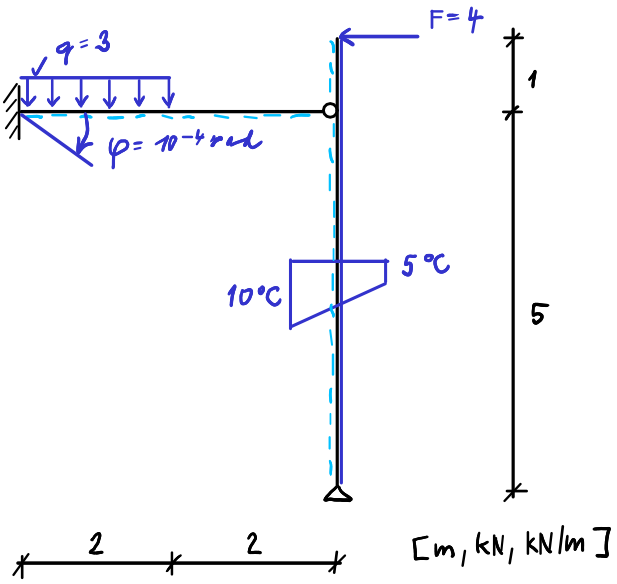
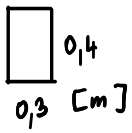


$E = 40\ GPa = 40 \cdot 10^6\ kPa$

$\alpha_T = 1 \cdot 10^{-5}\ °C^{-1}$



$A = 0{,}3 \cdot 0{,}4 = 0{,}12\ m^2$

$I = \frac{1}{12} \cdot 0{,}3 \cdot 0{,}4^3 = 1{,}6 \cdot 10^{-3}\ m^4$

$EI = 40 \cdot 10^6 \cdot 1{,}6 \cdot 10^{-3} = 64 \cdot 10^3\ kPa \cdot m^4$

$EA = 40 \cdot 10^6 \cdot 0{,}12 = 4{,}8 \cdot 10^6\ kPa \cdot m^2$

$\Delta t_0 = \frac{1}{2}(5 + 10) = 7{,}5\ °C$

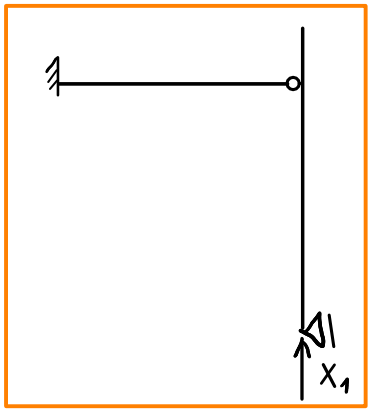
$\Delta t_1 = t_d - t_h = 10 - 5 = 5\ °C$

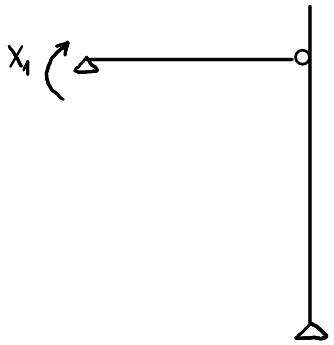
→ ŘEŠENÍ SILOVOU METODOU (ZANEDBEJTE VLIV POSOUVAJÍCÍCH SIL)

→ 1) STUPEŇ STAT. NEURČITOSTI, ZÁKLADNÍ SOUSTAVA

$n_s = 1$

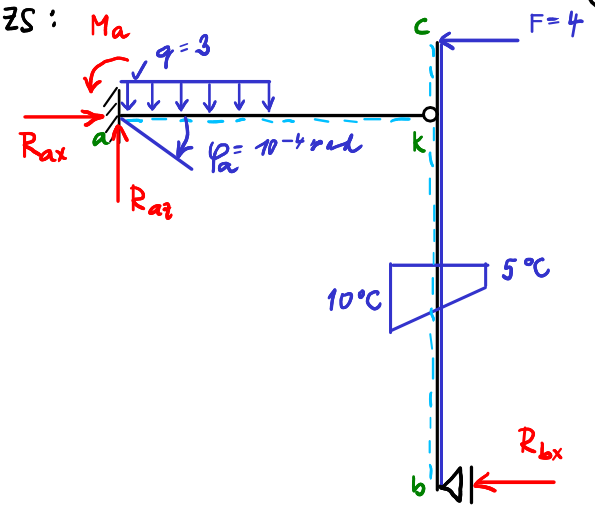
Volba ZSUS:

