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Identification of the effective bundle length in a multifilament yarn from the size effect response

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Abstract

The article proposes a method for characterizing the *in situ* interaction between filaments in a multifilament yarn. The stress transfer between neighboring filaments causes the reactivation of a broken filament at some distance from the break. The utilized statistical bundle models predict a change in the slope of the mean size effect curve once the specimen length becomes longer than the stress transfer length. This fact can be exploited in order to determine the stress transfer length indirectly using the yarn tensile test with appropriately chosen test lengths. The identification procedure is demonstrated using two test series of tensile tests with AR-glass and carbon yarns.

Keywords

chain-of-bundles model, stress transfer length, fiber-bundle-model, statistical size effect, testing of high-modulus multi-filament yarns

Introduction

Textile fabrics are being increasingly applied as reinforcement in concrete structures in civil engineering projects. In this application domain, alkali-resistant (AR) glass fibers and carbon fibers as well as aramid fibers and high-modulus polyethylene fibers are used as reinforcement. A common feature of these composite materials is the rather irregular structure of the bond between the yarn and the matrix. Due to the small filament diameter and the dense packing of filaments in the cross section the yarns do not get fully penetrated by the matrix. As a result, the bond between the filaments and the matrix develops only in the outer region of the yarn cross-section. This leads to a complex damage process in a loaded crack bridge. The effect of irregularity of the outer bond on crack bridge performance has been studied by the authors using the statistical fiber bundle model in Chudoba et al.[1] and Vořechovský and Chudoba.[2]

Due to the incomplete penetration of the matrix into the yarn, there is still a large fraction of filaments without any contact with the matrix. The filament–filament frictional stress, further referred to as the inner bond, is much lower than the bond shear stress transmitted by the outer bond between the filaments and the matrix. However, as documented in Hegger et al.,[3] the effect of the inner bond on the macroscopic performance of a reinforced tensile specimen cannot be neglected. While the outer bond affects the behavior locally at the length scale of a crack distance, the inner bond influences the failure process at the length scale of a structural element with a sufficiently large stress transfer (or anchorage) length. This can be documented by the significant contribution of the inner bond to the stress level in the post-cracking regime of a tensile specimen reinforced with AR-glass yarns.[4]

As a consequence, the interaction and damage effects for both the outer and the inner bond require a detailed mechanical characterization. While it is possible to study and characterize outer bond experimentally

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using the pull-out test of filaments and yarns from the matrix,[5] it is impossible to directly measure the *in situ* filament–filament interaction.

The key idea of the present approach is to exploit the fact that *in situ* filament interaction affects size effect behavior when the specimen length is larger than the stress transfer length, i.e., the length at which a broken filament recovers its stress. Such a yarn structure gets fragmented into a chain-of-bundles and behaves as a pseudo-composite. The change in the slope of the size effect curve during the transition from a single bundle to a chain-of-bundles observable in yarn tensile tests can be utilized for an indirect identification of interaction characteristics of filaments within yarn.

The article is organized as follows: First, a brief review of the related work on fiber bundle modeling and yarn characterization is provided. Then, the size effect response of a yarn during a tensile test is introduced and the structure of the mean size effect curve (MSEC) is described. The asymptotes of the MSEC are then briefly reviewed for a bundle without and with interaction. The procedure of identifying an effective bundle length is formulated and exemplified for real data with two types of yarns. After that, the limiting cases of the described identification procedure are discussed.

Related work

The indirect qualitative experimental observation of *in situ* filament interaction is made possible by imposing different levels of twist in the yarn tensile test. The authors experimentally studied the effect of increased *in situ* filament interaction on high-modulus multifilament yarns (carbon and AR-glass).[6] Multivariate experimental analysis was used to study the compound effect of the loading rate, test length, fineness, and twist.

A model with an analytical solution of the statistical properties of twisted bundles was described in Phoenix[7]; it examined the effect of random slack in yarns and cables in combination with the random breaking strains of the individual fibers. The model provides an explicit expression for the strength distribution of a bundle with a low number of fibers and, as an expansion of the classical Daniels' model, asymptotic results for a large number of fibers $n_f \to \infty$.

