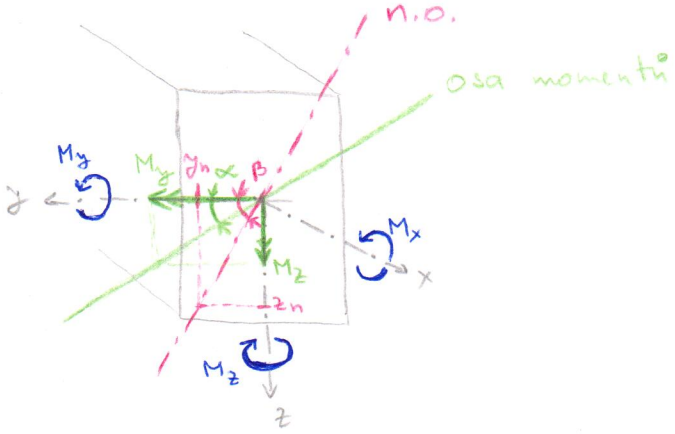
Prostorový ohyb

$N = 0$ (není tahové ani tlakové namáhání)
$M_y \neq 0$
$M_z \neq 0$

obecně:

$$\sigma_x = \frac{N}{A} + \frac{M_y}{I_y} \cdot z - \frac{M_z}{I_z} \cdot y$$

$$\Rightarrow \boxed{\sigma_x = \frac{M_y}{I_y} \cdot z - \frac{M_z}{I_z} \cdot y}$$



⇒ neutrální osa prochází těžištěm

(těžiště: $y=0, z=0$: $\sigma_x(y=0,z=0) = 0$)

→ sklon n.o. : úhel β

$$\operatorname{tg}\beta = \frac{z_n}{y_n}$$

$[y_n, z_n]$ - bod na n.o.

