cdot 10^6$

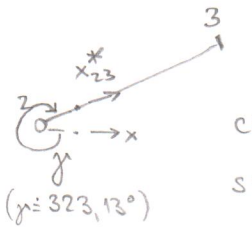
$\frac{3EI}{l^3} = \frac{3 \cdot 27 \cdot 10^9 \cdot 7{,}2 \cdot 10^{-3}}{5^3} = 4{,}67 \cdot 10^6$

$\frac{3EI}{l^2} = \frac{3 \cdot 27 \cdot 10^9 \cdot 7{,}2 \cdot 10^{-3}}{5^2} = 23{,}3 \cdot 10^6$

$$[K_{12}] = 10^6 \cdot \begin{array}{c} \\ u_1 \\ w_1 \\ \varphi_1 \\ \\ \\ \end{array} \begin{bmatrix} u_2 & w_2 \\ -(1300 \cdot 0 + 4{,}67 \cdot 1) & -(1300 - 4{,}67) \cdot 0 \cdot (-1) \\ -(1300 - 4{,}67) \cdot 0 \cdot (-1) & -(1300 \cdot 1 + 4{,}67 \cdot 0) \\ -23{,}3 \cdot (-1) & 23{,}3 \cdot 0 \\ 1300 \cdot 0 + 4{,}67 \cdot 1 & (1300 - 4{,}67) \cdot 0 \cdot (-1) \\ (1300 - 4{,}67) \cdot 0 \cdot (-1) & 1300 \cdot 1 + 4{,}67 \cdot 0 \\ 0 & 0 \end{bmatrix} \begin{array}{c} \\ \\ \\ \varphi_2 \\ \\ \\ \end{array}$$

$$= 10^6 \cdot \begin{array}{c} \\ u_1 \\ w_1 \\ \varphi_1 \\ \\ \\ \end{array} \begin{bmatrix} u_2 & w_2 \\ -4{,}67 & 0 \\ 0 & -1300 \\ 23{,}3 & 0 \\ 4{,}67 & 0 \\ 0 & 1300 \\ 0 & 0 \end{bmatrix} \begin{array}{c} \\ \\ \\ \varphi_2 \\ u_2 \\ w_2 \\ \end{array}$$

$[K_{23}]$ ... tab. 11.4 (c)



$(\gamma = 323{,}13°)$

$c = 0{,}8$

$s = -0{,}6$

$A_{23} = 0{,}4 \cdot 0{,}45 = 0{,}18\ m^2$

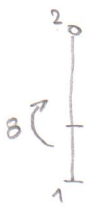
$I_{23} = \frac{1}{12} \cdot 0{,}4 \cdot 0{,}45^3 = 3{,}04 \cdot 10^{-3}\ m^4$

$\frac{EA}{\ell} = \frac{27 \cdot 10^9 \cdot 0{,}18}{5} = 972 \cdot 10^6$

$\frac{3EI}{\ell^3} = \frac{3 \cdot 27 \cdot 10^9 \cdot 3{,}04 \cdot 10^{-3}}{5^3} = 1{,}97 \cdot 10^6$

$\frac{3EI}{\ell^2} = \frac{3 \cdot 27 \cdot 10^9 \cdot 3{,}04 \cdot 10^{-3}}{5^2} = 9{,}85 \cdot 10^6$

$$[K_{23}] = 10^6 \cdot \begin{array}{c} \begin{matrix} u_2 & w_2 & \varphi_2 & u_3 & w_3 & \varphi_3 \end{matrix} \\ \begin{bmatrix} 972 \cdot 0{,}8^2 + 1{,}97 \cdot (-0{,}6)^2 & (972 - 1{,}97) \cdot 0{,}8 \cdot (-0{,}6) & 0 & -(972 \cdot 0{,}8^2 + 1{,}97 \cdot (-0{,}6)^2) & -(972 - 1{,}97) \cdot 0{,}8 \cdot (-0{,}6) & 9{,}85 \cdot (-0{,}6) \\ (972 - 1{,}97) \cdot 0{,}8 \cdot (-0{,}6) & 972 \cdot (-0{,}6)^2 + 1{,}97 \cdot 0{,}8^2 & 0 & -(972 - 1{,}97) \cdot 0{,}8 \cdot (-0{,}6) & -(972 \cdot (-0{,}6)^2 + 1{,}97 \cdot 0{,}8^2) & -9{,}85 \cdot 0{,}8 \end{bmatrix} \end{array} = 10^6 \cdot \begin{array}{c} \begin{matrix} u_2 & w_2 & \varphi_2 & u_3 & w_3 & \varphi_3 \end{matrix} \\ \begin{bmatrix} 623 & -466 & 0 & -623 & 466 & 5{,}91 \\ -466 & 351 & 0 & 466 & -351 & -7{,}88 \end{bmatrix} \end{array}$$



tab. 11.2 (d)

