TRC-SPECIMENS MODELED AS A CHAIN OF CRACKS BRIDGED BY BUNDLES

Study of impact of local scatter on global tensile strength

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Abstract: The present paper shows the correspondence between the short-range size effect occurring in crack bridge and the long-range size effect observed on a textile reinforced concrete (TRC) tensile specimens. In the analysis of the effects we exploit the fact that the specimen acts as a chain of crack bridges in its failure state. The cracks are bridged by bundles consisting of several thousands of filaments that are embedded in a cementitious matrix. We include the effect of scatter of filament properties over the bundle cross section in a crack bridge and study its influence on the ultimate failure of the reinforced specimen.

Key words: fiber bundle model, statistical size effect, chain of bundles

1. INTRODUCTION

The crucial aspect in determining the ultimate load carrying capacity of textile reinforced concrete (TRC) specimens is a correct description of the hot-spots of strain and damage, e.g. of the crack bridges. In the final stages of the tensile loading with a finished crack pattern the specimen may be viewed as a chain of crack bridges with its strength governed by the "weakest-link" concept. The experimental results on tensile TRC specimens show a significant loss of bundle efficiency that are usually ascribed to damage or low penetration of the bundle by the matrix combined with insufficient anchorage in the boundary layers (Hegger et al., 2005). This paper contributes to the discussions about the reasons for the strength reduction by studying the weakest link effect in a chain of crack bridges with scatter of strength.

The chained crack bridges are realized by multi-filament bundles exhibiting a considerable scatter of strength due to a highly heterogeneous nature of the material structure in the yarn, in the bond layer and in the cementitious matrix. As documented in Chudoba et al. (2005) on experimental and numerical studies, these sources of randomness can lead to a substantial reduction of the bundle strength especially for extremely short nominal lengths occurring in a case of a crack bridge. Therefore, the evaluation of the impact of varying material properties of bridges in a serial ordering on the global strength is inevitable.

In order to demonstrate the correspondence between the statistics of the global response and the local scatter in the crack bridge we first review the weakest link concept in Sec. 2. The simple modeling framework allows us to study the change of the ultimate strength with an increasing number of cracks N for different levels of scatter of local filament properties in the crack bridges. In Sec. 3 we use an analytical strain based fiber bundle model to evaluate the probability density function of a single crack bridge strength. Another example in this section uses a more sophisticated model including the debonding effect. Both examples document the general applicability of the procedure for evaluating the chain effect in a TRC loaded in tension.

2. GENERAL DETERMINATION OF TRC STRENGTH STATISTICS

As documented in Fig. 1 the tensile specimen exhibits very fine crack pattern. Obviously, the ultimate failure is governed by the weakest-link statistics. Therefore, the survival probability of the chain with N cracks may be obtained as a product of survival probabilities of the individual cracks:

$$1 - P_{f,N} = \left(1 - P_{f,1}\right)^N \tag{1}$$

where $P_{f,1} \equiv F_1(\sigma)$ represents the failure probability of a single crack bridge (cumulative strength distribution). The load level σ for a given number of crack bridges N and probability of failure $P_{f,N}$ can be computed with the inverse cumulative strength distribution function of a crack bridge:

$$\sigma = F_1^{-1}\left(1 - \sqrt[N]{1 - P_{f,N}}\right) \tag{2}$$

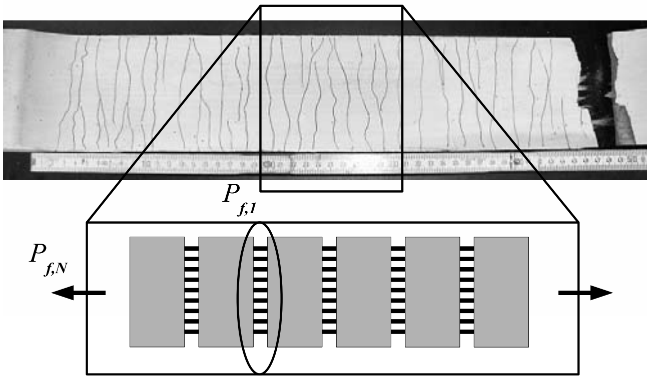

Figure 1. Crack pattern of a failed tensile specimen reductions by studying the weakest link effect in a chain of crack bridges with scatter of strength.

