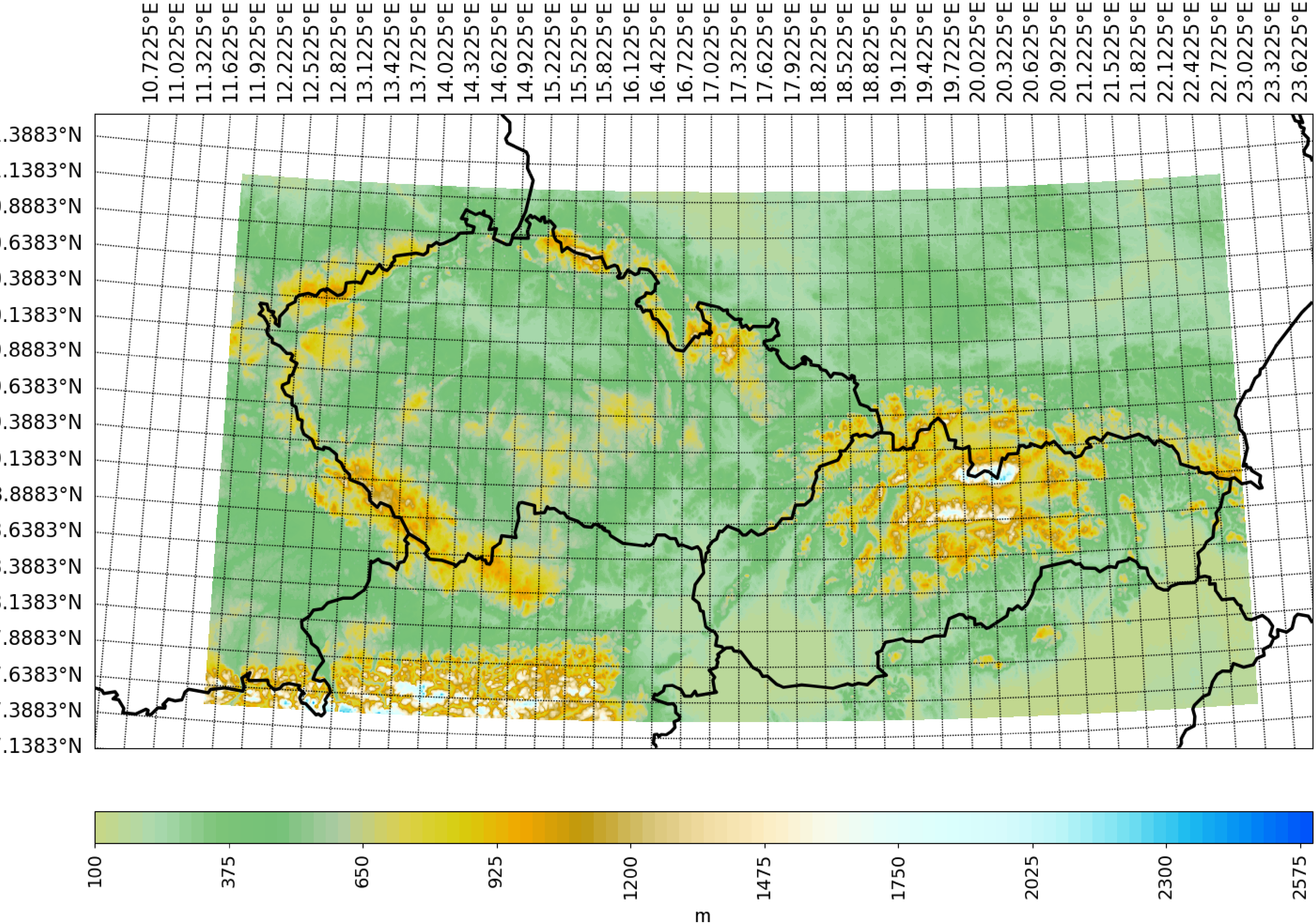
NUMERICKÝ VÝPOČET ROZDIELU GEOIDU A KVAZIGEODU PRE ÚZEMIE ČESKEJ A SLOVENSKEJ REPUBLIKY

THE NUMERICAL EVALUATION OF GEOID-QUASIGEOID SEPARATION FOR THE CZECH REPUBLIC AND THE SLOVAK REPUBLIC

Michal Buday
Viliam Vatrt
Radovan Machotka

DRUŽICOVÉ METODY V GEODÉZII A KATASTRU

BRNO UNIVERSITY OF TECHNOLOGY



10.7225°E
11.0225°E
11.3225°E
11.6225°E
11.9225°E
12.2225°E
12.5225°E
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13.1225°E
13.4225°E
13.7225°E
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14.6225°E
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21.8225°E
22.1225°E
22.4225°E
22.7225°E
23.0225°E
23.3225°E
23.6225°E
100
375
650
925
1200
1475
1750
2025
2300
2575
m