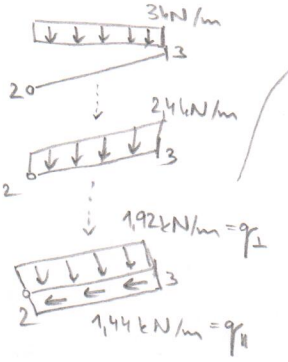
$a = 2\ m$

$b = 3\ m$

$\ell = 5\ m$

$M = 8\ kN$

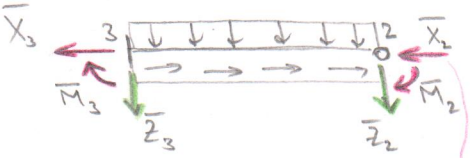
$$\{\bar{R}^*_{12}\} = \begin{Bmatrix} 0 \\ 1540 \\ 320 \\ 0 \\ -1540 \\ 0 \end{Bmatrix}$$



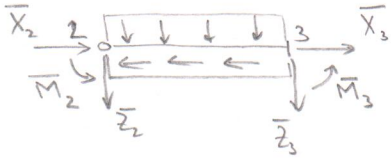
tab. 11.2 (a)

! v tabulce dány reakce pro prut vlevo vetknutý X máme prut vetknutý upravo !

ekvivalentní prut a reakce (symetrický)

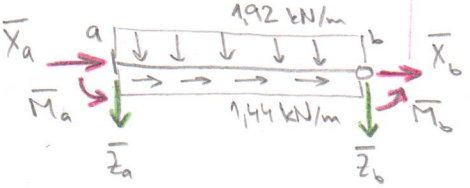


prut v našem modelu:



opačný směr

pomocný prut (z tabulek)



tab. 11.2 (a): $n = 1{,}44\ kN/m$

$q = 1{,}92\ kN/m$

$\ell = 5\ m$

$$\rightarrow \{\bar{R}^*_{ab}\} = \begin{Bmatrix} -3600 \\ -6000 \\ 6000 \\ -3600 \\ -3600 \\ 0 \end{Bmatrix} \begin{matrix} \bar{X}_a \\ \bar{Z}_a \\ \bar{M}_a \\ \bar{X}_b \\ \bar{Z}_b \\ \bar{M}_b \end{matrix}$$

$\bar{X}_2 = -\bar{X}_b$

$\bar{Z}_2 = \bar{Z}_b$

$\bar{M}_2 = -\bar{M}_b$

$\bar{X}_3 = -\bar{X}_a$

$\bar{Z}_3 = \bar{Z}_a$

$\bar{M}_3 = -M_a$

$$\Rightarrow \{\bar{R}^*_{23}\} = \begin{Bmatrix} 3600 \\ -3600 \\ 0 \\ 3600 \\ -6000 \\ -6000 \end{Bmatrix} \begin{matrix} \bar{X}_2 \\ \bar{Z}_2 \\ \bar{M}_2 \\ \bar{X}_3 \\ \bar{Z}_3 \\ \bar{M}_3 \end{matrix}$$

$$\{\bar{R}_{ij}\} = [T_{ij}]^T \cdot \{\bar{R}^*_{ij}\} \qquad [T_{ij}]^T = \begin{bmatrix} c & -s & 0 & \vdots & \emptyset \\ s & c & 0 & \vdots & \\ 0 & 0 & 1 & \vdots & \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ & & & \vdots & c \; -s \; 0 \\ & \emptyset & & \vdots & s \; \; c \; 0 \\ & & & \vdots & 0 \; 0 \; 1 \end{bmatrix}$$