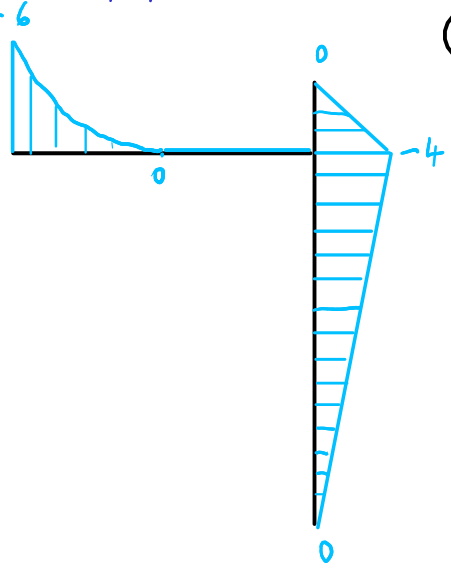


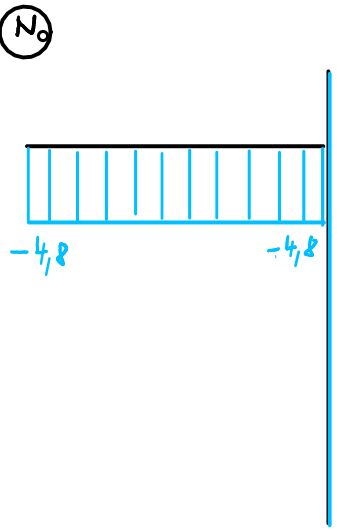


→ 2) ZATĚŽOVACÍ STAVY, VYKRESLENÍ N, V, M

0.ZS:







$\oplus\circlearrowleft\ \Sigma M_{ik}^{?} = 0: \quad F \cdot 1 - R_{bx} \cdot 5 = 0 \rightarrow R_{bx} = 0{,}8\ kN\ (\leftarrow)$

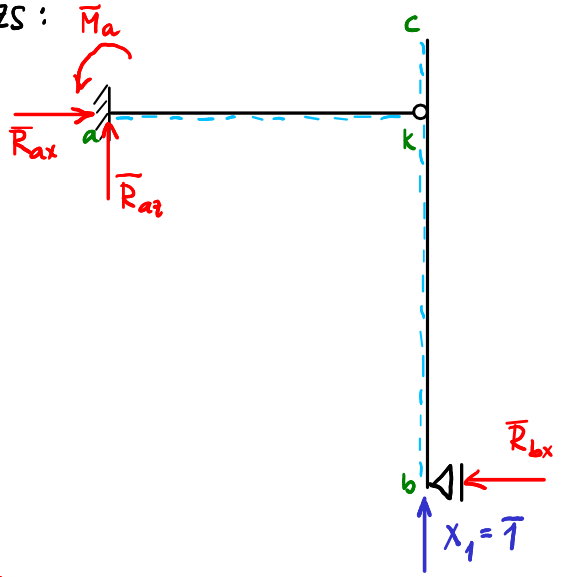
$\oplus\rightarrow\ \Sigma F_{ix} = 0: \quad R_{ax} - F - R_{bx} = 0 \rightarrow R_{ax} = 4{,}8\ kN\ (\rightarrow)$

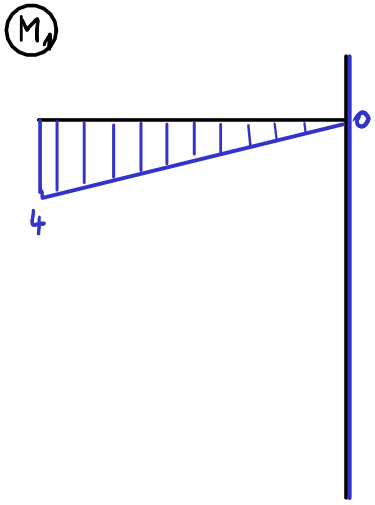
$\oplus\uparrow\ \Sigma F_{iz} = 0: \quad R_{az} - q \cdot 2 = 0 \rightarrow R_{az} = 6\ kN\ (\uparrow)$