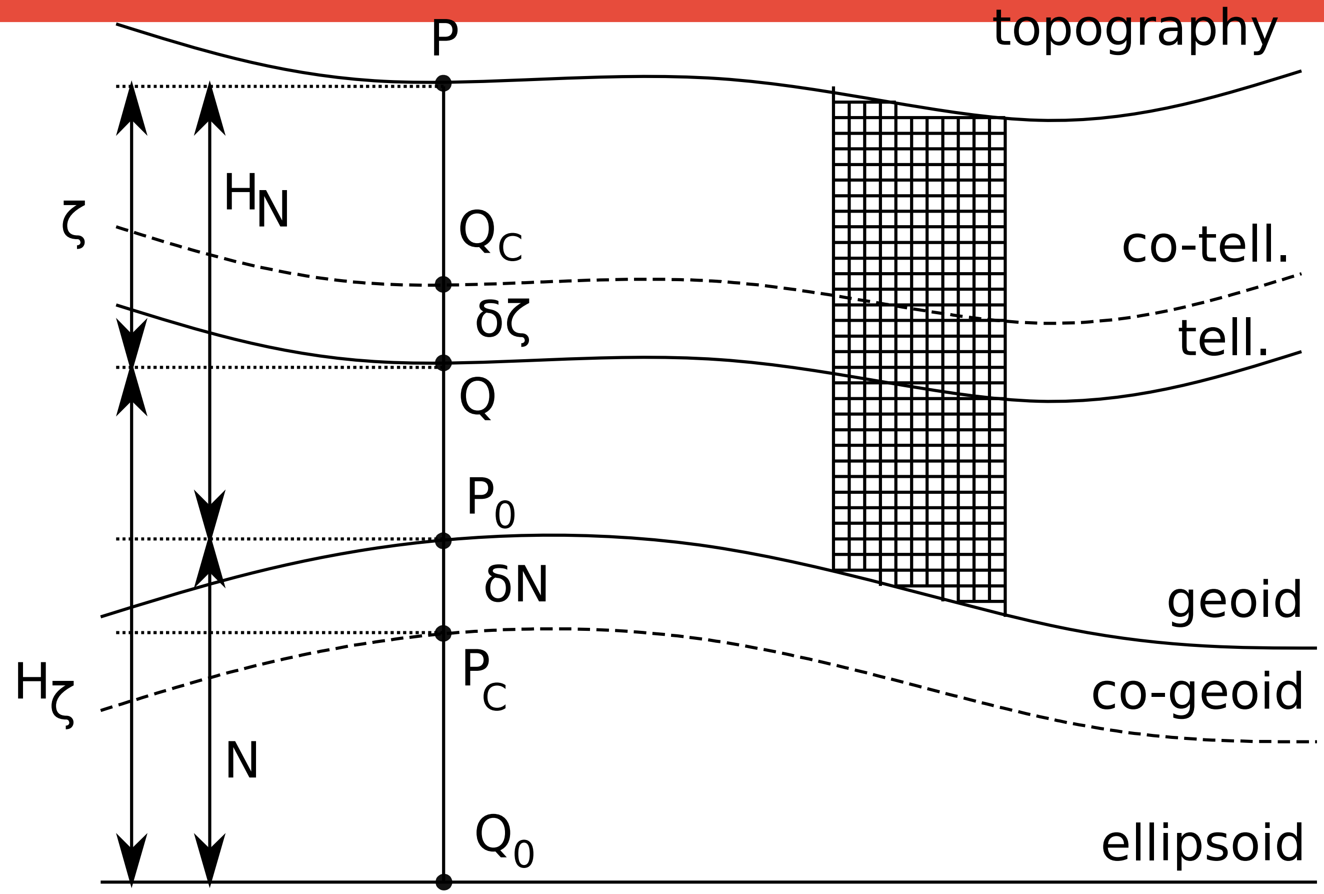
The Helmert-Stokes and Molodensky Heights overview

- The separation between the reference surfaces for orthometric and normal heights (geoid and quasigeoid) is typically in order of a few decimetres, in extreme cases it can reach 3 m

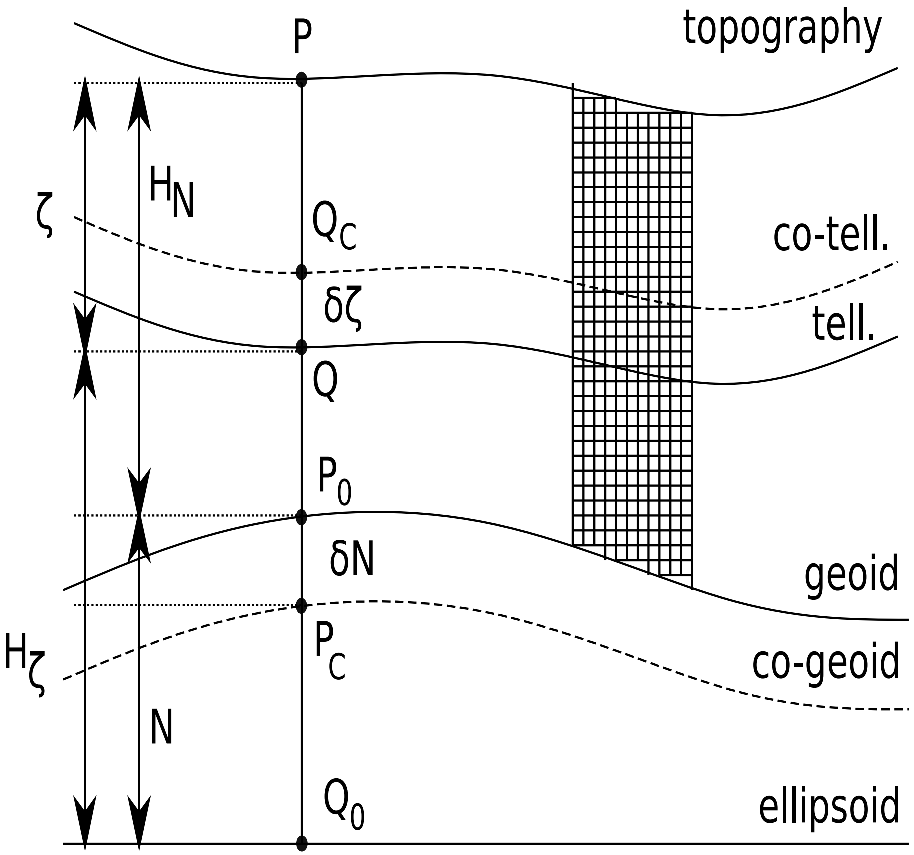
- The knowledge of geoid and quasigeoid separation is important for determination of the global or international vertical reference frame

- The key information is the knowledge about the mean gravity value averaged along the plumb line inside the topgraphy.

