

# IMPACT OF THE CONTRACTION RATE OF THE FLOW AND THE LOCAL AND CRITICAL VELOCITIES ON THE SCOUR AT ABUTMENTS

**Boriss Gjunsburgs<sup>1\*</sup>, Gints Jaudzems<sup>1</sup>, Elena Govsha<sup>1</sup>**

<sup>1</sup> Riga Technical University, Kipsalas Str. 6B/6A, Riga, LV-1048, Latvia

## Abstract

The contraction of the river floodplain by embankment and bridge structures, when some part of floodplain discharge is blocked, leads to streamline concentration near these structures, increase in velocity, vortex structure, turbulence, general scour, and local scours. The hydraulic contraction rate of the flow  $Q/Q_b$  increases at a rising stage of the flood due to flow redistribution between the channel and floodplain. The aim of this study is to estimate the impact of hydraulic contraction rate of the flow and, consequently, the local and critical velocities on the local scour at abutments. Experiments in a flume confirmed a considerable effect of the contraction rate of flow on the local velocity, backwater, and the depth, width, and volume of scour near the vertical abutment wall in steady and unsteady flows under clear water condition and in a uniform sand river bed: with increasing contraction rate of the flow, the local velocity, backwater values, and scour depth increase. At the same contraction rate, but at a different  $Fr$  number, the scour depth is different: with increase in the  $Fr$  number, the local velocity, backwater, scour depth, width, and volume increase. All the results obtained are confirmed by tests and methods suggested and are presented in figures and tables. The value of geometrical contraction of the flow, i.e., the ratio of channel width to length of the abutment  $L_a/L_c$ , as accepted by some authors, at different discharges, different flow distribution between the channel and floodplain, and different  $Fr$  number is constant and cannot be used for a step-by-step calculation of the depth of scour development in time during floods at the abutments.

## Key words

abutments; grain size; riverbed stratification; scour depth

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\*Corresponding author: Tel.: +371-67-089-253, Fax: +371-67-089-253,  
E-mail address: boriss.gjunsburgs@rtu.lv

## 1 INTRODUCTION

The contraction of bridge crossing (by embankments and bridge structures, such as abutments, piers, guide banks, and spur dikes) leads to a reduction of the river cross-section area and is the reason for the flow modification and emergence of a contraction scour near, in the vicinity, and under the bridge, as well as to local scour at the bridge structures.

Approaching the bridge contraction, the streamlines are bended by an embankment, and the flow goes parallel to it. The flow velocities along the extreme streamline drop almost to minimum values and then gradually increase, which is accompanied by circulation and development of different vortex structures. At the corner of the abutment, streamline concentration, sharp water level drop, rapid local increase of the velocity and local scour are observed (Gjunsburgs & Neilands [1]). The local velocity at the abutments depends on the contraction rate of the flow and is considerably higher than the approach flow velocity.

The effect of contraction rate of the flow, both geometrical and hydraulic, on the scour at engineering structures in flow was investigated by using different approaches.

In some studies, the impact of contraction flow rate is disregarded (Breusers and Raudkivi [2], Melville [3], Kothiyari and Ranga Raju [4], Oliveto and Hager [5], Coleman et al. [6] and others); in other studies, the contraction flow rate is calculated as a ratio of geometrical parameters, such as embankment length  $L_a$  to the channel width  $L_c$  or abutment length, or as a geometrical obstruction ratio coefficient (see Liu et al. [7], Gill [8], Floehlich [9], Lim [10], Rahman and Haque [11], Balio et al. [12], etc.). The hydraulic contraction rate of the flow was suggested by Rotenburg et al. [13] as a ratio between the total discharge  $Q$  and discharge under the bridge or the blocked part of it  $Q_b$  for the discharge in floodplain in natural conditions. Sturm and Janjua [14], Kouchhakzadeh and Townsend [15] show that the ratio of the flow obstructed by abutment,  $Q_a$ , to the flow at a specific width near the tip of abutment,  $Q_w$ , is a significant parameter in estimating the equilibrium scour depth.