A numerical approach using the Monte Carlo simulation of random filament strength was used in Realff et al.[8] to compute the strain–stress relationship of twisted blended yarns. The stress transfer length occurring in such a yarn structure was computed as a function of yarn strain, twist level (lateral pressure), the position of a filament within the bundle cross-section, and filament type. An advanced model for the statistical strength of twisted fiber bundles has been presented recently in Porwal et al.[9]

Another method of studying and observing the effect of filament interaction in a multifilament yarn is possible in terms of the size effect curve (the dependence of average yarn/bundle strength on length). The basic explanation for the dependence of strength on the filament length is the weakest-link concept. The strength variations of filaments are deemed to be caused by randomly distributed flaws and defects. This effect has been studied by many authors, see for example Gurvich et al.[10] and Pan et al.[11] On an average, the strength of a filament decreases with increasing length. This size effect can be captured by combining the classical Weibull distribution and associated extreme value theory with the chain-of-bundles model. Once the filament strength dependence on length is modeled, the effect of stress redistribution within a yarn of noninteracting filaments can be captured by the available fiber bundle models. The fiber bundles can then be linked in a series to reflect the fact that yarn with interacting filaments behaves like a chain-of-bundles. The propagation of the single-filament statistics through this series–parallel structure has been addressed in Harlow and Phoenix,[12] Harlow et al.,[13] Vořechovský,[14] and Watson and Smith.[15] We remark that in these models, a constant length of a bundle is assumed. The justification for this assumption is discussed in the sequel in the 'remarks' section.

The overall strength of a yarn with filament fragmentation has been related to the strength of a single bundle.[16] The length of such a bundle, referred to as critical length, corresponds to the stress transfer length. The variability of the stress transfer length along the yarn has been discussed in the later experimental investigation, in which the model was applied.[17]

The critical bundle length in both papers[16,17] was calculated by a formula which assumes that both the frictional coefficient and the local lateral pressure are known. It should be mentioned that in the cited papers no consideration has been given to size effect due to the scatter of the bundle strength for yarns longer than the interaction/critical length. As shall be discussed in the sequel, this assumption is justified only for yarns with a large number of filaments and a large number of bundles. The transition in the size effect curve from the parallel structure to the series–parallel structure has been recently used as an explanation for the low size effect observed for polyester yarns.[18]

Effect of filament interaction on the size effect of the yarn

Throughout the study, the only source of randomness considered is the variability in local filament strength. Filaments have an elastic response until sudden (brittle) failure occurs when they reach their strength. The local

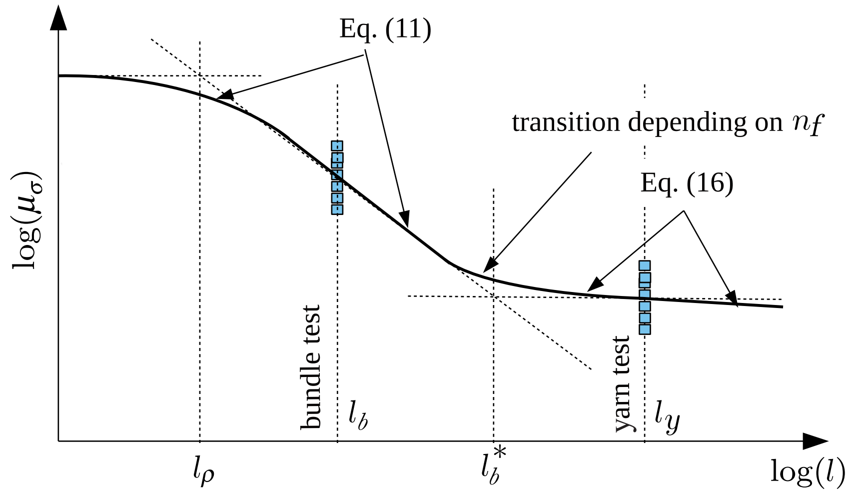

Figure 1. Mean size-effect curve in logarithmic scale with three distinguished asymptotes.

random strength (or breaking strain) X at a certain point along the filament length is considered to follow the Weibull distribution:

$$F_X(x) = P(X \leq x) = 1 - \exp\left[-\left(\frac{x}{s}\right)^m\right] \quad (1)$$

where s is the scale parameter and m the shape parameter depending only on the coefficient of variation (COV). The spatial distribution of random strength along a filament has a length scale l_ρ at which the strength variability diminishes.[2] As a consequence, for short specimens $l \ll l_\rho$, the strength realization can be considered constant along the filament and, therefore, the random filament strength is length-independent. On the other hand, for $l \gg l_\rho$, the local strength varies over the filament length. Therefore, the overall filament strength is defined by the minimum local strength along the filament length corresponding to the weakest link model and is well described by the Weibull extreme value distribution.[2]