The Helmert-Stokes and Molodensky Heights overview



$$N-\zeta=\frac{\bar{\gamma}-\bar{g}}{\bar{\gamma}}\cdot H\approx\frac{\Delta g_P^{BO}}{\bar{\gamma}}\cdot H+\frac{V^T(P_0)-V^T(P)}{\bar{\gamma}}$$

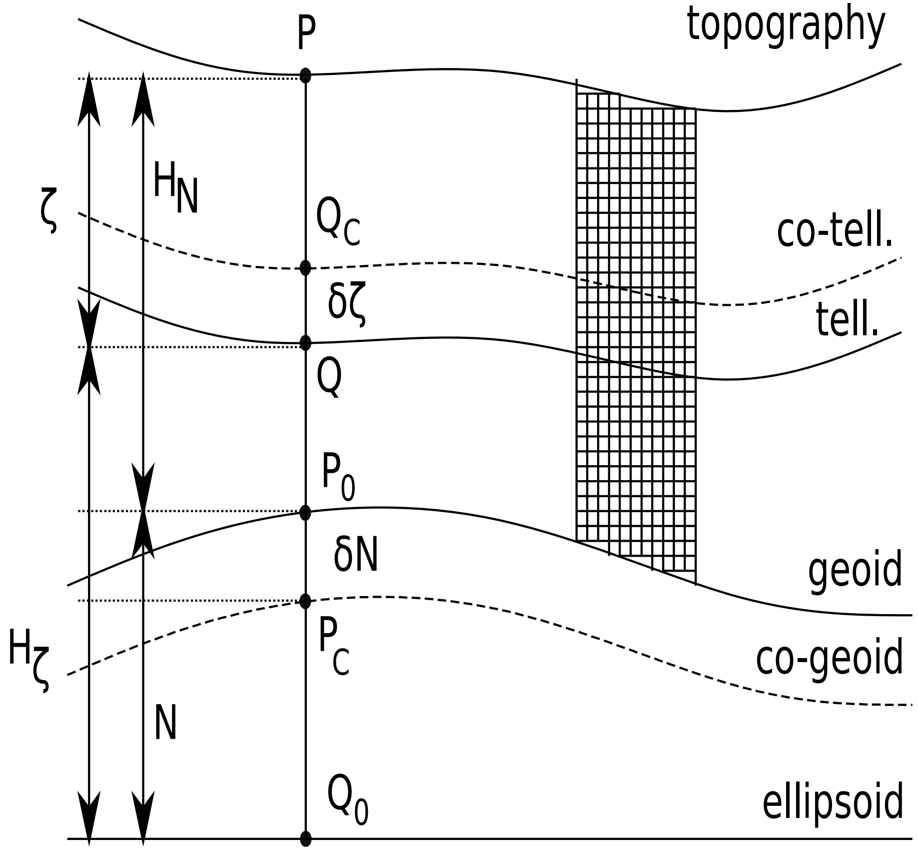


$$N^{cogeoid} = \frac{T(P_C)}{\gamma(Q_0)}$$

$$\Delta g^H = g^H(P_C) - \gamma(Q_0)$$

$$\zeta(P) = \frac{T(P)}{\gamma(Q)}$$

$$C(P) = -(W(P) - W_0)$$

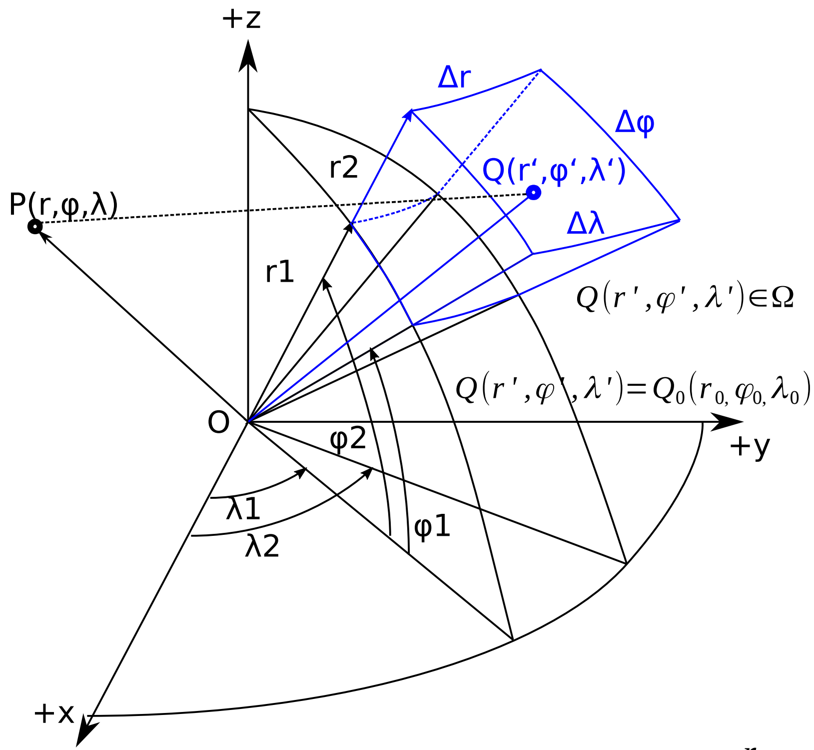


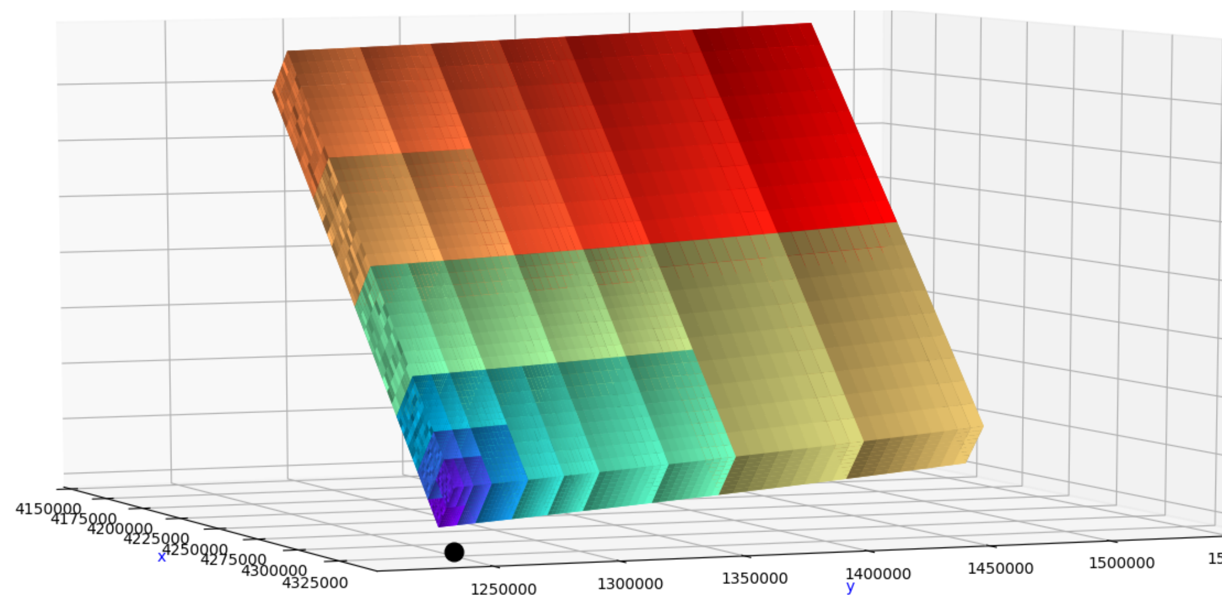
