Key Engineering Materials Vols. 488-489 (2012) pp 29-32
© (2012) Trans Tech Publications, Switzerland
doi:10.4028/www.scientific.net/KEM.488-489.29

The effect of mesh density in lattice models for concrete with incorporated mesostructure

Jan Eliáš[1,a], Miroslav Vořechovský[1,b]

[1]Institute of Structural Mechanics, Faculty of Civil Engineering, Brno University of Technology, Brno, Czech Republic

[a]elias.j@fce.vutbr.cz, [b]vorechovsky.m@fce.vutbr.cz

Keywords: lattice, brittle elements, mesh density dependency, concrete, grains, mesostructure

Abstract. This paper shows that mesh density dependency in brittle element lattice models cannot be completely removed by incorporating material mesostructure. This is concluded as a result of a series of simulations of fracturing in concrete using a 2D rigid-body-spring network. In the case of notched beams loaded in three-point-bending (TPB), adding finer grain structure led to a significant reduction in the dependency of the peak force on the mesh density. However, the energy dissipated during the fracture simulation was still affected significantly by the mesh density.

Introduction

Lattice models are a well established tool for fracture modeling and they appear to be very helpful, especially thanks to the increasing power of modern computers. In classical lattice models, the material is represented by a set of discrete elements inter-connected by springs. The use of a combination of simple constitutive models with a material structure incorporated from the meso-level [1,2], or alternatively of material parameters whose randomness somehow mimics this material structure [3,4], makes such models powerful tools able to model quasibrittle structural response. It is an alternative to the relatively complex constitutive laws applied in classical continuum mechanics-based models. The simplest models are those involving only elasto-brittle springs. Only this type of model is studied in this contribution.

The weak point of using purely brittle springs is the strong dependency of the results on network density. Since the network does not represent any real underlying structure, this dependency is understood as a bias which should be removed. If one insists on keeping the brittleness of elements, as we do (no softening of elements is incorporated), the mesh size dependency can be overcome e.g. by scaling the strengths of elements according to their lengths and a chosen internal length parameter [5] or, as is believed to be effective, by incorporating the material's inhomogeneities (voids, grains, microcracks), which introduces an internal length as well.

The paper presents a numerical study of the effect of grain microstructure projected on lattice elements in networks with varying densities (the grain layout properties to be used to classify elements is kept, but the network density is varied). Attention is particularly focused on how two of the most important characteristics are affected: the peak load and the dissipated energy. Conclusions are drawn from the presented series of simulations of fracturing in concrete using a 2D rigid-body-spring network.

Brief description of the model

Several lattice-type models can be found in the literature. In this research, the rigid-body-spring network developed by Kawai [6] is used. Fundamentally, the model is very similar to the one published by Bolander & Saito [7]. The fracture criteria are taken from the same article, i.e. a Mohr-Coulomb surface with tension cut-off is adopted. A more detailed description can be found in Eliáš [8]. Irregular geometry is used for the network in order to avoid directional bias. The meshing algorithm is based on Voronoi tessellation, which is performed on a set of pseudo-randomly placed triangulation nodes within the domain. The only restriction is that their minimal mutual distance equals a predefined parameter, $l_{\min}$.

When a notch is to be modeled, it is included by mirroring nodes along the notch line in the notch vicinity, see Figure 1b. Voronoi tessellation then creates a straight (notch) line and all springs lying on that line are subsequently removed to model the notch. In order to place the notch tip exactly at the desired coordinate, three points are placed with a prescribed distance from the tip. This procedure guarantees the exact location of the shared vertex of the three corresponding rigid bodies at the notch tip.