With these assumptions for a single filament, a qualitative profile of the MSEC of a multifilament yarn can be expected, as shown in Figure 1. Two different mechanisms of load transfer in a yarn can be distinguished depending on the yarn length. The two regions are separated by yarn length l_b^* at which the fiber fragmentation can occur. Here, we implicitly assume that the autocorrelation length of the random strength process along the filament is less than l_b^* where the mechanism changes. The two main regimes can be characterized as follows:

- For the range of lengths $l < l_b^*$, the yarn is acting as a bundle or a set of parallel, independent filaments with identical Weibull strength distribution. Its size effect behavior is described by Daniels[19], whose work was later corrected by Smith.[20] In such a bundle, a filament is assumed to break only once throughout its length and the associated released force is considered to be redistributed evenly among the surviving fibers. Two limiting behaviors can be distinguished for a bundle with independent filaments based on the dependence of the strengths of individual filaments on their length.
  – For very short lengths $l < l_\rho$, any realization of the random strength process along the filament can be considered a constant function. In other words, the realization of the local filament strength simplifies to a single random variable independent of the position along the filament. The consequence is that the left asymptote of the mean filament strength is a horizontal line at the level of the mean value of the local random filament strength. Therefore, the MSEC of the bundle also has a horizontal left asymptote. As the bundle length approaches the autocorrelation length l_ρ, the MSEC starts to decline from the left horizontal asymptote and turns slowly downwards in the direction of the middle asymptote dictated by the classical Weibull size effect. We remark that the only considered source of randomness is the local filament strength. The effect of random stiffness due to the varying cross-sections, lengths, and slack of filaments that leads to strength reduction for $l \to 0$ is not considered here and has been described by the authors in Chudoba et al.[1] and Vořechovský and Chudoba.[2]
  – bundles of a length greater than l_ρ but still less than l_b^* behave as bundles with independent fibers whose strength is described by the classical extreme value theory of independent strength along their length. The slope $-1/m$ of the middle asymptote is dictated solely by

the COV (or the shape parameter m) of the local filament random strength.[2]

- With increasing length, the filament–filament friction can recover the stress released upon filament breakage and allows for the fragmentation of a single filament along its length. The length $l_b^\star$ marks the transition from bundle behavior to the behavior of a chain-of-bundles. The slope of the MSEC for $l > l_b^\star$ becomes reduced and, in the limit, asymptotically approaches the slope $-1/(n_f m)$.[13] The particular shape of this transition depends on the number of filaments in the bundle.[12,21]

The transition zone from the bundle range to the chain-of-bundles range is of special interest. The change in the slope of the size effect curve reveals the length $l_b^\star$ at which the fragmentation starts. The idea of this article is to exploit this fact in order to identify the effective bundle length $l_b^\star$ within the tested yarn. The identification procedure tries to find an intersection between the two branches of the MSEC. The mathematical formulation of the two branches is summarized in the following two sections.

Bundle consisting of parallel independent filaments

The mean strength of a single Weibullian filament within a bundle is prescribed as

$$\mu_{\sigma_f} = s_0 \cdot \left(\frac{l_0}{l}\right)^{-1/m} \cdot \Gamma\left(1 + \frac{1}{m}\right) \tag{2}$$

with s_0, m denoting the scale and shape parameters of the Weibull distribution, respectively, and $\Gamma(\cdot)$ as the Gamma function.[22] The scale parameter s_0 is associated with the reference length l_0. As pointed out in Vořechovský and Chudoba,[2] the above power-law scaling predicts unlimited mean strength for $l \to 0$ and is therefore unrealistic. To impose an upper bound on the strength, a statistical length scale in the form of the autocorrelation length of a random strength process along the filament has been introduced in Vořechovský and Chudoba.[2] The MSEC can be reformulated as being dependent on length function $f_\rho(l)$ as

$$\mu_{\sigma_f} = s_0 \cdot f_\rho(l) \cdot \Gamma\left(1 + \frac{1}{m}\right). \tag{3}$$

The refined scaling function $f_\rho(l)$ accounting for the correlation length l_ρ has been suggested as either

$$f_\rho(l) = \left(\frac{l}{l_\rho} + \frac{l_\rho}{l_\rho + l}\right)^{-1/m} \tag{4}$$

or

$$f_\rho(l) = \left(\frac{l_\rho}{l_\rho + l}\right)^{1/m}. \tag{5}$$

We note that this length-scaling remains qualitatively unchanged for any arbitrary number of parallel filaments. Thus, in the sequel, the length dependency of the scaling parameter within the range $l_\rho < l_b < l_b^\star$ (Figure 1) shall be represented by the scaling function

$$s_b = s_0 \cdot f_\rho(l_b). \tag{6}$$

We also note that in the limit of $l \gg l_\rho$, the scaling in Equations (4) and (5), recovers the classical Weibull length-dependence $f_W(l) = \left(l_\rho / l\right)^{1/m}$.