$$W^H = U + T^H = W - \delta V^H$$

$$T^H = T - \delta V^H$$

$$\delta\zeta = \zeta^H - \zeta = -\frac{\delta V^H}{\bar{\gamma}}$$

Tesseroid geometry

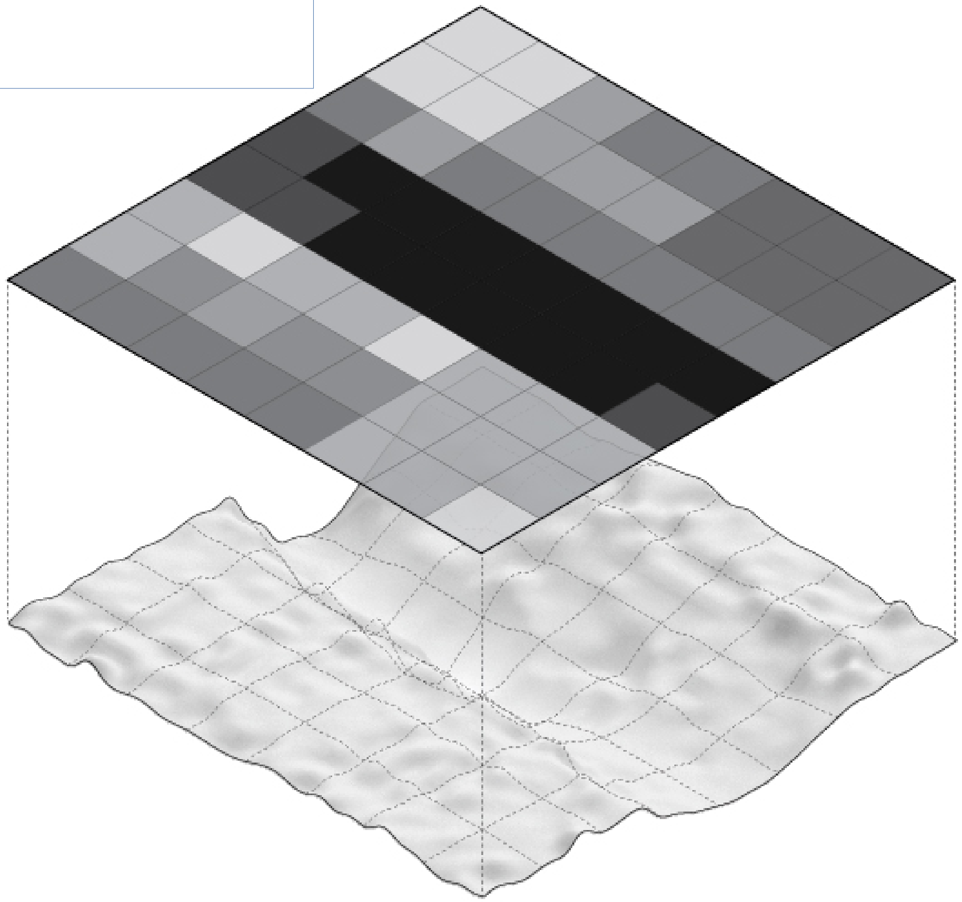




$$l(P,Q_0) \leq C \cdot \Delta x_i$$

$$V^{*}(P(r,\varphi,\lambda)) = G\rho \int_{r_1}^{r_2} \int_{\varphi_1}^{\varphi_2} \int_{\lambda_1}^{\lambda_2} \frac{r'^2 \cos\varphi' \, dr' \, d\varphi' \, d\lambda'}{\sqrt{r'^2 + r^2 - 2rr'(\sin\varphi \sin\varphi' + \cos\varphi \cos\varphi' \cos(\lambda' - \lambda))}}$$