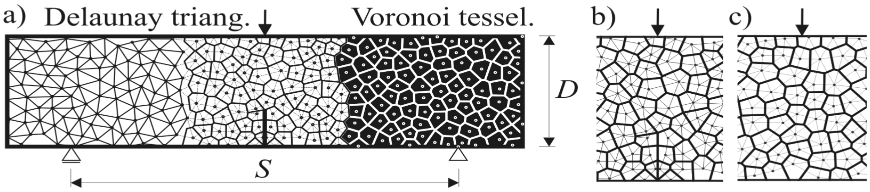

Figure 1: Specimens with and without a central notch (rel. notch depth $\alpha = 1/3$, depth $D = 0.1$ m, span $S = 0.4$ m). a) Illustration of the Delaunay triangulation corresponding to the dual graph of the Voronoi tessellation; b) and c) sketch of the central part of the specimen: with and without a notch.

Reduction of spurious mesh density effect via the incorporation of concrete mesostructure

In the homogeneous version of the model, the strength criterion defined by the breaking stress is identical for all springs. Also, the elastic normal and shear moduli are the same for all springs. The influence of network (mesh) density has to be understood as a spurious phenomenon because the mesh is arbitrary, artificial and does not arise from any real material structure. It is generally believed that the dependency of results on the network density might be removed by projecting material inhomogeneities onto the lattice. Incorporation of the grain structure introduces an internal length that decreases this dependency. In the following paragraphs this expectation is subjected to critical study which shows the limits of such a procedure.

The grain structure that is used here is generated by a computer algorithm using a Fuller curve. Typically, the maximal grain diameter d_{max} is chosen according to the real batch contents, and the minimal d_{min} according to the network density. The length of network elements should be at least three times smaller than d_{min} [9], otherwise the particles coalesce in the mesh. Grains smaller than d_{min} are ignored in the procedure. The larger the d_{min}, the smaller amount of grains included, yet changing d_{min} has no effect on coarse grains. Having small d_{min} enriches the model with small aggregates. Simulated grains are then projected onto the lattice to attribute springs with the three material phases – aggregate, matrix and the interfacial transitional zone (ITZ). These are distinguished according to the positions of nodes with respect to the mesostructure (see [9]). Each phase has a different strength and Young's modulus; values from [10] are used here.

The hypothesis tested the claim that the finer the mesostructure considered, the lower the mesh sensitivity of the model observed. To test the hypothesis, six different grain contents were generated. The first corresponds to a homogeneous case without any grains; the others differ by d_{min} (Figure 2); however, the maximum grain diameter d_{max} is kept equal to 32 mm. The mesh density varied from l_{min} equal to 2.5 mm up to a density of 0.625 mm. Note that not all the densities can be used for all the grain contents. The finer the grains, the finer the mesh required. For all possible combinations, 50 realizations of notched and unnotched TPB were simulated. In order to provide an idea of the model's behavior, Figure 3 shows average load-deflection diagrams for some of the considered densities and mesostructures. Notice the drastic strength decrease when one incorporates even only a few grains in the unnotched structure. The crack position can be sampled from many possible locations and thus the fracture always propagates along the weak ITZ. Such a steep drop in strength is not observed with notched structures: the crack propagates from the deep notch independently of the positions of the grains.

The results for the peak loads are shown in Figure 4. The homogeneous model of the notched TPB follows a straight line with a slope of 0.424. This slope differs from the expected 0.5 due to the probability of the occurrence of an inclined facet in the early stages of the crack [11, 12]. Models with grain content seem to reduce this dependency. The best results (almost a horizontal line) are achieved by the most detailed grain content. For this setting, the peak load hardly depends on the mesh density at all. In the unnotched case, the more detailed the structure, the less dependency is visible. However, for unnotched beams the trend is less apparent.

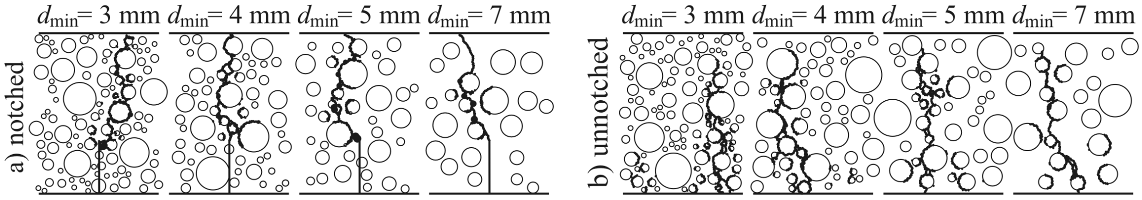

Figure 2: An example of crack patterns observed for (a) notched TPB and (b) unnotched TPB with the lattice model including concrete mesoscopic grain structure of varying fineness.