The aim of the present study is to estimate the effect of contraction rate of the flow and the local and critical velocities on the local scour at the abutments.

It is well known that, in the unsteady flow conditions, the discharge distribution between the channel  $Q_c$  and the floodplain  $Q_f$  undergoes changes during the flood and the contraction rate of the flow also changes in time and is different at any step of hydrograph. Under steady flow conditions, the redistribution of discharge due to scour in the channel and in floodplain under the bridge also leads to variations in contact rate. The geometrical contraction rate of the flow is constant under any flow conditions in flood and discharge redistribution because of scour. In this paper, the relative results of investigating the impact of the hydraulic contraction rate of the flow  $Q/Q_b$  (where  $Q$  is the flow discharge and  $Q_b$  is the discharge in the bridge opening under free surface flow conditions) are presented: the local scour  $h_s/h_f$  (where  $h_s$  is the depth of scour and  $h_f$  is the depth of water in the floodplain), the backwater value  $\Delta h/h_f$  (where  $\Delta h$  is the backwater value at the abutment), and the local velocity  $V_l/V_{ap}$  (where  $V_l$  is the local velocity of the flow near abutment and  $V_{ap}$  is the approach flow velocity); the effect of the  $Fr$  number of the flow on the scour development in time at the same contraction rate of the flow at the vertical wall of abutments in clear water conditions is also analyzed. The computer modeling shows that the hydraulic contraction rate of the flow due to discharge redistribution between the channel and floodplain affects significantly the depth of scour at the abutments.

According to the results of tests and computer modeling, the hydraulic contraction rate of the flow has a considerable effect on the local velocity, backwater, and scour depth, width, and volume at the vertical abutment wall in the case of steady and unsteady flow in clear water conditions and in the uniform sand river bed: with increase in the contraction rate of the flow, the local velocity, backwater values, and scour depth increase. At the same contraction rate, but at a different  $Fr$  number of the flow, the depth of scour is different: an increase in the  $Fr$  number leads to an increase in the local velocity, backwater, scour depth, width, and volume.

The geometrical contraction of the flow — the ratio between the channel width and length of the abutment  $L_d/L_c$ , upon different values of discharge distribution between the channel and flood plain and different  $Fr$  numbers in floods, is a constant value and cannot be used for a step-by-step calculation of the scour depth developed in time at the abutments during floods.

All the results obtained are confirmed by the tests and theoretical investigations and are presented in figures and tables.

## 2 EXPERIMENTAL SET-UP

The tests were carried out in a 3.5-m wide and 21-m long flume (Fig.1). The tests under free surface flow conditions were carried out to study the flow distribution between the channel and the floodplain.

The rigid bed tests were performed for different flow contractions and Froude numbers in order to investigate the changes in the velocity and water level in the vicinity of embankment, along it, and near the modeled abutments.

The aim of the sand bed tests was to study the scour process at the corner of the abutment, the changes in the local velocity, the effect of different flow parameters, the contraction rate, and the grain size of bed material on the scour near vertical abutment wall.

The openings of the bridge model were 50, 80, 120, and 200 cm. The flow contraction rate  $Q/Q_b$  varied respectively from 1.56 to 5.69 for a floodplain depth of 7 and 13 cm, and the Froude numbers varied from 0.078 to 0.134; the slope of the flume was 0.0012.

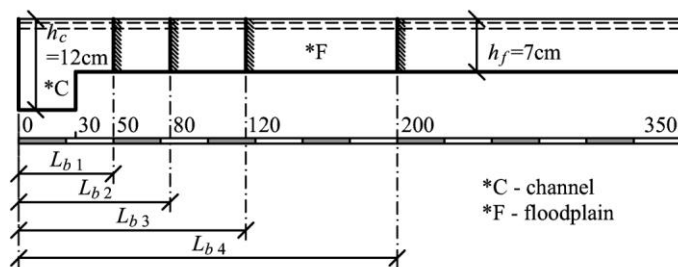


Fig. 1: Laboratory flume cross section with model bridge openings 50-200

The sand bed tests were carried out under clear-water conditions. The sand was placed 1 m up and down the contraction of the flume. The mean grain size was 0.24 and 0.67 mm. The condition that  $Fr_R = Fr_f$  was fulfilled, where  $Fr_R$  and  $Fr_f$  are the Froude numbers for the plain river and for the flume, respectively. The tests in the flume lasted for 7 hours, the length scale was 50 and the time scale was 7. With respect to the real conditions, the test time was equal to 2 days. This was the mean duration of time steps into which the flood hydrograph was

divided. Experimental data for the free surface flow conditions in a flume are presented in Table 1.