Digital Elevation model

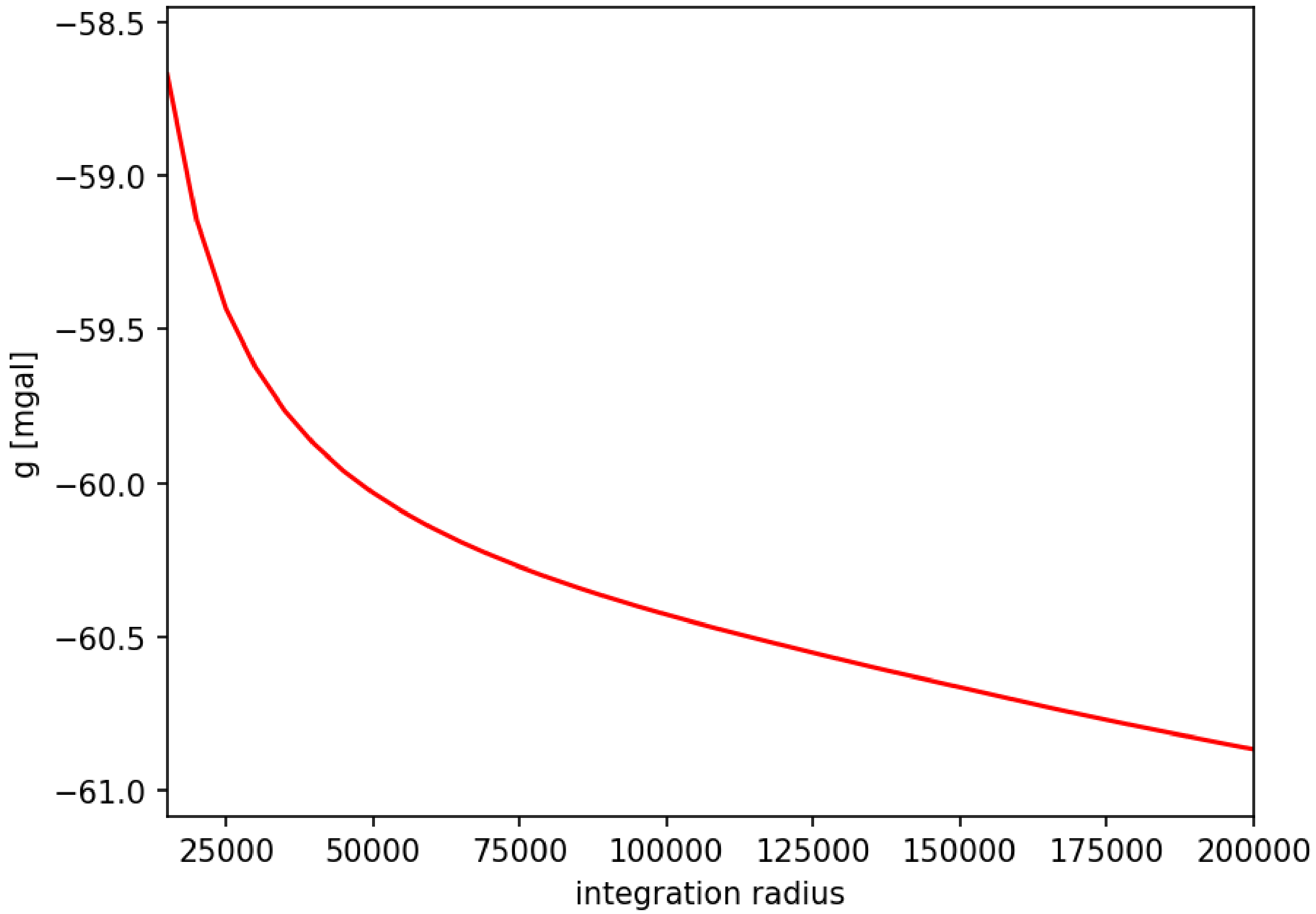


Taylor polynomial

$$V^*(r,\varphi,\lambda) = G\rho\Delta r\Delta\varphi\Delta\lambda\Big[K_{000} + \frac{1}{24}\Big(K_{200}\Delta r^2 + K_{020}\Delta\varphi^2 + K_{002}\Delta\lambda^2\Big) + O(\Delta^4/\ell_0^5)\Big]$$

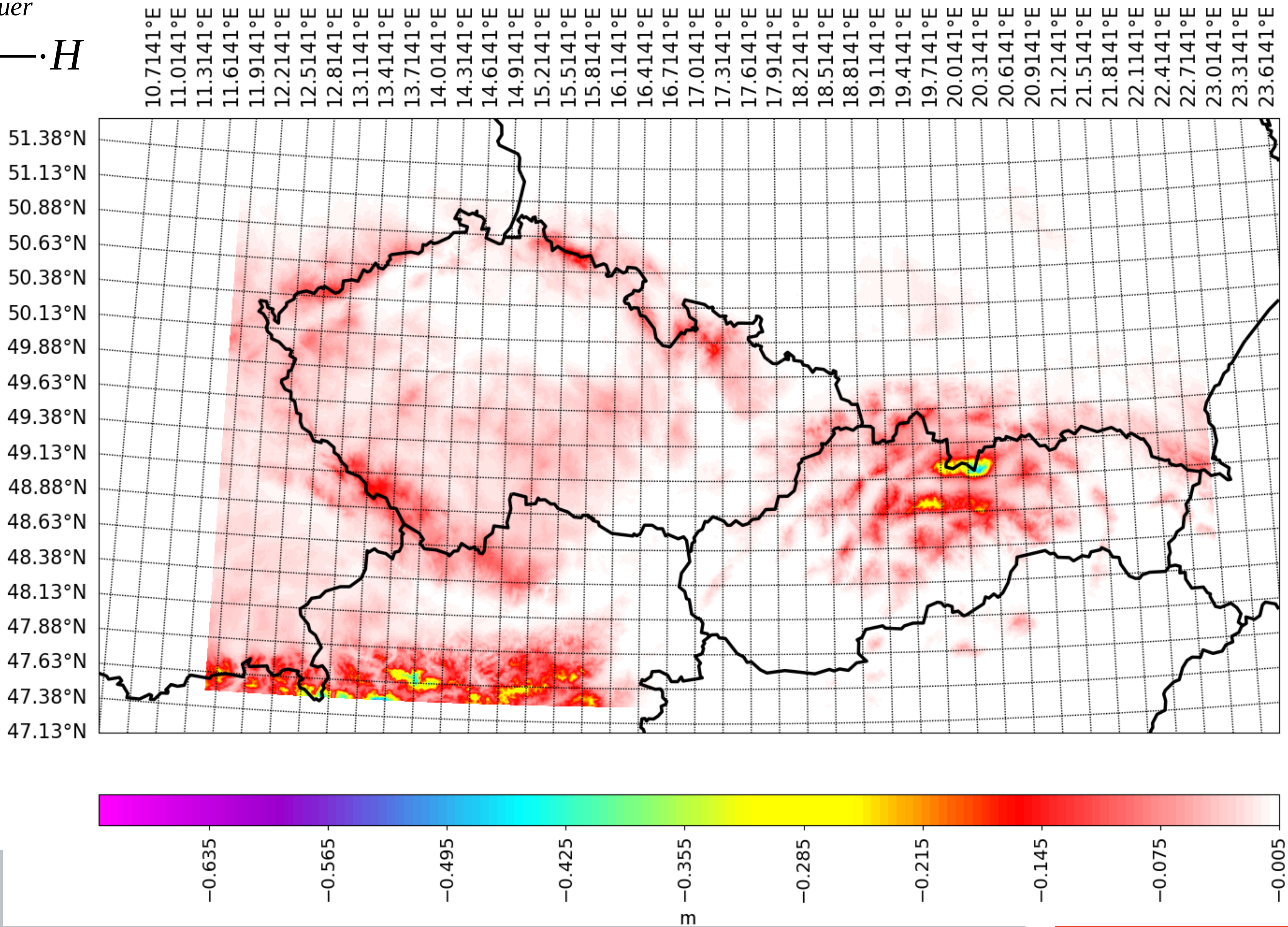
$$\begin{Bmatrix} V^*(r,\varphi,\lambda) \\ a_i^*(r,\varphi,\lambda) \\ M_{ij}^*(r,\varphi,\lambda) \end{Bmatrix} = \omega \begin{Bmatrix} K(P,Q) \\ L_i(P,Q) \\ N_{ij}(P,Q) \end{Bmatrix} + \frac{\omega}{24}\sum_{k=1}^{3} \Delta\xi_k{}^2 \begin{Bmatrix} \partial_k^2 K(P,Q) \\ \partial_k^2 L_i(P,Q) \\ \partial_k^2 N_{ij}(P,Q) \end{Bmatrix}\Bigg|_{\substack{r'=r_0 \\ \varphi'=\varphi_0 \\ \lambda'=\lambda_0}} + \begin{Bmatrix} O(\Delta^4/\ell_0^5) \\ O(\Delta^4/\ell_0^6) \\ O(\Delta^4/\ell_0^7) \end{Bmatrix}$$