Such a decomposition of the length effect allows for a simple scaling of the mean value

$$\mu_{\sigma_1} = \mu_{\sigma_0} \frac{f_\rho(l_1)}{f_\rho(l_0)} \tag{7}$$

that shall be used later in the identification procedure.

The cumulative distribution function of a random per fiber bundle strength x of a parallel coupling of filaments with independent identically distributed strength is given by the recursive formula for n_f number of filaments[19]

$$G_n(x) = \sum_{i=1}^{n_f} (-1)^{i+1} \binom{n_f}{i} F(x)^i G_{n-1}\left(\frac{n_f x}{n_f - i}\right), \; x \geq 0 \tag{8}$$

with $G_0(x) \equiv 1$ and $G_1(x)$ being equal to the cumulative distribution function of the strength $F(x)$ of a single fiber. The resulting bundle strength approaches the normal (or Gaussian) distribution as the number of filaments grows large ($n_f \to \infty$). Based on Daniels' analysis, the expected asymptotic mean bundle strength μ_{σ_b} with Weibull fibers is independent of n_f and is related to the filament properties as

$$\mu_{\sigma_b} = s_b \cdot m^{-1/m} \cdot c_m \quad \text{with} \quad c_m = \exp\left(-\frac{1}{m}\right) \tag{9}$$

with s_b obtained using Equation (6). The standard deviation γ_{σ_b} is given as

$$\gamma_{\sigma_b} = s_b \cdot m^{-1/m} \sqrt{c_m \cdot (1 - c_m)}. \tag{10}$$

We note that the (length-dependent) standard deviation of random yarn strength is scaled in the same way as the mean value is scaled in Equation (7). As a consequence, the COV of the bundle strength does not depend on the bundle length.

The decrease in the normalized mean bundle strength μ_{σ_b} with respect to the filament strength μ_{σ_f} is obvious from the comparison of Equations (9) and (2). Real bundles have a finite number of filaments n_f and the mean strength is thus only approaching the Daniels's asymptotic prediction. Smith found a way to eliminate the gap between the real strength distribution and Daniels's normal approximation by adjusting μ_{σ_b} to $\mu_{\sigma_{b,n_f}}$ using the actual number of filaments in the following way[20]

$$\mu_{\sigma_{b,n_f}} = \mu_{\sigma_b} + n_f^{-2/3} b \cdot \lambda. \tag{11}$$

In the case of Weibull filament distribution, the parameter b is given as:

$$b = s_b \cdot m^{-1/m-1/3} \exp[-1/(3m)]$$

and the coefficient $\lambda = 0.996$. This correction shifts the mean value of the bundle strength. The standard deviation corresponding to μ_{σ_b} given by Equation (10) is a fair approximation and does not need any further adjustment for a finite number of filaments n_f.

Chain-of-bundles strength

Filaments in real yarns exhibit a certain amount of frictional interaction that leads to multiple breaking of individual filaments. The distance between two breaks along a filament can only be larger than the stress transfer length sometimes called the ineffective or shielded length marking the distance around a break within which the filament does not contribute to the stress transfer of the bundle. Thus, from the statistical point of view, the yarn can be decomposed into a chain-of-bundles, each of a length corresponding to the stress transfer length. In particular, the yarn can be idealized as a one-dimensional chain of independent bundles sharing the same distribution of random strength $G(x)$.

The strength distribution $G(x)$ of each of these serially coupled bundles is described in the previous section. Obviously, the yarn strength consisting of serially coupled bundles with identically distributed and independent (IID) strengths is governed by the weakest bundle and thus is distributed as follows

$$H_{n_b,n_f}(x) = 1 - [1 - G_n(x)]^{n_b}, \quad x \geq 0. \tag{12}$$

The probabilistic distribution of the chain-of-bundles strength can have different shapes depending on the ratio between the number of filaments n_f, number of bundles n_b, and load level.[14,23] For small values of n_f, the lower (Weibull) tail of the bundle strength distribution approaches its mean value. On the other hand, for a large number of filaments, n_f, the Gaussian shape of the distribution reaches far into the lower tail of $G_n(x)$.