*Tab. 1: Experimental data for the open flow conditions in a flume*

Tests	$L$ (cm)	$h_f$ (cm)	$V$ (cm/s)	$Q$ (l/s)	$Fr$	$Re_c$	$Re_f$
L1	350	7	6.47	16.60	0.0780	7500	4390
L2	350	7	8.58	22.70	0.1030	10010	6060
L3	350	7	10.30	23.60	0.1243	12280	7190

The development of a scour was examined with different flow parameters in time intervals within one 7-h step and within two steps of the hydrograph, 7 hours each.

The tests were carried out with one floodplain model and one side contraction of the flow and with two identical or different floodplain models and two side contractions. The position of the main channel was varied for different tests.

### 3 METHOD

#### Scour development in multiple floods.

The differential equilibrium equation of the bed sediment movement in clear water was used, and a method for calculating the scour development with time at the bridge crossing structures (abutments and guide banks) during the multiple floods was elaborated and confirmed by experimental data (Gjunsburgs et al. [1], [16]).

Differential equation of equilibrium of the bed sediment movement for clear water conditions reads

$$\frac{dw}{dt} = Q_s \cdot \quad (1)$$

According to laboratory tests,  $w = 1/6 \pi m^2 h_s^3$ ;  $t$  = time;  $Q_s$  = sediment discharge out of a scour hole;  $m$  = steepness of a scour hole.

The left-hand part of Eq. (1) can be written as

$$\frac{dw}{dt} = \frac{1}{2} \pi m^2 h_s^2 \frac{dh_s}{dt} = a h_s^2 \frac{dh_s}{dt}, \quad (2)$$

where  $h_s$  = depth of scour;  $a = 1/2 \pi m^2$ . The sediment discharge  $Q_s$  was determined by the Levi [17] formula

$$Q_s = AB \cdot V_l^4, \quad (3)$$

where  $B = mh_s$  = width of a scour hole;  $V_l$  = local velocity at the abutment;  $A$  = a parameter in the Levi [17] formula:

$$A = \frac{5.62}{\gamma} \left( 1 - \frac{\beta V_0}{V_l} \right) \frac{1}{d^{0.25} h_f^{0.25}}, \quad (4)$$

where  $\gamma$  = specific weight of sediments;  $\beta$  = reduction coefficient of velocity  $V_0$  due to the vortex system;  $V_0$  = velocity required to start a sediment movement;  $V_l$  = local velocity;  $d$  = grain size of the bed material;  $h_f$  = water depth in the floodplain.

$$Q_s = A m h_s \cdot V_l^4 = b \frac{h_s}{k \left( 1 + \frac{h_s}{2h_f} \right)^4}, \quad (5)$$

where  $b = Am \cdot V_l^4$ ;  $k$  = coefficient of variations in discharge because of scour. During the flood, the hydraulic characteristics, contraction rate of the flow, velocities  $V_0$  and  $V_l$ , grain size in different bed layers, sediment discharge, as well as the depth and width of the scour, changed.

In the process of scour development, we have for  $A$ :

$$A = \frac{5.62}{\gamma} \cdot \left[ 1 - \frac{\beta V_0}{V_l} \left( 1 + \frac{h_s}{2h_f} \right)^{1.25} \right] \cdot \frac{1}{d^{0.25} \cdot h_f^{0.25} \left( 1 + \frac{h_s}{2h_f} \right)^{0.25}}. \quad (6)$$

Differential Eq. (1), according to formulas (2) and (5), can be presented as

$$a h_s^2 \frac{dh_s}{dt} = b \frac{h_s}{k \left( 1 + \frac{h_s}{2h_f} \right)^4},$$

or

$$D_i \cdot h_s \left( 1 + \frac{h_s}{2h_f} \right)^4 dh_s = dt. \quad (7)$$

where  $D_i = a/b$ .