$$\{\bar{R}_{12}\} = [T_{12}]^T \cdot \{\bar{R}^*_{12}\} = \begin{bmatrix} & & & & \\ & & & & \\ & \emptyset & & 0\;1\;0 & \\ & & & -1\;0\;0 & \\ & & & & \end{bmatrix} \cdot \begin{Bmatrix} & \\ & \\ 0 & \\ -1540 & \\ & \end{Bmatrix} = \begin{Bmatrix} & \\ & \\ -1540 & \\ 0 & \\ & \end{Bmatrix} \begin{matrix} \\ \\ \bullet\ \bar{X}_2\ (u_2) \\ \bullet\ \bar{Z}_2\ (w_2) \\ \\ \end{matrix}$$

$$\{\bar{R}_{23}\} = [T_{23}]^T \cdot \{\bar{R}^*_{23}\} = \begin{bmatrix} 0{,}8 & 0{,}6 & 0 & \vdots & \\ -0{,}6 & 0{,}8 & 0 & \vdots & \emptyset \\ & & & \vdots & \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ & & & \vdots & \\ & & & \vdots & \end{bmatrix} \cdot \begin{Bmatrix} 3600 \\ -3600 \\ \cdots \\ \\ \\ \end{Bmatrix} = \begin{Bmatrix} 720 \\ -5040 \\ \cdots \\ \\ \\ \end{Bmatrix} \begin{matrix} \\ \\ \\ \bullet\ \bar{X}_2\ (u_2) \\ \bullet\ \bar{Z}_2\ (w_2) \end{matrix}$$

④ $[K]$, $\{\bar{R}\}$

$$[K] = 10^6 \cdot \begin{bmatrix} u_2 & w_2 \\ 4{,}67 + 623 & 0 - 466 \\ 0 - 466 & 1300 + 351 \end{bmatrix} \begin{matrix} u_2 \\ w_2 \end{matrix} = 10^6 \cdot \begin{bmatrix} 628 & -466 \\ -466 & 1650 \end{bmatrix}$$

$$\{\bar{R}\} = \begin{Bmatrix} -1540 + 720 \\ 0 - 5040 \end{Bmatrix} = \begin{Bmatrix} -820 \\ -5040 \end{Bmatrix} \begin{matrix} \bar{X}_2\ (u_2) \\ \bar{Z}_2\ (w_2) \end{matrix}$$

⑤ $\{F\}$, $\{r\}$ … u_2, w_2

$$\{F\} = \{S\} - \{\bar{R}\} = \begin{Bmatrix} 10000 \\ 0 \end{Bmatrix} - \begin{Bmatrix} -820 \\ -5040 \end{Bmatrix} = \begin{Bmatrix} 10800 \\ 5040 \end{Bmatrix}$$

$$[K] \cdot \{r\} = \{F\}$$

$$10^6 \cdot \begin{bmatrix} 628 & -466 \\ -466 & 1650 \end{bmatrix} \cdot \begin{Bmatrix} u_2 \\ w_2 \end{Bmatrix} = \begin{Bmatrix} 10800 \\ 5040 \end{Bmatrix} \Rightarrow \begin{matrix} 628 \cdot 10^6\, u_2 - 466 \cdot 10^6\, w_2 = 10800 \\ -466 \cdot 10^6\, u_2 + 1650 \cdot 10^6\, w_2 = 5040 \end{matrix}$$

$$\rightarrow 628 \cdot 10^6\, u_2 = 10800 + 466 \cdot 10^6 \cdot w_2$$

$$u_2 = \frac{10800 + 466 \cdot 10^6 \cdot w_2}{628 \cdot 10^6}$$

$$\rightarrow -466 \cdot 10^6 \cdot \frac{10800 + 466 \cdot 10^6\, w_2}{628 \cdot 10^6} + 1650 \cdot 10^6\, w_2 = 5040$$

$$\Rightarrow w_2 = 1{,}00 \cdot 10^{-5}\ m$$

$$\Rightarrow u_2 = \frac{10800 + 466 \cdot 10^6 \cdot 1 \cdot 10^{-5}}{628 \cdot 10^6}$$

$$u_2 = 2{,}46 \cdot 10^{-5}\ m$$

6. $\{r_{ij}\}$

$$\{r_{12}\} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ \hline 2{,}46\cdot10^{-5} \\ 1\cdot10^{-5} \\ 0 \end{Bmatrix} \qquad \{r_{23}\} = \begin{Bmatrix} 2{,}46\cdot10^{-5} \\ 1\cdot10^{-5} \\ 0 \\ \hline 0 \\ 0 \\ 0 \end{Bmatrix}$$