As known from extreme value theory, the minimum of IID Gaussian variables, here representing the strength of a chain-of-bundles with dominating Gaussian distribution, approaches the Gumbel statistical function[24] as n_b grows large

$$H_{n_b,n_f}(x) = 1 - \exp\left[-\exp\left(\frac{x - b_{n_b,n_f}}{a_{n_b,n_f}}\right)\right], \quad x \geq 0 \quad n_b \to \infty, \tag{13}$$

where

$$a_{n_b,n_f} = \frac{\gamma_{\sigma_b}}{\sqrt{2\omega}} \tag{14}$$

$$b_{n_b,n_f} = \mu_{\sigma_{b,n_f}} + \gamma_{\sigma_b}\left[\frac{\ln(\omega) + \ln(4\pi)}{\sqrt{8\omega}} - \sqrt{2\omega}\right] \tag{15}$$

and $\omega = \ln(n_b)$. The mean value of yarn strength is then $\mu_{\sigma_y} = b_{n_b,n_f} - \eta \cdot a_{n_b,n_f}$ and the median equals $b_{n_b,n_f} + \ln(\ln(2)) \cdot a_{n_b,n_f}$. Here, $\eta \approx 0.5772$ denotes the Euler–Mascheroni constant. The strength distribution given in Equation (13) is very accurate for a number of bundles greater than approximately 300. Therefore, for lower numbers of bundles $n_b \in (1; 300)$, the authors suggest using the recently proposed[14] cubic regression of the mean values calculated numerically from Equation (12)

$$\mu_{\sigma_y} = \mu_{\sigma_b} - \gamma_{\sigma_b}\left(-0.007\omega^3 + 0.1025\omega^2 - 0.8684\omega\right), \tag{16}$$

where μ_{σ_b} and γ_{σ_b} are the mean bundle strength and standard deviation, respectively. This approximation describes the transition from the mean value of the Gaussian distribution of a single bundle to the mean value of the Gumbel distribution of a chain-of-bundles.

As already mentioned, for the strength distribution of bundles consisting of a low number of filaments n_f, the left Weibull tail reaches close to the mean value. As a consequence, the Weibull shape of the distribution becomes significant also for the distribution of the chain-of-bundles strength. Yarns consisting of a very large number of such bundles (of the order of 10^3 bundles with eight parallel filaments) have a Weibull strength distribution with a Weibull modulus obtained solely by multiplying the number of filaments n_f by the Weibull modulus of a single filament m.[12]

For the considered types of multifilament yarns consisting of several hundreds of filaments and a low number of bundles per meter (approximately 5 for

AR-Glass, 2400 tex), it is sufficient to use the approximating Equation (16) or the median value obtained from

$$\sigma_y^{50} = \mu_{\sigma_{b,n_f}} + \gamma_{\sigma_b}\Phi^{-1}\left(1 - 0.5^{1/n_b}\right). \quad (17)$$

Here, $\Phi^{-1}(\cdot)$ stands for the inverse standard Gaussian cumulative distribution function (percent point function) and $n_b = l_y/l_b$ the number of bundles of which the yarn consists.

Evaluation of the effective bundle length

Let us assume that two sets of strength data $\mu_{\sigma_b}^{test}$ and $\mu_{\sigma_y}^{test}$ are available for two respective test lengths falling into the different length ranges identified in Figure 1, i.e., $l_b^{test} < l_b^\star$ and $l_y^{test} > l_b^\star$. Apart from the known specimen lengths and the measured mean strengths, knowledge of the Weibull modulus value m of the tested material and the value of the autocorrelation length l_ρ is required. The estimation of the transitional length $l_b^\star$ is then performed using the following procedure.