Integration radius

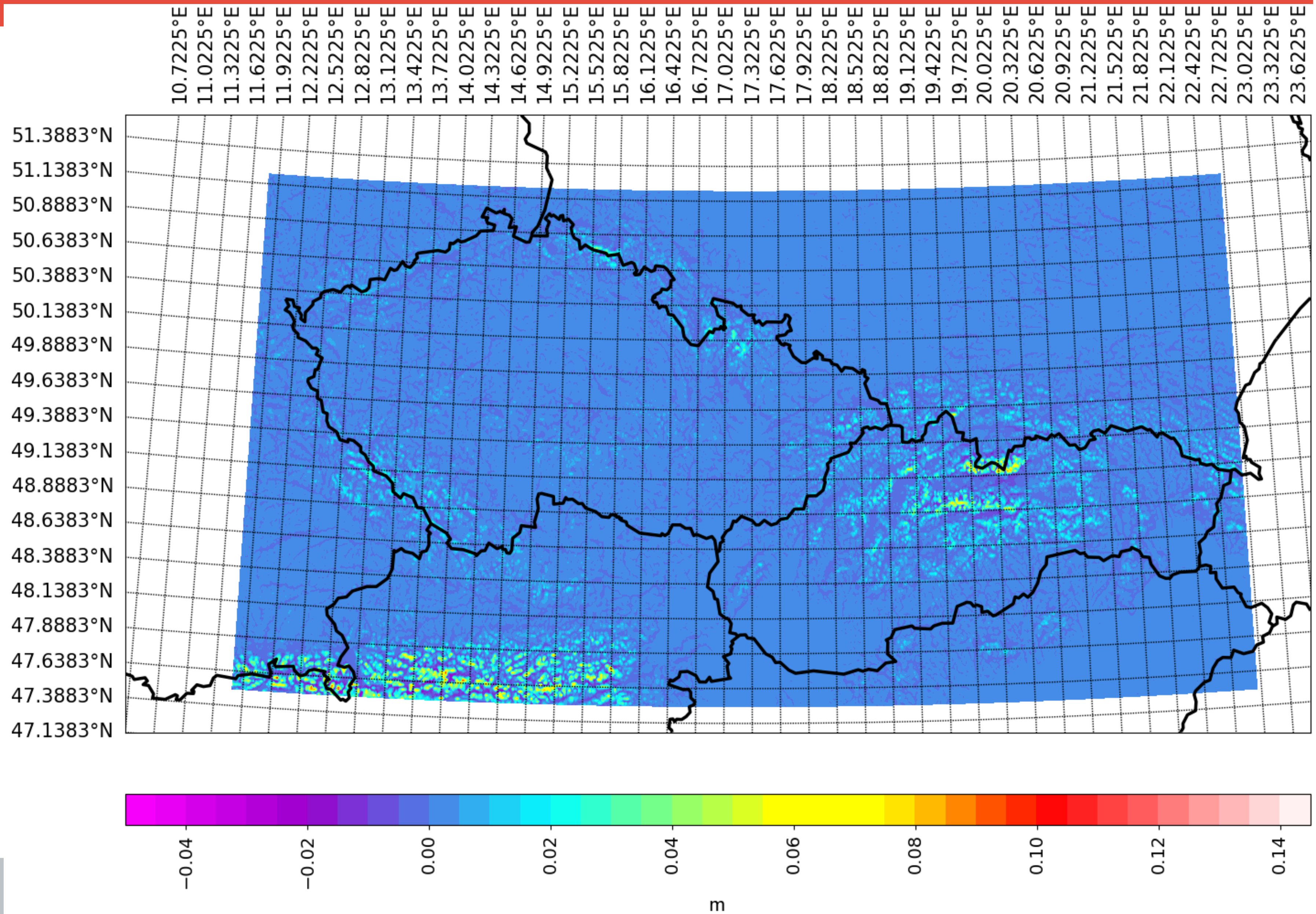


Geoid to quasigeoid separation

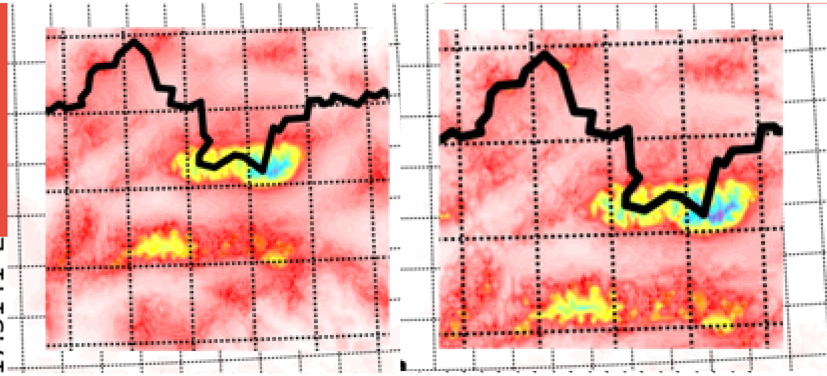
$$\frac{\Delta g_P^{Bouguer}}{\bar{\gamma}} \cdot H$$