After integration,

$$t = D_i \int_{x_1}^{x_2} h_s \left( 1 + \frac{h_s}{2h_f} \right)^4 dh_s. \quad (8)$$

In our tests, the hydrograph was divided into time steps, and each step was divided into small time intervals. It was assumed that, inside a time interval,  $D$  is constant.

The integration with new variables  $x = 1 + h_s/2h_f$ ,  $h_s = 2h_f(x-1)$ , and  $dh_s = 2h_f dx$  yields

$$N_i = \frac{t_i}{4D_i h_f^2} + N_{i-1}, \quad (9)$$

where  $N_i = 1/6x_i^6 - 1/5x_i^5$ ,  $t_i$  is the time interval,  $D_i$  is a constant parameter in a steady-flow time step, and  $h_f$  is the flow depth in floodplain.

Using the graph  $N = f(x)$  for calculated  $N_i$ , we determine the values of  $x_i$  and scour depth at the end of time interval:

$$h_s = 2h_f(x-1) \cdot k_m \cdot k_s \cdot k_\alpha, \quad (10)$$

where  $k_m$  is a coefficient depending on the side-wall slope of the abutment;  $k_s$  is a coefficient depending on the abutment shape (Richardson and Davis [18]); and  $k_\alpha$  is a coefficient depending on the angle of flow crossing (Richardson and Davis [18]).

According to the method used, the scour development in time depends on the flow hydraulics contraction rate, the river-bed parameters, side-wall slope, flow crossing angle, as well as on the probability, sequence, frequency, and duration of multiple floods (Gjunsburgs et al. [16]).

### Local and critical velocities

At the corner of the abutments, the streamline concentration, a sharp drop in water level, and a rapid increase in velocity were observed.

To calculate the local velocity, we used the Bernoulli equation for two cross sections of an extreme unit streamline. The local velocity at the nose corner of abutments, for the plain river bed, was found from the following formula (Gjunsburgs and Neilands [1]):

$$V_l = \varphi \sqrt{2g\Delta h}, \quad (11)$$

where  $\varphi$  = velocity coefficient, which depends on contraction rate of the flow;  $\Delta h$  = backwater value, which depends on contraction rate of the flow (Rotenburg et al. [13]). The results of tests obtained for different values of opening width, discharge, depth, and Froude numbers of the flow are in good agreement with the formula for maximum backwater value.

The critical velocity  $V_0$  in the plain river bed is determined by the Studenitcnikov [19] formula:

$$V_0 = \beta 1.15 \sqrt{g} d^{0.25} h_f^{0.25}, \text{ or } V_0 = \beta \cdot 3.6 d_i^{0.25} h_f^{0.25}, \quad (12)$$

where  $\beta$  = reduction coefficient of the critical velocity in bended flow;  $d$  = mean grain size on the top of the river bed;  $h_f$  = water depth in the floodplain.

To find the changes in local velocity due to scour, we calculated the discharge across the width of a scour hole before and after some scour depth  $Q_f = kQ_{se}$ , where  $Q_f$  is the discharge

across the width of the scour hole with the plain bed and  $Q_{se}$  is discharge with an arbitrary depth of scour:

$$m \cdot h_s \cdot h_f \cdot V_l = k \left( m \cdot h_s h_f + \frac{m \cdot h_s}{2} h_s \right) V_{lt}, \quad (13)$$

where  $m$  = steepness of the scour hole;  $h_s$  = scour depth;  $h_f$  = depth of flow in the floodplain;  $V_l$  = local velocity in the plain river bed;  $V_{lt}$  = local velocity at any scour depth.

The local velocity  $V_{lt}$  at any scour depth is determined from Eq. (13):

$$V_{lt} = \frac{V_l}{k \left( 1 + \frac{h_s}{2h_f} \right)}. \quad (14)$$

A comparison between the tests data for the local velocity at any scour depth and those calculated by Eq. (14) gave good results.