1. The mean strength $\mu_{\sigma_b}^{test}$ estimated as the average strength for the length l_b^{test} is substituted into Equations (9) and (11) in order to obtain the scaling parameter s_b of the Weibull distribution for the tested length

$$s_b = \mu_{\sigma_b}^{test} \cdot \left[m^{-1/m} \cdot c_m + n_f^{-2/3} \cdot m^{-(1/m+1/3)} \exp\left(-\frac{1}{3m}\right)\lambda \right]^{-1}. \quad (18)$$

2. With the scaling parameter s_b at hand, the corresponding standard deviation γ_{σ_b} is evaluated using Equation (10). It is important to emphasize that we use the theoretical scatter of the bundle strength to identify the slope of the MSEC in the range of lengths $l \in \langle l_\rho; l_b^\star \rangle$ instead of the measured value of scatter. Note that in a typical yarn, the number of filaments n_f is very large and thus the theoretical scatter of the bundle strength is very small (asymptotically, it is inversely proportional to the square root of n_f). Usage of the theoretical scatter of the bundle strength is justified by the fact that the experimentally obtained standard deviation is increased by sources of randomness other than the scatter of local strength along the filaments. Obviously, this was also the case in the performed tests, as the measured levels of scatter did not correspond to the slopes of the MSEC for the two tested types of yarns. This discrepancy was ascribed to the manual production of the specimens and clamps.[6] An analytical solution explaining the variability due to additional sources of scatter at the yarn level will be proposed by the authors in another paper. Let us finally remark that even if a realistic measurement of the scatter of the yarn strength due to random filament strength were possible, much larger sample size would be required for a statistically significant estimate of the second statistical moment as compared to the estimate of the mean yarn strength.
3. The obtained bundle characteristics are scaled to the unknown length $l_b^\star$ using Equation (7) and exploiting the fact that standard deviation scales identically with the mean value

$$\mu_{\sigma_b}^\star = \mu_{\sigma_b}^{test} \cdot \frac{f(l_b^\star)}{f(l_b^{test})} \quad \text{and} \quad \gamma_{\sigma_b}^\star = \gamma_{\sigma_b}^{test} \cdot \frac{f(l_b^\star)}{f(l_b^{test})}.$$

4. The chaining effect involved in the experimental data is now expressed using Equation (16) for the unknown bundle length $l_b^\star$ as

$$\mu_{\sigma_y}^{test} = \mu_{\sigma_b}^\star - \gamma_{\sigma_b}^\star\left(-0.007\omega_\star^3 + 0.1025\omega_\star^2 - 0.8684\omega_\star\right) \quad (19)$$

where $\omega_\star$ represents the logarithm of the number of bundles in series $\omega_\star = \ln(l_y^{test}/l_b^\star)$. The nonlinear Equation (19) is then solved for $l_b^\star$ using numerical root-finding methods.

In order to demonstrate the identification procedure on real data, two test series with different yarn types (carbon and AR-glass) have been conducted. The input data and the results of the evaluation are summarized in Table 1. The resulting effective bundle length for AR-glass yarns is one-third larger than that of the carbon yarn, with a higher amount of frictional interaction detected within the carbon yarn. This trend is in agreement with the observation of the postpeak behavior in the tensile test. The level of stress transmitted by friction in the postpeak regime is significantly higher for carbon yarns than for AR-glass yarns.

Remarks

Due to the limitations of the experimental setup, the described procedure can be considered valid only within a certain range of test parameters, or more precisely, within a certain range of test lengths. The following limiting cases must be considered when designing the test series with the goal of identifying the effective stress transfer length.

- The identification procedure is valid only if the x-coordinate of the intersection point of the two size effect curves for a single bundle and a chain-of-bundles, respectively, is within the two test lengths l_b^{test} and l_y^{test}, i.e., if the sought length $l_b^\star \in \langle l_b^{test};\ l_y^{test} \rangle$.

- If the autocorrelation length is of the same order as the test length ($l_b^{test} \approx l_\rho$), the estimation of $l_b^\star$ becomes sensitive to slight changes in l_ρ. In particular, for the identification summarized in Table 1, the autocorrelation length $l_\rho = 1.0\,\text{mm} \ll l_b^{test} = 50\,\text{mm}$ was assumed leading to $l_b^\star = 142.1\,\text{mm}$ for carbon yarn. When assuming the autocorrelation length in the same length range as the short test length, e.g., $l_\rho = l_b^{test} = 50\,\text{mm}$, the estimated bundle length is $l_b^\star = 226.0\,\text{mm}$. The size effect curve obtained for this parameter combination is shown in Figure 2 in the user interface of the implemented software module. A possible remedy would be to add further test(s) to the range between l_b^{test} and l_y^{test} and to make the autocorrelation length a part of the fitting procedure.
- The identification procedure does not account for the case that the measured strength for l_b^{test} is distorted by the nonuniform loading of filaments due to irregularities in the yarn clamping. These effects lead to a reduction in strength for short specimens, as described in Vořechovský and Chudoba.[2] This case can be handled by simply rejecting short tests with a drop in mean strength. For the tested AR-glass yarns, the strength reduction could be observed experimentally for the test lengths $l_b^{test} < 40\,\text{mm}$.
- The identification is carried out using the mean values obtained from the experiments. The information on scatter is included using the Weibull modulus of the raw material identified independently. The scatter is only evaluated for bundles of length $l \leq l_b^\star$ with the goal to extrapolate the part of MSEC governed by the chain-of-bundles model.

