Discussion of "Mechanism behind the Size Effect Phenomenon" by Xiaozhi Hu and Kai Duan

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In the January issue of this *Journal*, the authors published a paper focused on effect of size on the nominal strength of structures made of heterogeneous materials. The authors identified the source of the size effect to be the boundary effect. The authors also compare their theory with sources identified and formulas derived earlier by Bažant and his collaborators.

The same issue of the *Journal* contains another paper by Yu et al. (2010) focused on weaknesses of the boundary effect model. This paper already discusses the subject matter of the theories under comparison.

The purpose of this discussion is to point out a misprint in Eq. (7) of the original paper. The authors refer to the paper by Bažant and Pang (2007) as a source of their Eq. (7). In my opinion, this is incorrect. The authors probably have in mind Eq. (9) from the paper by Bažant, Vořechovský, and Novák (2007), which is focused on the combined energetic-statistical size effect on nominal strength.

The discussed paper continues with an explanation of the meaning of the constants in the formula that is adopted from Bažant and Yu (2009).

In my opinion, the misprint in Eq. (7) of the discussed paper appears in the numerator of the first addend. The numerator should contain l_s not D_b. The equation continues with an incorrect deduction of asymptotic behavior. The problem probably originates from Eq. (8) in the paper by Bažant and Yu (2009), who, unfortunately, also misprinted the formula in the same way. The discusser would like to stop this confusing misprint from spreading further.

The original form of the equation reads [see Eq. (9) of Bažant et al. (2007)]

$$\sigma_N(D) = f_r^\infty \left[\left(\frac{l_s}{D + l_s} \right)^{rn/m} + \frac{rD_b}{D + l_p} \right]^{1/r} \tag{1}$$

whereas the authors and also Bažant and Yu (2009) incorrectly write

$$\sigma_N(D) = f_r^\infty \left[\left(\frac{D_b}{D + l_s} \right)^{rn/m} + \frac{rD_b}{D + l_p} \right]^{1/r} \tag{2}$$

In the original form, the first addend accounts for the probabilistic part of the phenomenon whereas the second addend accounts for the energetic part—it controls the transition from the plastic limit corresponding to the relatively ductile behavior of a small structure to the brittle failure of a large structure. The first addend corresponds to the mean value of probabilistic strength and reaches the upper bound of "1" when the structural size D tends to zero. The supposed asymptotic behavior for $D \to 0$ expressed in Eq. (7) of the original paper is therefore wrong. On the other extreme (D tending to infinity), the addend resembles the classical Weibull-type size effect with reference length l_s.

The whole formula represents an approximation of the mean value of nominal strength $\sigma_N(D)$, which is considered to be a random variable. The constant l_s has the role of statistical characteristic length, and in theory, its origin differs from D_b, although the lengths might be comparable. This is thoroughly discussed by Bažant et al. (2007).

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References

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Closure to "Mechanism behind the Size Effect Phenomenon" by Xiaozhi Hu and Kai Duan

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The paper "Mechanism behind the Size Effect Phenomenon" was written to address several misinterpretations (Bazant and Yu 2009) of the boundary effect model developed by writers in the past few years. The first part of Eq. (7) in our paper is from Bazant and Yu (2009) and is as follows:

$$\sigma_N = f_r^\infty \left[\left(\frac{D_b}{l_s + D} \right)^{rn/m} + \frac{rD_b}{l_p + D} \right]^{1/r}$$

$$= \begin{cases} f_r^\infty \left[\left(\frac{D_b}{l_s} \right)^{rn/m} + \frac{rD_b}{l_p} \right]^{1/r} = \text{constant} \cdot f_r^\infty \quad \text{if } D \to 0 \\ f_r^\infty \left[\left(\frac{D_b}{D} \right)^{rn/m} + \frac{rD_b}{D} \right]^{1/r} \propto f_r^\infty \cdot D^{-n/m} \quad \text{for large } D \\ \qquad (\text{Weibull SE}) \end{cases}$$

where D_b, l_s, l_p, r, n, and m = empirical constants that can be adjusted to fit experimental data, according to Bazant and Yu (2009), and f_r^∞ = assumed asymptotic strength if the size D goes to infinity. The second part of Eq. (7) is subsequently given by the writers of the original paper for $D \to 0$ and $D \to \infty$.

Vořechovský, a coauthor of Bazant, Vořechovský, and Novak (2007), pointed out that Bazant and Yu (2009) made an error and the first part of the original equation should have the following form:

$$\sigma_N = f_r^\infty \left[\left(\frac{l_s}{D + l_s} \right)^{rn/m} + \frac{rD_b}{D + l_p} \right]^{1/r}$$

$$= \begin{cases} f_r^\infty \left[1 + \frac{rD_b}{l_p} \right]^{1/r} = \text{constant} \cdot f_r^\infty & \text{if } D \to 0 \\ f_r^\infty \left[\left(\frac{l_s}{D} \right)^{rn/m} + \frac{rD_b}{D} \right]^{1/r} \propto f_r^\infty \cdot D^{-n/m} & \text{for large } D \\ & (\text{Weibull SE}) \end{cases}$$

However, this revised equation does not change the outcomes for $D \to 0$ or $D \to \infty$ because the second part of the preceding equation is virtually the same as that of Eq. (7) in our paper. Therefore, our conclusions remain unchanged regardless of whether Eq. (7) or the original equation (Bazant et al. 2007) is used.

One of the fundamental errors of Bazant's size effect model is that the assumed asymptotic strength f_r^∞ adopted for a very large D or a "homogeneous" material without any solid experimental evidence also determines the nominal strength of a very small test sample or a highly heterogeneous composite. This is one of our concluding remarks in the paper. Clearly, Vořechovský's discussion does not change the conclusion.

References

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