The critical velocity  $V_{0t}$  varies in time and, at any scour depth, can be determined through the mean depth of flow  $h_m = h_f (1 + h_s / 2h_f)$  near the abutments

$$V_{0t} = \beta \cdot 3.6 d_i^{0.25} h_f^{0.25} \left( 1 + \frac{h_s}{2h_f} \right)^{0.25}, \quad (15)$$

where  $d_i$  = grain size of the bed;  $V_0 = 3.6 d_i^{0.25} h_f^{0.25}$  is the critical velocity in the plain bed (Studenitcnikov [19]).

The scour development stops when the local velocity  $V_{lt}$  (Eq. 14) becomes equal to the critical velocity  $\beta V_{0t}$  (Eq. 15). According to the test results and methods presented, it is found that the local velocity at the abutments considerably depends on the hydraulic contraction rate of the flow.

## 4 RESULTS

Table 2 presents some tests results for different values of contraction rate of the flow. It is seen that, with increase in the hydraulic contraction rate of the flow, the backwater value, local velocity, and scour depth increase.

*Tab. 2: Comparison of some experimental and calculation results at different values of contraction rate  $Q/Q_b$*

Test	$Q/Q_b$	$\Delta Z_{test}$ (cm)	$\Delta h_{calc}$ (cm)	$\Delta Z_{test} / \Delta h_{calc}$	$V_{l\ test}$ (cm/s)	$V_{l\ calc}$ (cm/s)	$V_{l\ test} / V_{l\ calc}$	$h_s\ test$ (cm)	$h_s\ calc$ (cm)	$h_s\ test / h_s\ calc$
AL1	5.27	2.40	2.20	1.09	39.3	36.80	1.07	13.33	13.30	1.00
AL4	3.66	1.36	1.19	1.14	36.0	33.04	1.09	9.79	10.10	0.97
AL7	2.60	0.58	0.60	0.97	25.1	26.76	0.94	5.99	5.94	1.01
AL10	1.56	0.32	0.35	0.91	21.6	22.08	0.98	2.97	3.08	0.96

With development of scour in time, under steady flow conditions, the local velocity reduces, but the critical one increases. The velocities become more similar with increasing depth of scour. The relative depth of scour increases with the contraction rate of the flow (Fig. 2).

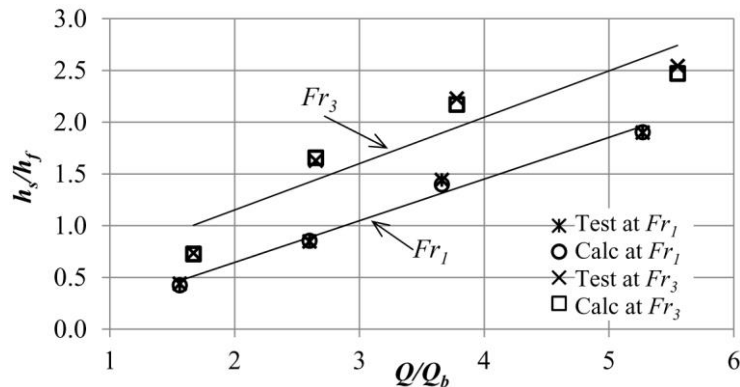


Fig. 2: Relative scour depth  $h_s/h_f$  vs the contraction rate of the flow,  $Q/Q_b$ , obtained in tests and calculation.

Based on the method presented, a computer modeling was carried out for estimating the effect of contraction rate of the flow on the depth of scour at different values of discharge distribution between the channel and floodplain.

Table 3 presents the effect of contraction rate of the flow,  $Q/Q_b$ , at different discharge distributions between the channel and floodplain, on the backwater, local velocity, and scour depth. With increase in the discharge in the channel and its reduction in the floodplain, under steady flow conditions, the hydraulic contraction rate of the flow, the backwater value, local velocity, scour depth at 7 h, and the equilibrium depth of scour were found to decrease.