In case of normally distributed strength of a crack bridge (short filament bundle) the load level of a chain of bridges with failure probability $P_{f,N}$ reads simply:

$$\sigma = \Phi^{-1}\left(1-\sqrt[N]{1-P_{f,N}}\right) \tag{3}$$

This formula provides a general procedure for estimating the strength statistics of TRC with *N* crack bridges, each with the failure probability distribution $P_{f,1} \equiv F_1(\sigma)$. This distribution is obtained using the statistical strain-based bundle model (Phoenix and Taylor 1973) as follows

1. Given a single-filament response function (constitutive law) $q(e;\boldsymbol{\theta})$ as a function of strain e and a vector of random (or deterministic) quantities $\boldsymbol{\theta}$ with their corresponding distribution functions $G_{\theta,i}(\theta_i)$ compute the mean response of a filament within the bundle (normalized bundle force) as a *k*-fold integral over *k*-number of nondeterministic variables of the model $q(e;\boldsymbol{\theta})$:

$$\mu_{\boldsymbol{\theta}}(e;\boldsymbol{\theta}) = \int_{\boldsymbol{\theta}} q(e;\boldsymbol{\theta})\,\mathrm{d}G_{\boldsymbol{\theta}}(\boldsymbol{\theta}) \tag{4}$$

2. Find the local maximum of the mean response (bundle strain $e = e^*$ at which the maximum force is attained). This can be done either by seeking the stationary point of $\mu_{\boldsymbol{\theta}}(e;\boldsymbol{\theta})$ in case of analytical expression, see examples in Chudoba et al. (2005) and Vořechovský & Chudoba (2005) or numerically by seeking the peak force value.

3. Evaluate the mean bundle response function at bundle strain $e = e^*$ to get the mean bundle strength: $\mu_\theta^* = \mu_\theta\left(e^*\right)$ and compute the bundle strength variance as:

$$\Gamma_\theta^* = \Gamma_\theta\left(e^*, e^*\right) = \int_{\boldsymbol{\theta}} \left[q\left(e^*; \boldsymbol{\theta}\right) - \mu_{\boldsymbol{\theta}}^* \right]^2 \, \mathrm{d}G_{\boldsymbol{\theta}}\left(\boldsymbol{\theta}\right) \tag{5}$$

4. Estimate the whole cumulative density function $P_{f,1}$. In most cases it suffices to consider Gaussian distribution for the "middle" part (core) (0.1 to 0.9 percentiles) so that the probability of failure at a given load level σ reads:

$$P_{f,1}\left(\sigma\right) \equiv F_1\left(\sigma\right) = \Phi\left(\frac{\sigma - \mu_\theta^*}{\sqrt{\Gamma_\theta^*}} \right) \tag{6}$$

where $\Phi\left(\bullet\right)$ = standard Gaussian cumulative distribution function.

Note that the distribution (of bundle strength $P_{f,1}$) can be estimated numerically by means of Monte Carlo simulation. One can evaluate the bundle response N_{sim} times and save the peak forces (of all simulations). Then the CDF of bundle strength can be estimated by an empirical cumulative histogram of the peak sample.

3. EXAMPLES FOR SELECTED CRACK BRIDGE MODELS IN A CHAIN

In order to demonstrate the procedure and the significance of the size effect in a TRC specimen we now provide two examples of a crack bridge model with the response function represented both analytically and numerically.

Example 1. Bundle model with perfect clamping: The scatter of filament stiffness parameters leads to a reduced strength of a bundle with a very short length. In a crack bridge the strength reduction is in particular caused by the scatter of filament lengths and their delayed activation (slack). Besides that, the scatter of filament strength across the bundle leads to a further reduction of the bundle strength (Smith and Phoenix 1981). In order to demonstrate the effect on an example we consider a crack bridge with a scatter of filament lengths. In particular, we introduce the relative difference of the filament length with respect to the nominal length as $\lambda = \left(l - l_i\right)/l$ with a uniform distribution, i.e. $G_\lambda\left(\lambda\right) = \lambda / \lambda_{\mathrm{max}}$ where $0 \le \lambda \le \lambda_{\mathrm{max}}$. The other parameters of the filament, i.e. Young's modulus E, area A and breaking strain ξ are considered constant. For the chosen distribution, it is possible to derive analytical formulas for the bundle mean strength and its variance (see Chudoba et al. 2005) at a given control strain e as

$$\begin{aligned}\mu_\lambda(e;\lambda) &= \int_\lambda q(e;\lambda)\,\mathrm{d}G_\lambda(\lambda) \\ &= \begin{cases} EAe\ln(1+\lambda_{\max})/\lambda_{\max} & 0 \le e \le \xi\ (\text{linear}) \\ EAe\dfrac{\ln(1+\lambda_{\max})-\ln(e/\xi)}{\lambda_{\max}} & e > \xi\ (\text{nonlinear}) \end{cases}\end{aligned} \tag{7}$$

The calculated normalized mean load strain diagrams are exemplified in Fig. 2 for several levels of scatter represented by $\lambda_{\max}$. Obviously, the crack bridge strength/efficiency rapidly decreases with an increasing $\lambda_{\max}$.