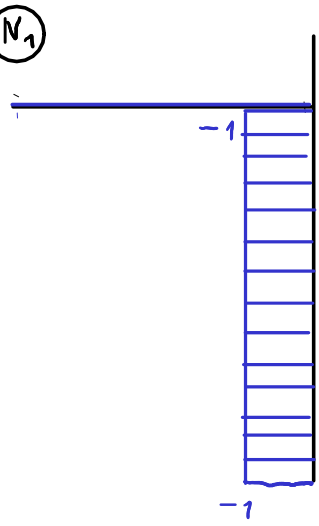
$\oplus\circlearrowleft\ \Sigma M_{ia} = 0: \quad M_a - q \cdot 2 \cdot 1 + F \cdot 1 - R_{bx} \cdot 5 = 0 \rightarrow M_a = 6\ kNm\ (\circlearrowleft)$

$\oplus\circlearrowleft$ KO: $\Sigma M_{ik}^{L} = 0: \quad M_a - R_{az} \cdot 4 + q \cdot 2 \cdot 3 = 0$ ✓

1. ZS :







$\oplus\circlearrowleft \sum M_{ik}^{P} = 0 : \quad -\bar{R}_{bx} \cdot 5 = 0 \rightarrow \bar{R}_{bx} = 0$

$\oplus\rightarrow \sum F_{ix} = 0 : \quad \bar{R}_{ax} - \bar{R}_{bx} = 0 \rightarrow \bar{R}_{ax} = 0$

$\oplus\uparrow \sum F_{iz} = 0 : \quad \bar{R}_{az} + X_1 = 0 \rightarrow \bar{R}_{az} = -1\ kN \ (\downarrow)$

$\oplus\circlearrowleft \sum M_{ia} = 0 : \quad \bar{M}_a - \bar{R}_{bx} \cdot 5 + X_1 \cdot 4 = 0 \rightarrow \bar{M}_a = -4\ kNm \ (\circlearrowleft)$

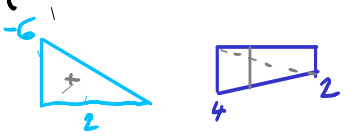
$\oplus$ KO : $\sum M_{ik}^{L} = 0 : \quad \bar{M}_a - \bar{R}_{az} \cdot 4 = 0$ ✓

→ 3) SOUSTAVA PŘETVÁRNÝCH ROVNIC, VYČÍSLENÍ PŘETVÁRNÝCH KOEFICIENTŮ

$$\boxed{\delta_{1,0} + \delta_{1,1} \cdot X_1 = 0} \qquad / w_b = 0 /$$

$$\delta_{1,0} = \int \frac{M_1 M_0}{EI} dx + \int \frac{N_1 N_0}{EA} dx + \int M_1 \alpha_T \Delta t_1 \, dx + \int N_1 \alpha_T \Delta t_0 \, dx - \sum \bar{M}_a \varphi_a$$

$$\delta_{1,0} = \frac{1}{64 \cdot 10^3} \cdot \left\{ \left(\tfrac{1}{2} \cdot (-6) \cdot 2 \right) \cdot \left[\tfrac{1}{3} \cdot 2 + \tfrac{2}{3} \cdot 4 \right] + \left(\tfrac{2}{3} \cdot 2 \cdot \tfrac{1}{8} \cdot 3 \cdot 2^2 \right) \cdot \left[\tfrac{1}{2} \cdot 2 + \tfrac{1}{2} \cdot 4 \right] \right\} +$$



$$+ \frac{1}{4,8 \cdot 10^6} \cdot \{ \phi \} + 10^{-5} \cdot 5 \cdot \phi + 10^{-5} \cdot 7,5 \cdot (-1) \cdot 5 - (+4 \cdot 10^{-4})$$