n.o. : $\sigma_x = 0$

bod na n.o. : $\frac{M_y}{I_y} \cdot z_n - \frac{M_z}{I_z} \cdot y_n = 0$

$$\frac{M_y}{I_y} \cdot z_n = \frac{M_z}{I_z} \cdot y_n$$

$$\frac{z_n}{y_n} = \frac{M_z}{I_z} \cdot \frac{I_y}{M_y}$$

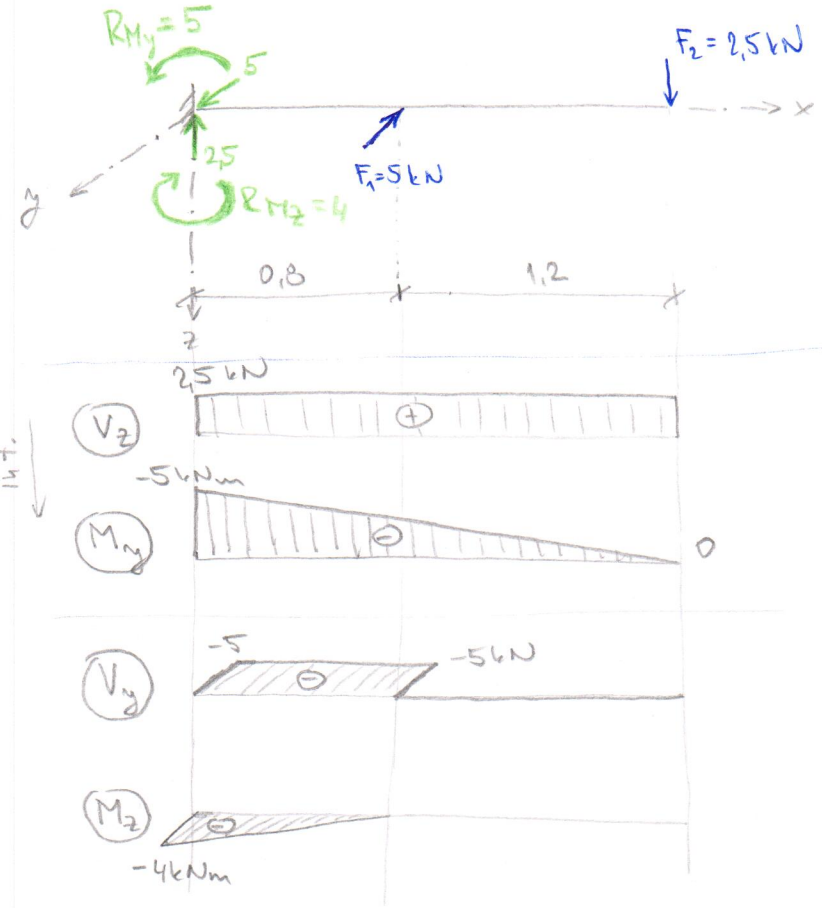
Sklon osy momentů: úhel α

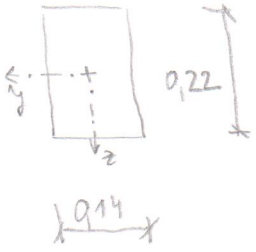
$$\operatorname{tg}\alpha = \frac{M_z}{M_y}$$

→ sklon n.o. (β): $\operatorname{tg}\beta = \frac{z_n}{y_n} = \frac{M_z}{M_y} \cdot \frac{I_y}{I_z} = \operatorname{tg}\alpha \cdot \frac{I_y}{I_z}$

⇒ n.o. není shodná s momentovou osou (vliv různých tuhostí v „y" a „z" směru)

Př.) Vyšetřete průběh σ_x na průřezu ve vetknutí





reakce

$R_y = F_1 = 5kN$

$R_z = F_2 = 2,5kN$

$R_{My} = F_2 \cdot 2 = 5kNm$

$R_{Mz} = F_1 \cdot 0,8 = 4kNm$

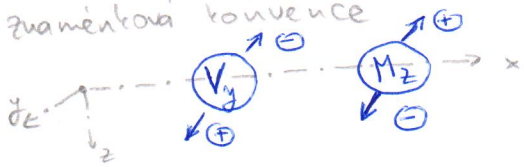
$$\sigma_x = \frac{M_y}{I_y} \cdot z - \frac{M_z}{I_z} \cdot y$$

ve vetknutí: $M_y = -5kNm = -5000\ Nm$

$M_z = -4kNm = -4000\ Nm$

rovina xy: V_y, M_z

znaménková konvence

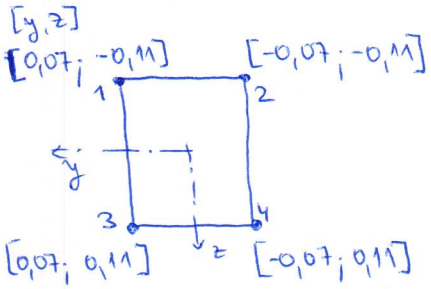


int.: $-V_y$, M_z

$$I_y = \frac{1}{12} bh^3 = \frac{1}{12} \cdot 0,14 \cdot 0,22^3 = 124,23 \cdot 10^{-6}\ m^4$$

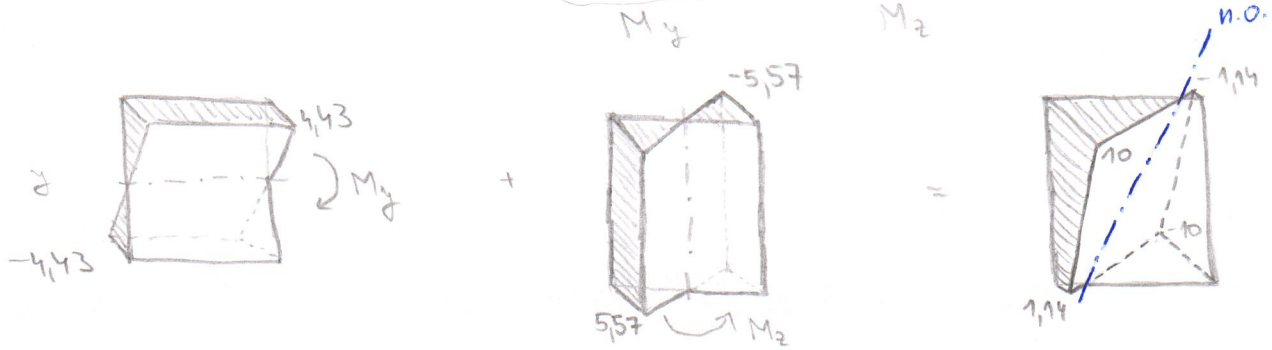
$$I_z = \frac{1}{12} hb^3 = \frac{1}{12} \cdot 0,22 \cdot 0,14^3 = 50,31 \cdot 10^{-6}\ m^4$$

$$\sigma_x = \frac{-5000}{124,23 \cdot 10^{-6}} \cdot z - \frac{-4000}{50,31 \cdot 10^{-6}} \cdot y = \underbrace{-40,25 \cdot 10^6\, z}_{\text{vliv } M_y} + \underbrace{79,51 \cdot 10^6\, y}_{\text{vliv } M_z}$$