In the derivation of the crack bridge strength distribution we shall exploit the fact that the maximum mean bundle strength is attained at the global strain $e^* = \xi$ for the uniform distributions with $\lambda_{\max} \le 1.71$ and has the simple form:

$$\begin{aligned}\mu_\lambda(e^*) &= \int_\lambda q(e^*;\lambda)\,\mathrm{d}G_\lambda(\lambda) \\ &= EA\xi\ln(1+\lambda_{\max})/\lambda_{\max}\end{aligned} \tag{8}$$

The corresponding variance (see Phoenix and Taylor 1973) is obtained as

$$\begin{aligned}\Gamma_\lambda^* &= \int_\lambda \left[q(e^*;\lambda)-\mu_\lambda(e^*)\right]^2 \mathrm{d}G_\lambda(\lambda) \\ &= (EA\xi)^2\left[\frac{1}{1+\lambda_{\max}}-\frac{\ln^2(1+\lambda_{\max})}{\lambda_{\max}^2}\right]\end{aligned} \tag{9}$$

With reference to the central limit theorem we can expect the convergence of the crack bridge strength distribution to the normal distribution, see Eq. 6. The verification of the convergence to the Gaussian distribution has been done by Monte Carlo simulation in connection with the deterministic bundle model described in Chudoba et al. (2005).

With the crack strength distribution at hand we can approach to the quantification of the chain statistics. Using Eq. (3) we quantify the interaction of the short-range size effect due to a local scatter $(\lambda_{\max})$ with the chaining of crack bridges in a tensile specimen. In Fig. 4 we plot the crack bridge efficiency (reduction of strength with respect to a perfect crack bridge) for the failure probability $P_{f,N}$ = 0.5 (median) and the levels of the scatter parameter $\lambda_{\max}$ studied previously in Fig. 2.

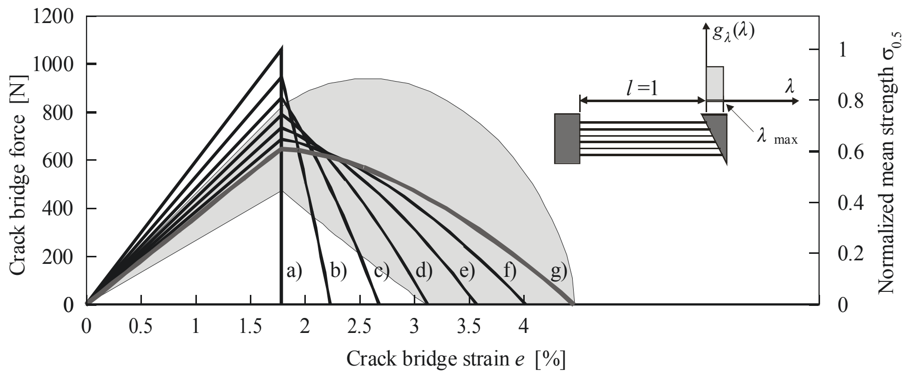

Figure 2. Mean force-strain diagrams of one crack bridge with uniform distribution of additional fiber length $\lambda \in (0, \lambda_{\max})$. $\lambda_{\max}$= a) 0.0, b) 0.25, c)0.5, d) 0.75, e)1.0, f) 1.25 and g) 1.5 plotted with a scatterband (mean ± standard deviation).

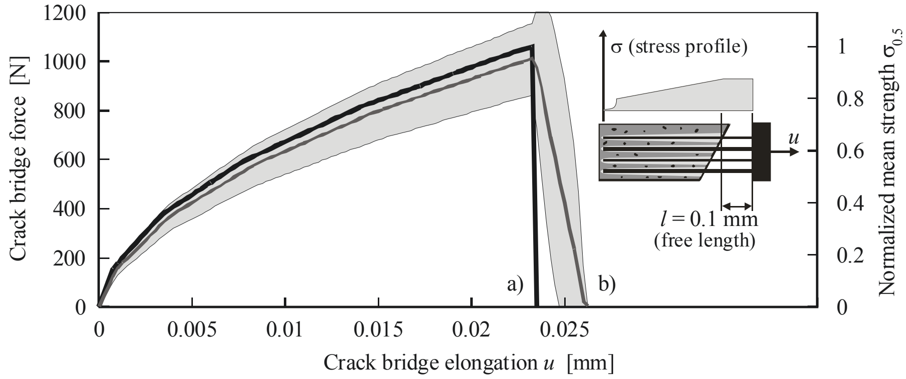

Figure 3. Mean force-strain diagrams ± one standard deviation of one crack bridge with debonding model and with uniform distribution of additional fiber length $\lambda \in (0, \lambda_{\max})$. $\lambda_{\max}$ = a) 0.0, b) 1.5.

