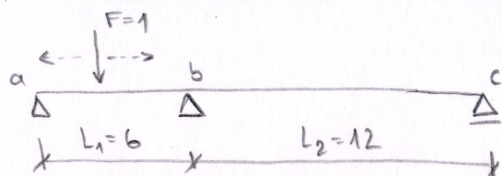


# PČ spojitých nosníků - výpočet pořadnice PČ

(2)



Pro PČ momentu v místě podpěry:

trimomentová rovnice

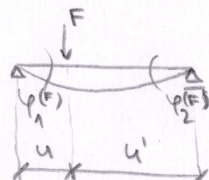
$$\varphi_{ba} + \varphi_{bc} + M_b (\alpha_{ba} + \alpha_{bc}) = 0$$

$$\alpha = \frac{L}{3EI} \quad \begin{cases} \alpha_{ba} = \frac{2}{EI} \\ \alpha_{bc} = \frac{4}{EI} \end{cases}$$

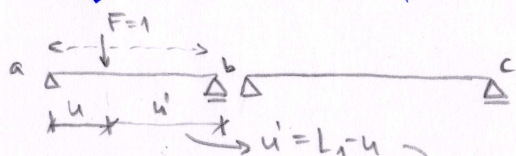
$$M_b = - \frac{\varphi_{ba} + \varphi_{bc}}{\alpha_{ba} + \alpha_{bc}}$$

$$\varphi_1^{(F)} = \frac{F \cdot u \cdot u'}{6EI \cdot L} (L + u')$$

$$\varphi_2^{(F)} = \frac{F \cdot u \cdot u'}{6EI \cdot L} (L + u)$$



**I** - polyblivě zat. F na prutu a-b:



$$\varphi_{ba}^{(F)} = \frac{F \cdot u \cdot u'}{6EI \cdot L_1} (L_1 + u) = \frac{F \cdot u(6-u)}{6EI \cdot 6} (6+u) =$$

pozn.: pro vykreslení PČ... F=1

$$= \frac{1 \cdot u(36-u^2)}{36EI}$$

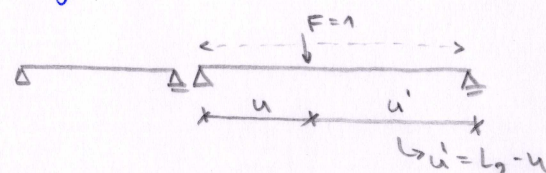
$\varphi_{bc}$ : nulové zatížení prutu b-c

$$\Rightarrow \varphi_{bc} = 0$$

$$M_{b,I} = - \frac{\varphi_{ba} + \varphi_{bc}}{\alpha_{ba} + \alpha_{bc}} = - \frac{\frac{u(36-u^2)}{36EI} + 0}{\frac{2}{EI} + \frac{4}{EI}} =$$

$$= - \frac{36u - u^3}{36EI} \cdot \frac{EI}{6} = - \frac{36u - u^3}{216}$$

**II** - polyblivě zat. F na prutu b-c



$\varphi_{ba}$ : nulové zatížení prutu a-b  $\Rightarrow \varphi_{ba} = 0$

$$\varphi_{bc}^{(F)} = \frac{F \cdot u \cdot u'}{6EI \cdot L_2} (L_2 + u') = \frac{F \cdot u(12-u)}{6EI \cdot 12} (12+12-u) =$$

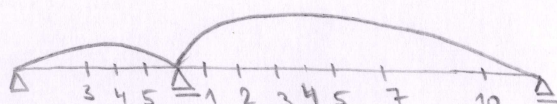
$$= \frac{1 \cdot u(12-u)(24-u)}{72 \cdot EI}$$

$$M_{b,II} = - \frac{\varphi_{ba} + \varphi_{bc}}{\alpha_{ba} + \alpha_{bc}} = - \frac{0 + \frac{u(12-u)(24-u)}{72 \cdot EI}}{\frac{2}{EI} + \frac{4}{EI}} =$$