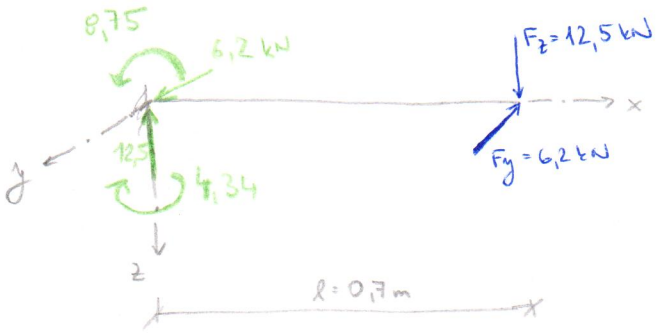


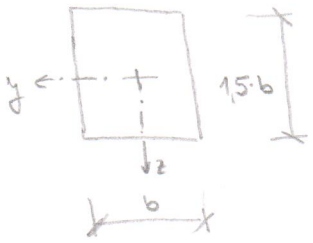
$\sigma_1 = -40,25 \cdot 10^6 \cdot (-0,11) + 79,51 \cdot 10^6 \cdot 0,07$

	M_y	M_z	
$=$	$4,43 \cdot 10^6$	$+ 5,57 \cdot 10^6$	$= 10 \cdot 10^6 = 10\ MPa$
$\sigma_2 =$	$4,43 \cdot 10^6$	$- 5,57 \cdot 10^6$	$= -1,14\ MPa$
$\sigma_3 =$	$-4,43 \cdot 10^6$	$+ 5,57 \cdot 10^6$	$= 1,14\ MPa$
$\sigma_4 =$	$-4,43 \cdot 10^6$	$- 5,57 \cdot 10^6$	$= -10\ MPa$



Př.] Navrhněte b (s ohledem na σ_x)





$f_d = 12$ MPa

reakce

$R_y = F_y = 6,2$ kN

$R_z = F_z = 12,5$ kN

$R_{My} = F_z \cdot 0,7 = 8,75$ kNm

$R_{Mz} = F_y \cdot 0,7 = 4,34$ kNm

$$\sigma_{max} = \frac{|M_y|}{I_y} \cdot z + \frac{|M_z|}{I_z} \cdot y$$

$$I_y = \frac{1}{12} b h^3 = \frac{1}{1}$$

$$I_z = \frac{1}{12} h b^3$$

$$\sigma_{max} = \frac{|M_y|}{\frac{1}{12} b h^3} \cdot \frac{h}{2} + \frac{|M_z|}{\frac{1}{12} h b^3} \cdot \frac{b}{2} = \frac{6 \cdot |M_y|}{b h^2} + \frac{6 \cdot |M_z|}{h b^2} \qquad h = 1,5 \cdot b$$

$$\sigma_{max} = \frac{6 \cdot |M_y|}{b \cdot (1,5b)^2} + \frac{6 \cdot |M_z|}{1,5b \cdot b^2} = \frac{4}{b^3} \left(\frac{|M_y|}{1,5} + |M_z| \right) = \frac{4}{b^3} \left(\frac{8,75 \cdot 10^3}{1,5} + 4,34 \cdot 10^3 \right) = \frac{40,693 \cdot 10^3}{b^3}$$

$$|\sigma_{max}| \leq f_d$$

$$\frac{40,693 \cdot 10^3}{b^3} \leq 12 \cdot 10^6 \rightarrow b^3 \geq \frac{40,693 \cdot 10^3}{12 \cdot 10^6} \Rightarrow b \geq 0,1502 \text{ m}$$

$$h = 1,5 \cdot b = 0,2254 \text{ m}$$

Návrh: $b = 0,15$ m, $h = 0,23$ m

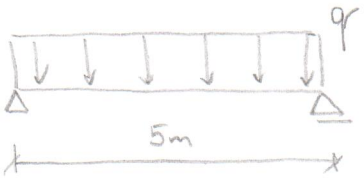
→ Posudek: $\sigma_{max} = \frac{6 \cdot 8,75 \cdot 10^3}{0,15 \cdot 0,23^2} + \frac{6 \cdot 4,34 \cdot 10^3}{0,23 \cdot 0,15^2} = 11,648$ MPa