Tab. 3: Influence of the discharge distribution between the channel and floodplain on the scour depth

$Q_c\%$	$Q_f\%$	$L_a$ (cm)	$Q$ (l/s)	$Q/Q_b$	$\Delta h$ (cm)	$V_l$ (cm/s)	$h_s$ (7 h) (cm)	$h_{equil}$ (cm)	$L_a/b$
5%	85%	300	27.14	5.55	5.00	52.44	20.30	26.80	6
20%	80%	300	27.14	4.00	3.18	52.12	18.32	24.05	6
30%	70%	300	27.14	2.91	2.30	49.79	15.72	20.26	6
40%	60%	300	27.14	2.29	1.85	47.61	14.55	18.60	6
50%	50%	300	27.14	1.88	1.55	44.24	13.05	16.40	6
60%	40%	300	27.14	1.60	1.30	40.80	11.48	14.18	6
80%	20%	300	27.14	1.23	0.83	32.90	7.98	9.38	6

The geometrical contraction rate  $L_a/b$  is constant at any redistribution of discharge between the channel and floodplain; it does not affect the backwater value, local velocity, and scour depth at 7 h or in equilibrium stage.

The relative local velocity  $V_l/V_{ap}$  increases with the contraction rate of the flow (Fig. 3). With increase in the  $Fr$  number, the ratio between the local and approach velocities also increases.



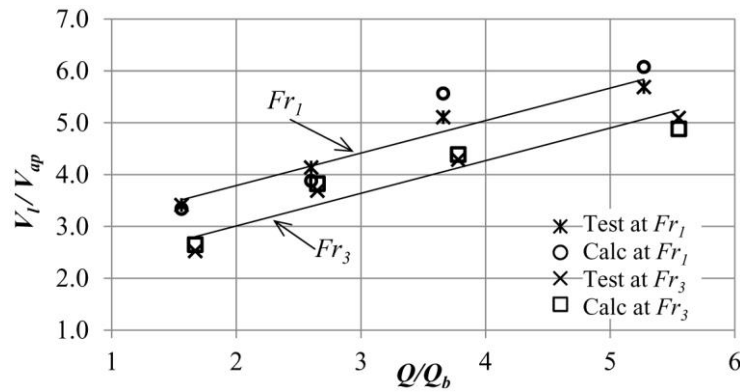


Fig. 3: Relative local velocity vs contraction rate of the flow.

## 5 CONCLUSIONS

The contraction of river flow by the bridge crossing (embankments and bridge structures: abutments, piers, guide banks, and spur dikes) leads to a reduction of the cross-sectional area. This factor is the reason for the flow modification, the scour contraction near, in the vicinity of, and under the bridge, and the local scour at the bridge structures. The hydraulic contraction rate of the flow  $Q/Q_b$  increases in flood, at its rising stage, and varies with redistribution of the flow between the channel and floodplain. In the steady flow conditions, with different discharge distribution between the channel and floodplain, the contraction rate of the flow changes.

The results obtained in tests by the method presented and the data of computer modeling confirmed that the hydraulic contraction rate of the flow considerably affects the depth, width, and volume of a scour hole at the engineering structures in flow. An increase in the contraction rate of the flow rate increases the relative local depth of scour  $h_s/h_f$  (Fig. 2) at the vertical wall abutments under clear water conditions, the ratios between the backwater value and water depth in the floodplain,  $\Delta h/h_f$ , and between the local velocity at the abutment and the approach flow velocity,  $V_l/V_{ap}$ , (Fig. 3); the increased  $Fr$  number of the flow (at the same contraction rate) increases the depth, width, and volume of a scour hole at the abutment.

The geometrical contraction of the flow, i.e., the ratio of the channel width to length of the abutment,  $L_a/L_c$ , at different discharge distributions between the channel and flood plain and different  $Fr$  number in floods has a constant value and cannot be used for a step-by-step calculation of the scour depth developed in time during floods at the abutments. The effect of contraction rate of the flow on the value of scour depth at the abutments cannot be neglected in calculating the scour near bridge structures in flow in plain rivers.

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