$$= - \frac{u \cdot (u^2 - 36u + 288)}{72 \cdot EI} \cdot \frac{EI}{6} =$$

$$= - \frac{u^3 - 36u^2 + 288u}{432}$$

**M<sub>b</sub>**



$$M_{b,I}(u=3) = - \frac{36 \cdot 3 - 3^3}{216} = -0,375$$

$$M_{b,I}(u=4) = - \frac{36 \cdot 4 - 4^3}{216} = -0,370$$

$$M_{b,I}(u=5) = -0,255$$

$$M_{b,II}(u=1) = -0,586$$

$$M_{b,II}(u=2) = -1,019$$

$$M_{b,II}(u=3) = -1,313$$

$$M_{b,II}(u=4) = -1,482$$

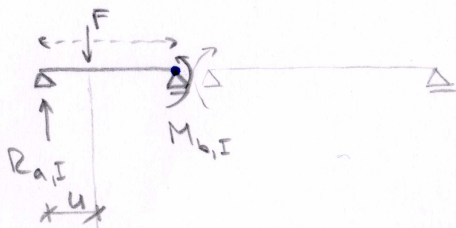
$$M_{b,II}(u=5) = -1,539$$

$$M_{b,II}(u=7) = -1,377$$

$$M_{b,II}(u=10) = -0,648$$

## PČ reakce $R_a$

① - zat. F na prutu a-b



$$R_{a,I} = \frac{F \cdot (L_1 - u) + M_{b,I}}{L_1} = \frac{1 \cdot (6 - u) + \left(-\frac{36u - u^3}{216}\right)}{6}$$

$$= \frac{u^3 - 252u + 1296}{1296}$$

$$R_{a,I}(u=0) = 1$$

$$R_{a,I}(u=1) = 0,806$$

$$R_{a,I}(u=2) = 0,617$$

$$R_{a,I}(u=3) = 0,438$$

$$R_{a,I}(u=4) = 0,271$$

$$R_{a,II}(u=2) = -0,170$$

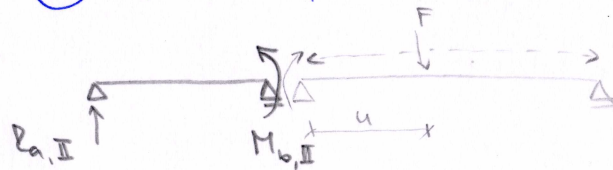
$$R_{a,II}(u=4) = -0,247$$

$$R_{a,II}(u=6) = -0,250$$

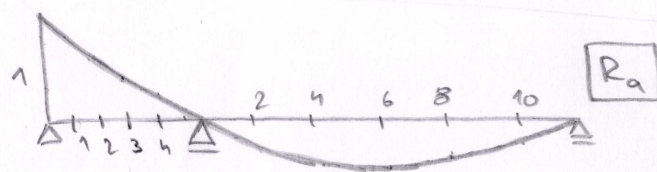
$$R_{a,II}(u=8) = -0,198$$

$$R_{a,II}(u=10) = -0,108$$

② - zat. F na prutu b-c



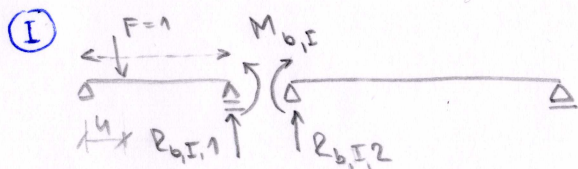
$$R_{a,II} = \frac{M_{b,II}}{L_1} = \frac{-\frac{u^3 - 36u^2 + 288u}{432}}{6} = -\frac{u^3 - 36u^2 + 288u}{2592}$$



• extrém  $\Leftrightarrow$  derivace = 0

$$\frac{d}{du} \left( -\frac{u^3 - 36u^2 + 288u}{2592} \right) = 0 \Rightarrow u = \dots$$

## PČ reakce $R_b$



$$R_{b,I} = R_{b,I,1} + R_{b,I,2}$$

$R_{b,I,1}$  analogicky jako  $R_{a,I}$ :

$$R_{b,I,1} = \frac{F \cdot u - M_{b,I}}{L_1} = \frac{u + \frac{36u - u^3}{216}}{6}$$

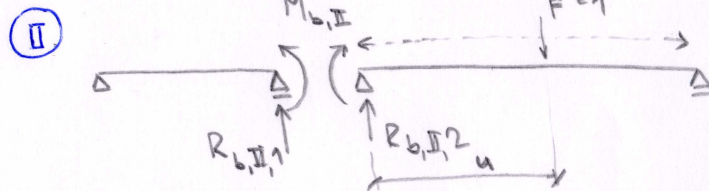
$$= \frac{252u - u^3}{1296}$$

$R_{b,I,2}$  analogicky jako  $R_{a,II}$ :

$$R_{b,I,2} = \frac{-M_{b,I}}{L_2} = \frac{\frac{36u - u^3}{216}}{12} = \frac{36u - u^3}{2592}$$

$$R_{b,I} = \frac{252u - u^3}{1296} + \frac{36u - u^3}{2592} = \frac{540u - 3u^3}{2592}$$

$$= \frac{180u - u^3}{864}$$



$$R_{b,II} = R_{b,II,1} + R_{b,II,2}$$

$R_{b,II,1}$  analogicky jako  $R_{a,II}$

$$R_{b,II,1} = -\frac{M_{b,II}}{L_1} = \frac{\frac{u^3 - 36u^2 + 288u}{432}}{6} = \frac{u^3 - 36u^2 + 288u}{2592}$$