Example 2. Bundle model with debonging: The procedure for evaluating the total strength described in Sec. 2 can be used with more complex idealizations of a crack bridge taking into account further failure and damage mechanisms. In addition to filament rupture considered in the previous model we now include the influence of debonding between filament and matrix. The response function $q(e;\boldsymbol{\theta})$ is represented by a finite element idealization of the shear lag with a cohesive interface between the filament and matrix. In order to evaluate the integrals (Eqs. 7 and 8) a general numerical integration tool has been implemented to obtain the statistical characteristics of a crack bridge strength (Konrad et al., 2006). We remark that the crack bridge model provides the possibility to study the impact of the variability in any of the model parameter(s) on the statistics of the overall bundle response (load displacement diagram), not only on the length that is used in this paper.

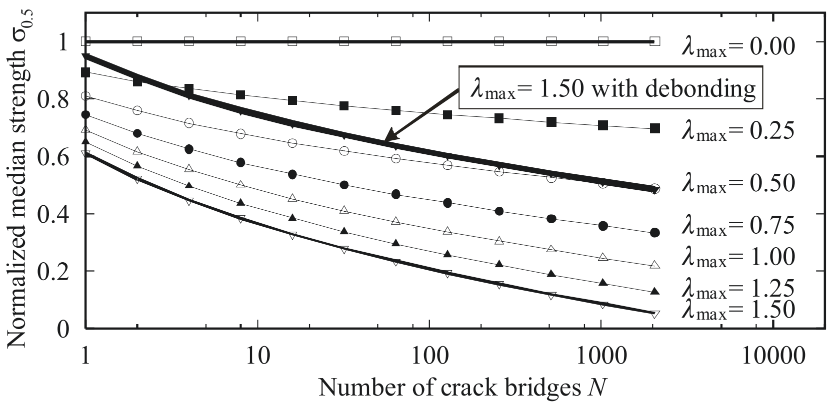

Figure 4. Median chain strength for varying number of cracks and scatter λ_{max} ($P_{f,N}$ = 0.5)

With $G_{\lambda}(\lambda)$ defined as in the example 1, we now include the debonding of a filament from the matrix as an additional effect. Again we keep all other parameters including the interface characteristics, constant. The resulting load-displacement diagram displayed in Fig. 3b for $\lambda_{max} = 1.5$ shows a significantly higher mean crack bridge strength (1008 N, see Fig. 3b) than in the case of a perfect bond (648 N, see Fig. 2g). The reason for such an increased strength is the homogenizing effect of the debonding causing a more uniform stress distribution across the bundle (more filaments can act simultaneously before they break). We also note that there is no significant reduction in the scatter of strength: 173.99 N for perfect bond and 145.08 N with included debonding.

The performance of a chain of crack bridges with and without debonding is compared for $\lambda_{max} = 1.5$ in the semi-logarithmic plot in Fig. 4. Due to a similar amount of scatter, the slope of the two size-effect curves is almost the same. In other words, while the local debonding improves the mean strength by introducing stress redistribution during the failure process, the decay of strength with the increasing number of cracks remains almost the same. The size effect curve is simply shifted upwards.

4. CONCLUSIONS

The paper shows the correspondence between local scatter in a crack bridge of a textile-reinforced tensile specimen and the resulting reduction of the specimen tensile strength. The presented approach provides a rough estimation and explanation of the strength reduction of tensile specimens with dry yarn reinforcement with a high amount of imperfections in the bundle structure and in its bond to the cementitious matrix. If we consider a usual range of lengths of tensile structural elements of order of magnitude 1-6 m and average crack distance of 0.01-0.02 m, the realistic range of N is 50-600. As demonstrated by the two examples, the local scatter in a crack bridge significantly affects the load bearing capacity of textile reinforced specimens and, thus, the statistical size-effect resulting from the weakest link failure must be an inherent part of dimensioning rules for the discussed type of composite. While the size effect was demonstrated with a single source of randomness, in reality the crack bridge exhibits several mutually interacting sources of randomness that have been deliberately disregarded here.

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