$\bar{M}_a = 4$; $\varphi_a = 10^{-4}$

$$\delta_{1,0} = \frac{-14}{64 \cdot 10^3} - 3,75 \cdot 10^{-4} - 4 \cdot 10^{-4}$$

$$\underline{\delta_{1,0} = -9,9375 \cdot 10^{-4}}$$

$$\delta_{1,1} = \int \frac{M_1 M_1}{EI} dx + \int \frac{N_1 N_1}{EA} dx$$

$$\delta_{1,1} = \frac{1}{64 \cdot 10^3} \left\{ \left(\tfrac{1}{2} \cdot 4 \cdot 4 \right) \cdot \left[\tfrac{2}{3} \cdot 4 \right] \right\} + \frac{1}{4,8 \cdot 10^6} \cdot \{ (5 \cdot (-1)) \cdot [-1] \} = \frac{64}{3 \cdot 64 \cdot 10^3} + \frac{5}{4,8 \cdot 10^6}$$

$$\underline{\delta_{1,1} = 3,34375 \cdot 10^{-4}}$$

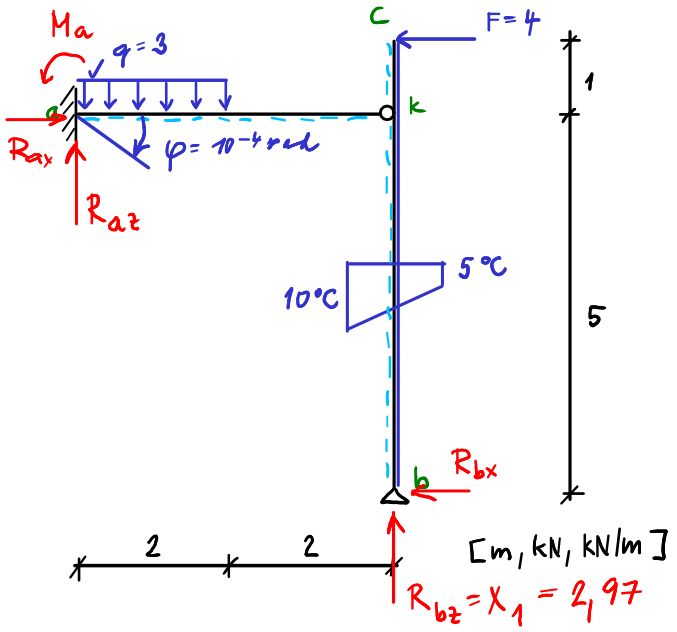
→ 4) ŘEŠENÍ SOUSTAVY PŘETVÁRNÝCH ROVNIC

$$\delta_{1,0} + \delta_{1,1} \cdot X_1 = 0$$

$$-9{,}9375 \cdot 10^{-4} + 3{,}34375 \cdot 10^{-4} \cdot X_1 = 0$$

$$X_1 \doteq 2{,}972\,kN \quad (\uparrow)$$

→ 5) VYKRESLENÍ VNITŘNÍCH SIL:



$\oplus\circlearrowleft \sum M_{ik}^{?} = 0: \quad F \cdot 1 - R_{bx} \cdot 5 = 0 \rightarrow R_{bx} = 0{,}8\,kN \ (\leftarrow)$

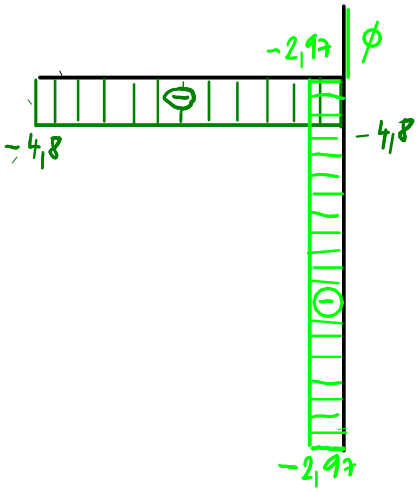
$\oplus\rightarrow \sum F_{ix} = 0: \quad R_{ax} - F - R_{bx} = 0 \rightarrow R_{ax} = 4{,}8\,kN \ (\rightarrow)$

$\oplus\uparrow \sum F_{iz} = 0: \quad R_{az} - q \cdot 2 + R_{bz} = 0 \rightarrow R_{az} = 3{,}03\,kN \ (\uparrow)$