Table 1. Summary of yarn properties, experimental data, and the evaluated bundle lengths for carbon and AR-glass yarns

Property	Unit	Symbol	Carbon	AR-glass
Fineness	(tex)	–	1600	2400
Number of filaments	(–)	n_f	24,000	1600
Weibull modulus	(–)	m	5.00	4.52
Autocorrelation length	(mm)	l_ρ	1.0	1.0
Test length I	(mm)	l_b^{test}	50.0	100.0
Measured strength I	(MPa)	$\mu_{\sigma_b}^{test}$	1955.8	1038.0
Test length II	(mm)	l_y^{test}	500.0	500.0
Measured strength II	(MPa)	$\mu_{\sigma_y}^{test}$	1586.9	882.8
Identified bundle length	(mm)	$l_b^\star$	142.1	201.8

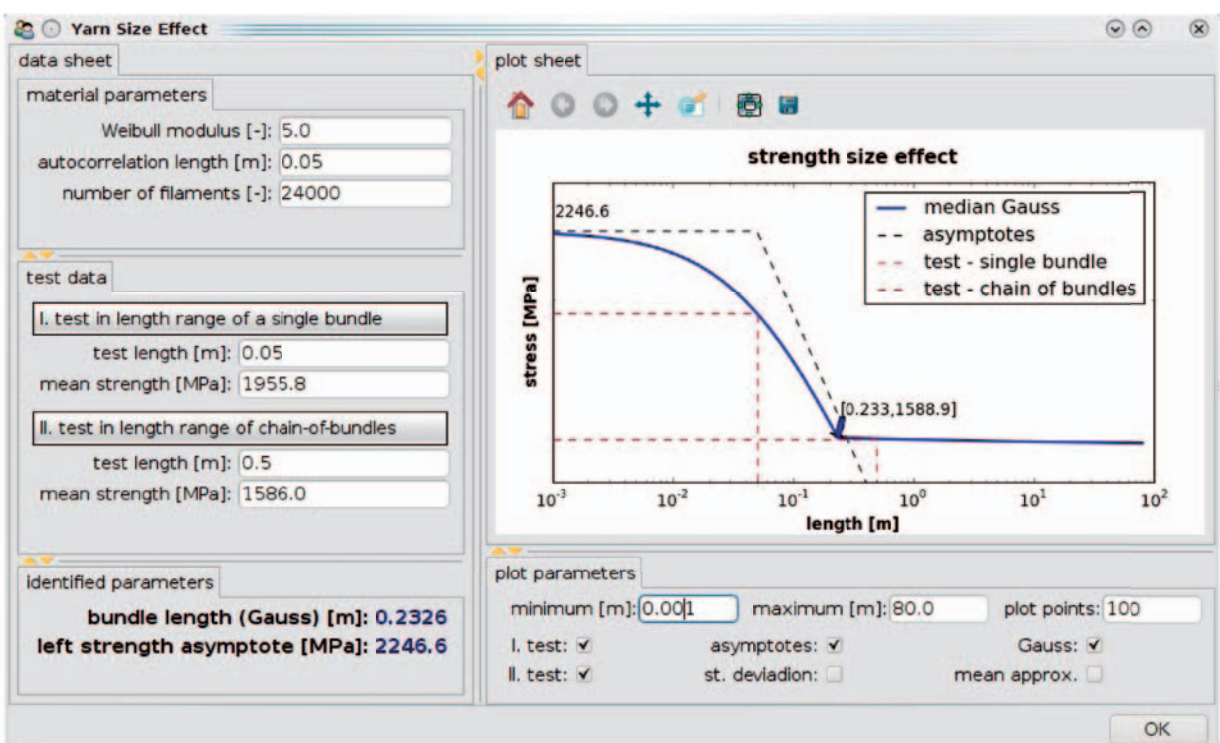

Figure 2. Example of identification with the implemented module.