$R_{b,II,2}$  analogicky k  $R_{a,I}$

$$R_{b,II,2} = \frac{F \cdot (L_2 - u) - M_{b,II}}{L_2} = \frac{(12 - u) + \frac{u^3 - 36u^2 + 288u}{432}}{12}$$

$$= \frac{5184 - 432u + u^3 - 36u^2 + 288u}{12 \cdot 432}$$

$$= \frac{u^3 - 36u^2 - 144u + 5184}{5184}$$

$$R_{b,II} = \frac{u^3 - 36u^2 + 288u}{2592} + \frac{u^3 - 36u^2 - 144u + 5184}{5184}$$

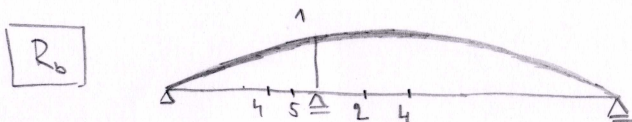
$$= \frac{3u^3 - 108u^2 + 432u + 5184}{5184} = \frac{u^3 - 36u^2 + 144u + 1728}{1728}$$

$$R_{b,I} (u=4) = 0,759$$

$$R_{b,II} (u=2) = 1,088$$

$$R_{b,I} (u=5) = 0,897$$

$$R_{b,II} (u=4) = 1,037$$



extrém  $\Leftrightarrow$  derivace = 0

derivace:  $\frac{d}{du} \left( \frac{u^3 - 36u^2 + 144u + 1728}{1728} \right) = 0$

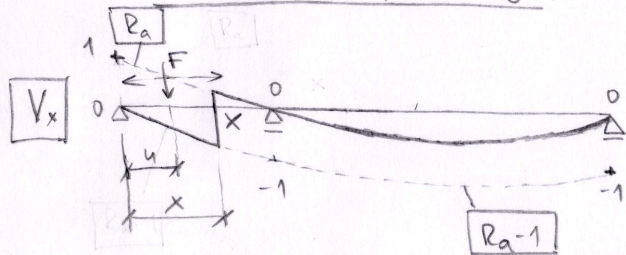
$$\frac{3u^2 - 72u + 144}{1728} = \frac{u^2 - 24u + 48}{576}$$

$$\left\{ \begin{array}{l} \frac{u^2 - 24u + 48}{576} = 0 \\ \Leftrightarrow u^2 - 24u + 48 = 0 \end{array} \right.$$

~~$$u_1 = 21,798$$~~

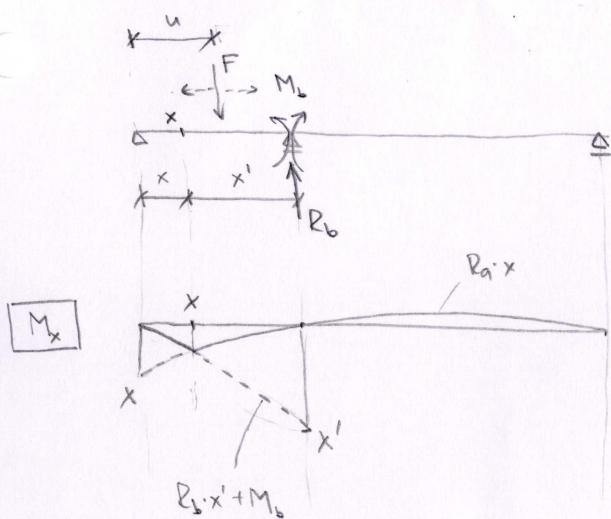
$$u_2 = 2,202 \text{ m}$$

PČ posouvající síly



$$V_x: \begin{cases} u \in (0, x) \Rightarrow V_x = R_a - F = R_{a,I} - 1 \\ u > x \Rightarrow V_x = R_a \begin{cases} u \in (x, b) \Rightarrow V_x = R_{a,I} \\ u \in (b, c) \Rightarrow V_x = R_{a,II} \end{cases} \end{cases} \quad (F=1)$$

PČ momentů (vnitřní síly)



$$M_x: \begin{cases} u \in (0, x) \Rightarrow M_x = R_b \cdot x' + M_b \\ u > x \Rightarrow M_x = R_a \cdot x \end{cases}$$