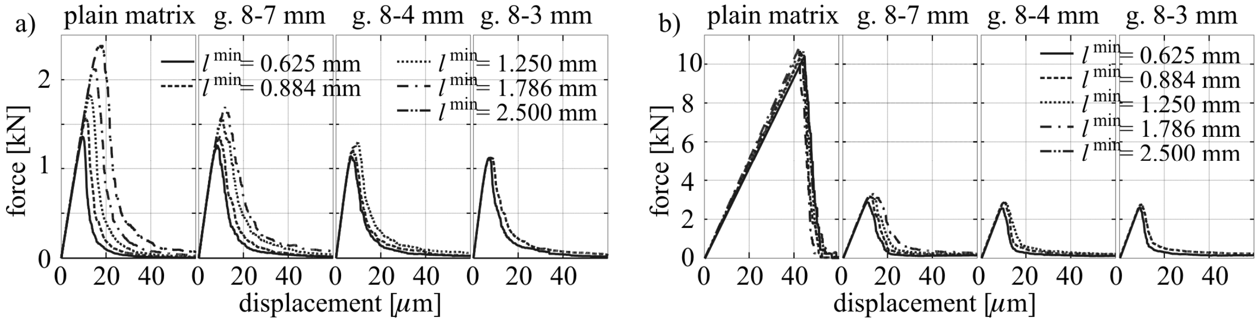

Figure 3: Average load-displacement diagrams of 50 realizations of (a) notched TPB and (b) unnotched TPB for some of the considered densities and mesostructures.

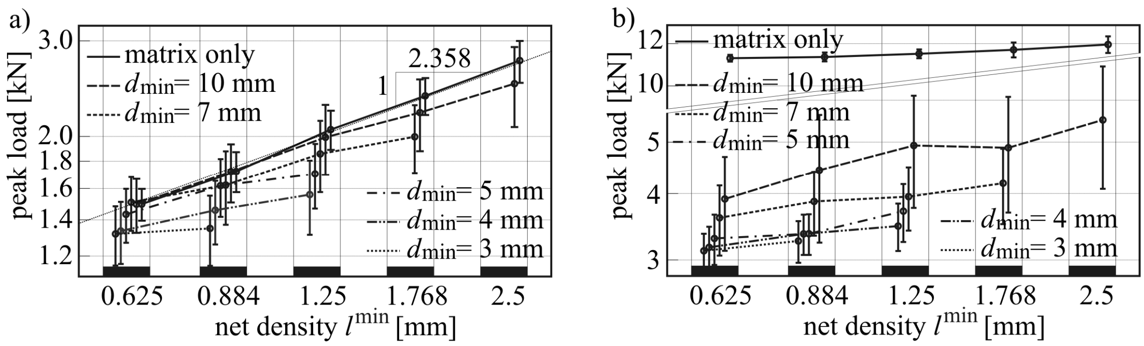

Figure 4: Dependence of maximum load for (a) notched TPB and (b) unnotched TPB on mesh density for varying incorporated grain structure.