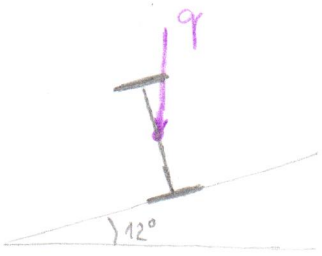
$$|\sigma_{max}| \leq f_d$$

$$11,648 \text{ MPa} \leq 12 \text{ MPa} \quad \checkmark$$

⇒ navržený průřez: $b = 0,15$ m, $h = 0,23$ m

Př.: Navrhněte I profil





	q_k	γ
1) stálé zat.	2 kN/m	1,35
2) proměnné	3 kN/m	1,5

(je bráněno klopení)

$f_d = 210$ MPa

habidka profilů:

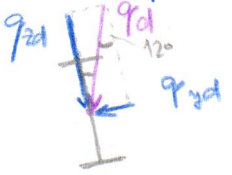
	W_y [m³]	W_z [m³]
I 200	$214 \cdot 10^{-6}$	$25{,}9 \cdot 10^{-6}$
I 220	$278 \cdot 10^{-6}$	$33{,}1 \cdot 10^{-6}$
I 240	$353 \cdot 10^{-6}$	$41{,}5 \cdot 10^{-6}$

$$\sigma_{max} = \frac{|M_y|}{I_y} \cdot z + \frac{|M_z|}{I_z} \cdot y$$

$$\frac{I_y}{z} = W_y \qquad \frac{I_z}{y} = W_z$$

$$\sigma_{max} = \frac{|M_y|}{W_y} + \frac{|M_z|}{W_z}$$

$$q_d = q_{d1} + q_{d2} = q_{k1} \cdot \gamma_1 + q_{k2} \cdot \gamma_2 = 2 \cdot 1{,}35 + 3 \cdot 1{,}5 = 2{,}7 + 4{,}5 = 7{,}2 \text{ kN/m}$$



$$\sin 12° = \frac{q_{yd}}{q_d} \Rightarrow q_{yd} = q_d \cdot \sin 12° = 1{,}497 \text{ kN/m}$$

$$\cos 12° = \frac{q_{zd}}{q_d} \Rightarrow q_{zd} = q_d \cdot \cos 12° = 7{,}043 \text{ kN/m}$$

prostě podepřený nosník po celé délce zatížený konst. spoj. zatížením
→ max M je uprostřed a má hodnotu $\frac{1}{8} q l^2$

$$|M_y| = \frac{1}{8} q_{zd} l^2 = \frac{1}{8} \cdot 7{,}043 \cdot 5^2 = 22{,}01 \text{ kNm}$$

$$|M_z| = \frac{1}{8} q_{yd} l^2 = \frac{1}{8} \cdot 1{,}497 \cdot 5^2 = 4{,}678 \text{ kNm}$$

volba profilu:

I200: $\sigma_{max} = \frac{22{,}01 \cdot 10^3}{214 \cdot 10^{-6}} + \frac{4{,}678 \cdot 10^3}{25{,}9 \cdot 10^{-6}} = 283{,}5 \text{ MPa} > f_d\ (= 210 \text{ MPa})$ … nevyhovuje

I220: $\sigma_{max} = \frac{22{,}01 \cdot 10^3}{278 \cdot 10^{-6}} + \frac{4{,}678 \cdot 10^3}{33{,}1 \cdot 10^{-6}} = 220{,}5 \text{ MPa} > f_d$ … nevyhovuje

I240: $\sigma_{max} = \frac{22{,}01 \cdot 10^3}{353 \cdot 10^{-6}} + \frac{4{,}678 \cdot 10^3}{41{,}5 \cdot 10^{-6}} = 175{,}1 \text{ MPa} < f_d$ … vyhovuje ✓

⇒ navržený profil je I 240