7. $\{\hat{R}_{ij}\} \rightarrow \{\hat{R}^*_{ij}\}$, $\{R^*_{ij}\}$

$$\{\hat{R}_{ij}\} = [K_{ij}] \cdot \{r_{ij}\}$$

$$\{\hat{R}_{12}\} = [K_{12}] \cdot \{r_{12}\} = 10^6 \cdot \begin{bmatrix} u_1 & w_1 & \varphi_1 & u_2 & w_2 & \varphi_2 \\ & & & -4{,}67 & 0 & \\ & & & 0 & -1300 & \\ & & & 2{,}33 & 0 & \\ & & & 4{,}67 & 0 & \\ & & & 0 & 1300 & \\ & & & 0 & 0 & \end{bmatrix} \cdot \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 2{,}46\cdot10^{-5} \\ 1\cdot10^{-5} \\ 0 \end{Bmatrix} = \begin{Bmatrix} -115 \\ -13000 \\ 573 \\ 115 \\ 13000 \\ 0 \end{Bmatrix}$$

$$\{\hat{R}_{23}\} = 10^6 \cdot \begin{bmatrix} 623 & -466 & & & & \\ -466 & 351 & & & & \\ 0 & 0 & & & & \\ -623 & 466 & & & & \\ 466 & -351 & & & & \\ 5{,}91 & -7{,}88 & & & & \end{bmatrix} \cdot \begin{Bmatrix} 2{,}46\cdot10^{-5} \\ 1\cdot10^{-5} \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 10700 \\ -7950 \\ 0 \\ -10700 \\ 7950 \\ -224 \end{Bmatrix}$$

$$\{\hat{R}^*_{12}\} = [T_{12}] \cdot \{\hat{R}_{12}\} \qquad [T_{12}] = \begin{bmatrix} c & s & 0 & & \\ -s & c & 0 & & \varnothing \\ 0 & 0 & 1 & & \\ & \varnothing & & c & s & 0 \\ & & & -s & c & 0 \\ & & & 0 & 0 & 1 \end{bmatrix}$$

$$\{\hat{R}^*_{12}\} = \begin{bmatrix} 0 & -1 & 0 & & & \\ 1 & 0 & 0 & & \varnothing & \\ 0 & 0 & 1 & & & \\ & & & 0 & -1 & 0 \\ & \varnothing & & 1 & 0 & 0 \\ & & & 0 & 0 & 1 \end{bmatrix} \cdot \begin{Bmatrix} -115 \\ -13000 \\ 573 \\ 115 \\ 13000 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 13000 \\ -115 \\ 573 \\ -13000 \\ 115 \\ 0 \end{Bmatrix}$$

$$\{R^*_{12}\} = \{\bar{R}^*_{12}\} + \{\hat{R}^*_{12}\} = \begin{Bmatrix} 0 \\ 1540 \\ 320 \\ 0 \\ -1540 \\ 0 \end{Bmatrix} + \begin{Bmatrix} 13000 \\ -115 \\ 573 \\ -13000 \\ 115 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 13000 \\ 1430 \\ 893 \\ -13000 \\ -1430 \\ 0 \end{Bmatrix}$$

$$\{\hat{R}^*_{23}\} = \begin{bmatrix} 0{,}8 & -0{,}6 & 0 & & & \\ 0{,}6 & 0{,}8 & 0 & & \varnothing & \\ 0 & 0 & 1 & & & \\ & & & 0{,}8 & -0{,}6 & 0 \\ & \varnothing & & 0{,}6 & 0{,}8 & 0 \\ & & & 0 & 0 & 1 \end{bmatrix} \cdot \begin{Bmatrix} 10700 \\ -7950 \\ 0 \\ -10700 \\ 7950 \\ -224 \end{Bmatrix} = \begin{Bmatrix} 13300 \\ 60 \\ 0 \\ -13300 \\ -60 \\ -224 \end{Bmatrix}$$

$$\{R^*_{23}\} = \begin{Bmatrix} 3600 \\ -3600 \\ 0 \\ 3600 \\ -6000 \\ -6000 \end{Bmatrix} + \begin{Bmatrix} 13300 \\ 60 \\ 0 \\ -13300 \\ -60 \\ -224 \end{Bmatrix} = \begin{Bmatrix} 16900 \\ -3540 \\ 0 \\ -9700 \\ -6060 \\ -6220 \end{Bmatrix}$$

8. průběhy vnitřních sil