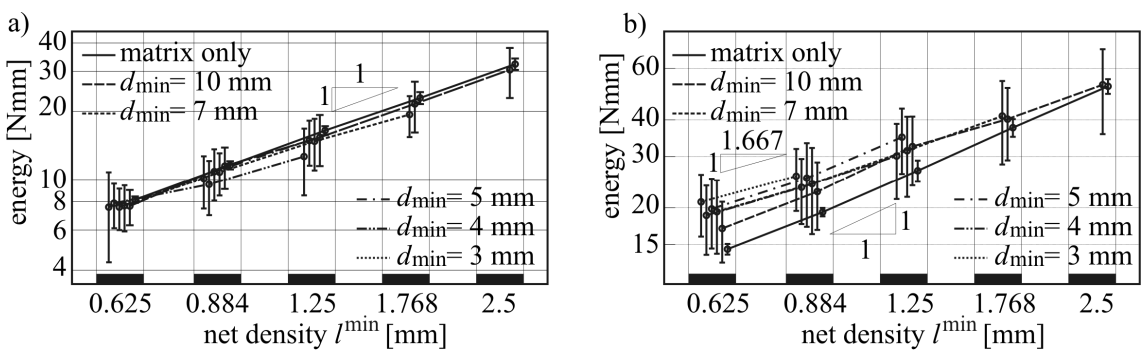

Figure 5: Dependence of the area under the load- displacement curve for (a) notched TPB and (b) unnotched TPB on mesh density for varying incorporated grain structure.

The second monitored parameter is the area under load-displacement curves, which represents energy. No substantial reduction in the dependence of this energy on the mesh density (by the incorporation of grain structure) was observed for the notched structure. Fig. 5a documents that all the lines share approximately the same rising slope. The energy dependency is only slightly reduced for the unnotched structure, see Fig. 5b. However, the line plotted for the finest resolution is far from being horizontal.

Summary

The effect of the discretization of brittle elements in lattice models for concrete was studied. Mechanical properties were assigned to the elements according to their correspondence to the three phases of concrete, namely matrix, aggregates and the interfacial transitional zone. Projecting of the concrete mesostructure onto the lattice has up to now been understood as a regularization technique that should overcome network mesh density dependency. The effect of such "regularization" was studied on both notched and unnotched beams loaded in three point bending. We report that even though the peak force dependence is reduced by the incorporation of mesostructure (and almost disappears in notched structures with the most detailed grain content), the strong dependence of the dissipated energy on mesh density is not removed.

Acknowledgement. This outcome has been achieved with financial support provided by the Czech Science Foundation under project no. P105/10/J028, and also by the Czech Ministry of Education, Youth and Sports under project no. 1M06005 (Civak). This support is gratefully acknowledged.

References

[1] E. Schlangen: *Experimental and numerical analysis of fracture processes in concrete* (Ph.D. thesis, Delft University of Technology, Netherlands 1993).

[2] J.E. Bolander, H. Hikosaka, W.-J. He: Eng. Computations Vol. 15 (1998), p. 1094–1116.

[3] P. Grassl, Z.P. Bažant: J. Eng. Mech. Vol. 135 (2009), p. 85–92.

[4] M.J. Alava, P.K.V.V. Nukala, S. Zapperi: Int. J. Fracture Vol. 154 (2008), p. 51–59.

[5] A. Jagota, S. J. Bennison: Modelling Simul. Mater. Sci. Eng. Vol. 3 (1995), p. 485–501.

[6] T. Kawai: Nucl. Eng. Des. Vol. 48 (1978), p. 207–229.

[7] J.E. Bolander, S. Saito: Eng. Fract. Mech. Vol. 61 (1998), p. 569–591.

[8] J. Eliáš: *Discrete simulation of fracture processes of disordered materials* (Ph.D. thesis, Brno University of Technology, Faculty of Civil Engineering, Brno, Czech Rep. 2009).

[9] J.G.M. van Mier: *Fracture Processes of Concrete: Assessment of Material Parameters for Fracture Models* (CRC Press, Boca Raton, Florida 1997).

[10] E.P. Prado, J.G.M. van Mier: Eng. Fract. Mech. Vol. 70 (2003), p. 1793–1807.

[11] M. Vořechovský, J. Eliáš, In: Proceedings of the Tenth International Conference on Computational Structures Technology, held in Valencia, Spain, Civil-Comp Press, Stirlingshire, Scotland, ISBN 978-1-905088-38-6, (2010) pp. 268, full papers on CD-ROM (20 pages).

[12] M. Vořechovský, J. Eliáš, In: EURO-C 2010, *Computational Modelling of Concrete Structures*, edited by Bicanic et al, CRC Press/Balkema, The Netherlands, ISBN 978-0-415-58479-1, (2010), p. 419–428.