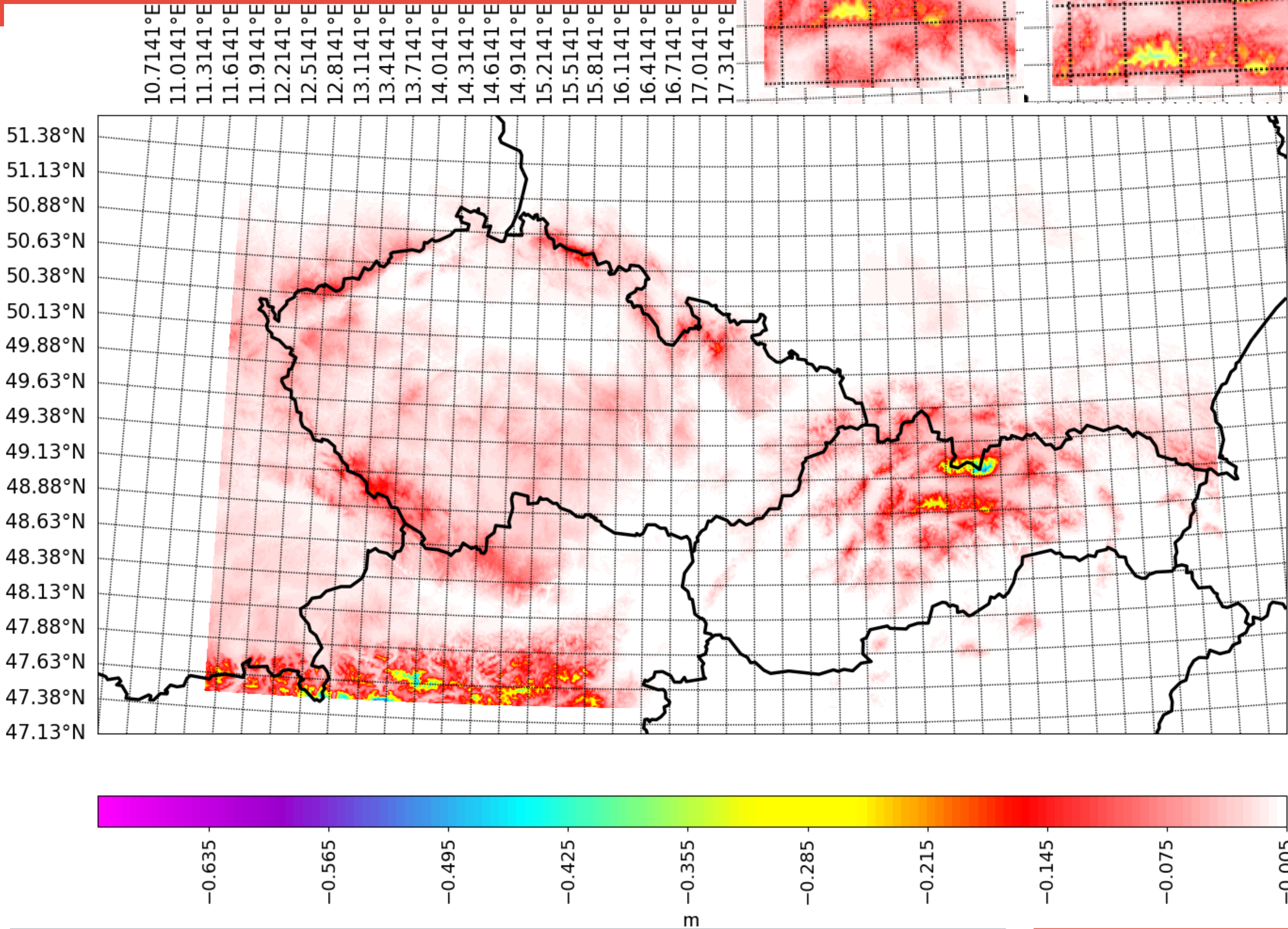


$$\frac{V^T(P_0)-V^T(P)}{\bar{\gamma}}$$

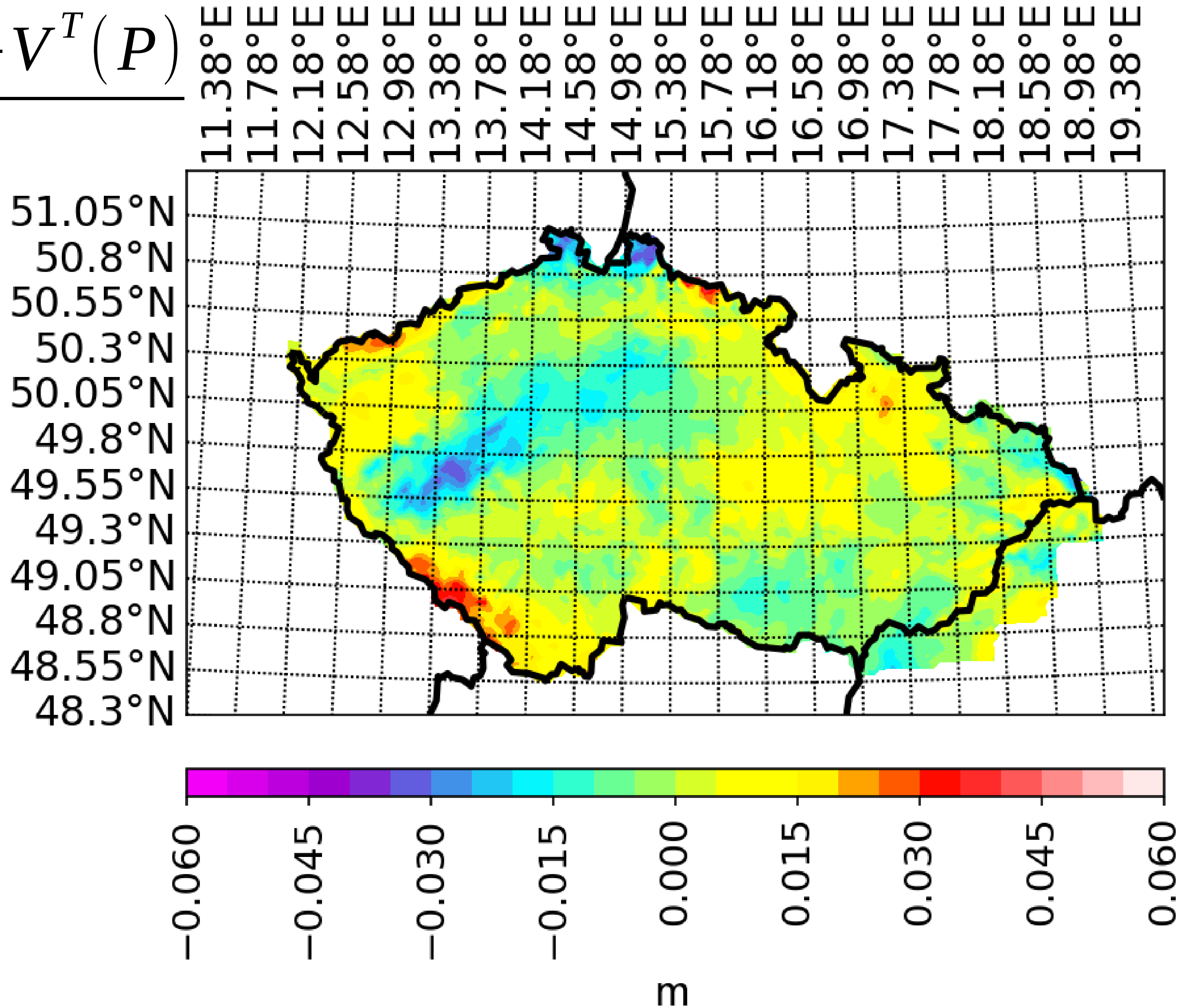


Sum of last two terms

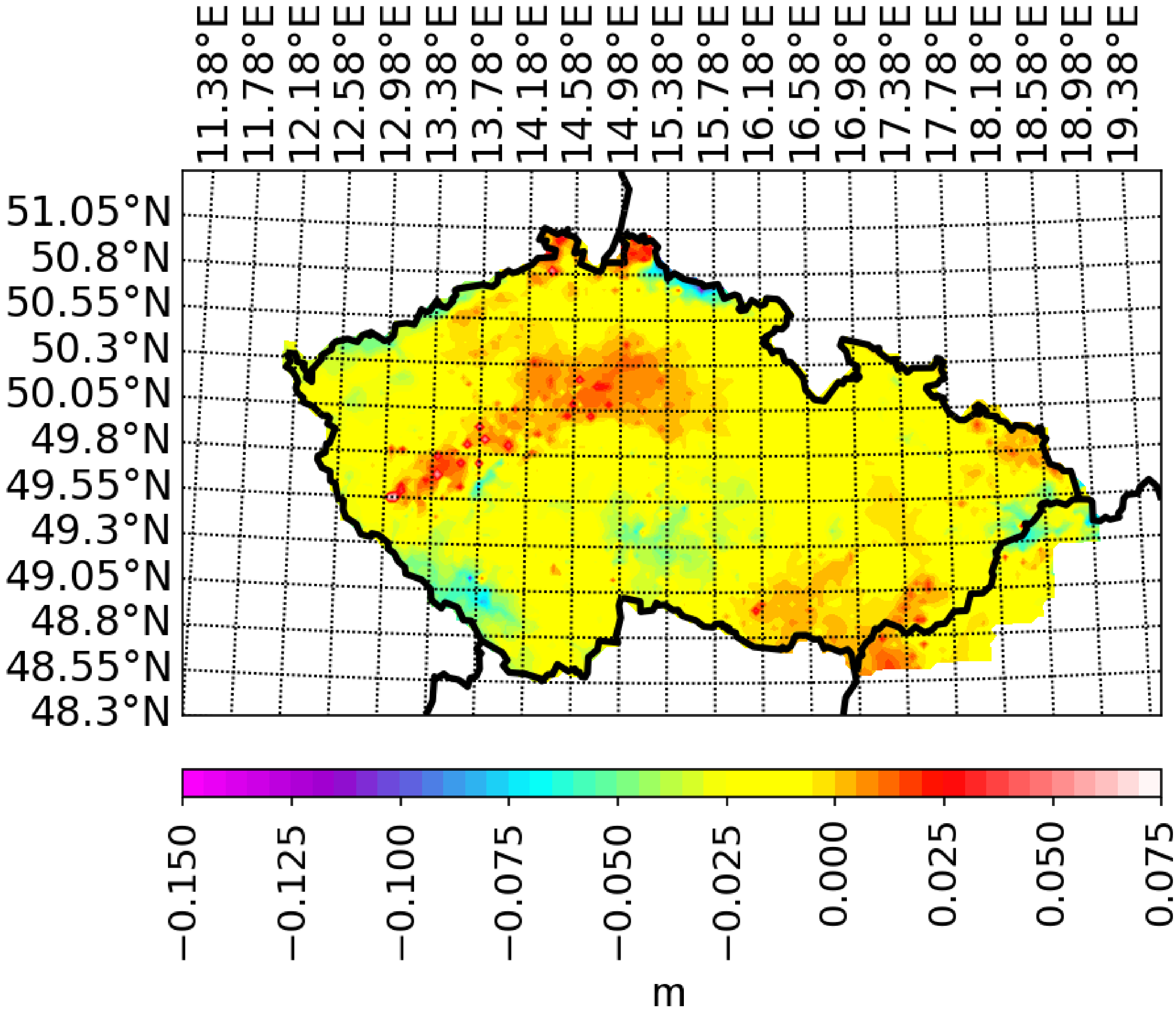




$$\frac{V^T(P_0)-V^T(P)}{\bar{\gamma}}$$



$$\frac{\Delta g_P^{Bouguer-refined}}{\bar{\gamma}} \cdot H$$



Conclusion

- The Bouguer anomaly should be refined

- For cm precision the effect of a condensation layer is required

- The global gravity field can be obtained from GGMF (low degrees 120,180,....)

Thank you for your attention

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