Another point to mention is that bundle length has been identified as a deterministic value. It might be argued that it exhibits some scatter along the yarn, i.e., that the bundles in a yarn have different lengths.[17] The justification for the assumption of constant bundle length can be constructed by realizing that the actual bundle length corresponds to the stress transfer length which in turn depends on the spatially variable filament–filament friction. In particular, two directions of spatial scatter of friction can be distinguished: along and across the yarn.

- Along the yarn: As the level of filament friction is relatively low, the stress transfer length needed to recover the breaking stress is large, in the order of centimeters. On the other hand, the length-scale of spatial variation of the filament–filament friction due to irregular packing of the yarn is in the order of micrometers. Realizing that the stress transfer length represents the sum of many local frictional links along the filament, we can expect that the local scatter of friction becomes homogenized at the scale of the stress transfer length. Therefore, the scatter of stress transfer length can be regarded as very small.

  More precisely, the fluctuating friction intensity along a single filament can be idealized as n_c number of frictional cells with a constant level of friction, each represented by an independent, identically distributed (IID) random variable given by the mean μ_c and variance $\sigma_c^2 > 0$. The COV in each cell is $\mathrm{cov}_c = \sigma_c/\mu_c$. As stated above, the number of frictional cells n_c along a filament within the mean stress transfer length that have a significant difference in friction level is very high. Therefore, the mean value μ_f of the sum of these frictional contributions along the yarn defining the stress transfer length converges to $n_c \cdot \mu_c$ and, according to central limit theorem (CLT), the variance is equal to $\sigma_f^2 = n_c \cdot \sigma_c^2$. As a consequence, the COV $\mathrm{cov}_f = \sigma_c/(\sqrt{n_c} \cdot \mu_c) = \mathrm{cov}_c/\sqrt{n_c}$ rapidly diminishes at the length-scale of the stress transfer length with large n_c.
- Across the yarn: The scatter of friction due to variable filament surface roughness or lateral pressure within the yarn cross-section diminishes as the number of filaments n_f grows large.

  Formally, the bundle length can be idealized as the average of the filament stress transfer lengths within the cross-section. All stress transfer lengths of individual filaments can be viewed as IID random variables characterized by mean μ_f and variance $\sigma_f^2 > 0$. The CLT then states that as the sample size n_f increases, the distribution of the sample average approaches the normal distribution with the mean $\mu_l = \mu_f$ and the variance $\sigma_l^2 = \sigma_f^2/n_f$ irrespective of the shape of the distribution of the random variable. The COV of the stress transfer length is given by $\mathrm{cov}_l = \mathrm{cov}_c/\sqrt{n_c n_f}$. Thus, in the case of applied yarns, the scatter of the filament transfer length can be assumed to be very small.

Based on these considerations, the variance of stress transfer length should become insignificant and, therefore, the assumption of the constant bundle length $l_b^\star$ along the yarn seems to be justified in the context of the experimental identification.

It should be noted that the redistribution pattern included in the applied chain-of-bundles model is based on the global load sharing rule. As the chaining of bundles for lengths $l > l_b^\star$ is due to frictional stress along the filaments, it should also cause a more local redistribution of stresses upon a filament break. This issue is not included in the applied chain-of-bundles approximation of the MSEC.

Conclusions

This article presents a possible method of utilizing the available statistical fiber bundle models and chain-of-bundles model for the systematic identification of effective bundle length within tested yarns. Within this length, a filament is assumed to break only once. The proposed identification procedure uses the dependence of the yarn strength on the test length. In particular, it exploits the MSECs predicted by the fiber bundle model and by the chain-of-bundles model. The intersection between the two branches of the MSEC corresponds to the length of a single bundle $l_b^\star$ which is assumed to be an inherent property of a yarn.

The identified effective bundle length can be seen as a comparative value for the level of redistribution within the bundle. Two yarn types (carbon 1600 tex, and AR-glass 2400 tex) have been chosen to demonstrate the identification procedure.

In spite of its simplicity, the present approach demonstrates the idea and at least for the two studied cases delivers plausible values. In the long run, the approach acts as motivation to undertake further work in two directions: First, the industrial testing devices used should be improved in order to provide automatic testing of high-modulus multifilament yarns with varied lengths. Second, more advanced modeling of the MSEC transition between the bundle range and chain-of-bundles range would enable further theoretical conclusions to be drawn regarding the redistribution mechanisms between the filaments within the yarn. In particular, a random-field simulation accounting for effects such as the positioning of filaments within the bundle[8] and the transition from the global to the local load sharing with possibly variable bundle length would shed more light onto the MSEC transition from the single bundle to the chain-of-bundles.

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