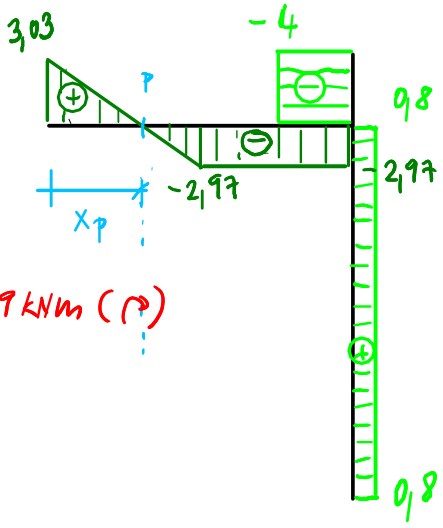
$\oplus\circlearrowleft \sum M_{ia} = 0: \quad M_a - q \cdot 2 \cdot 1 + F \cdot 1 - R_{bx} \cdot 5 + R_{bz} \cdot 4 = 0 \rightarrow M_a = -5{,}89\,kNm \ (\curvearrowleft)$

$\oplus\circlearrowleft$ KO: $\sum M_{ik}^{L} = 0: \quad M_a - R_{az} \cdot 4 + q \cdot 2 \cdot 3 = 0$ ✓

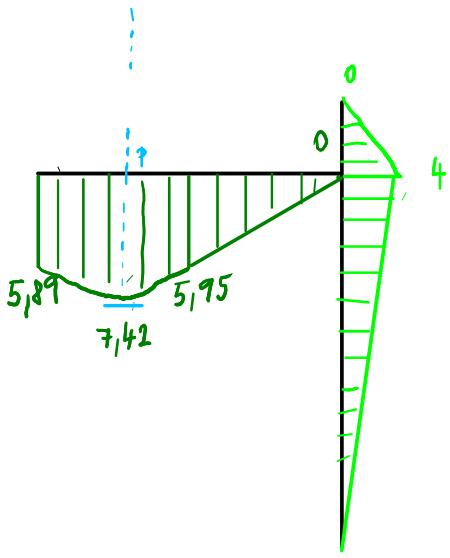
(N)



(V)

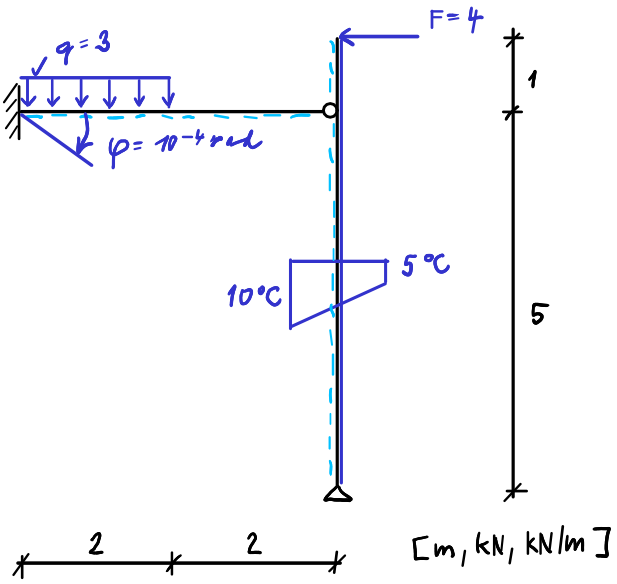


(M)



$$x_p = \frac{V_p^{L}}{q} = \frac{3{,}03}{3} = 1{,}01\,m$$

$$M_p = 5{,}89 + 3{,}03 \cdot 1{,}01 - 3 \cdot \frac{1{,}01^2}{2} = 7{,}42\,kNm$$



$E = 40\ GPa = 40 \cdot 10^6\ kPa$

$\alpha_T = 1 \cdot 10^{-5}\ °C^{-1}$

$A = 0{,}3 \cdot 0{,}4 = 0{,}12\ m^2$

$I = \frac{1}{12} \cdot 0{,}3 \cdot 0{,}4^3 = 1{,}6 \cdot 10^{-3}\ m^4$

$EI = 40 \cdot 10^6 \cdot 1{,}6 \cdot 10^{-3} = 64 \cdot 10^3\ kPa \cdot m^4$

$EA = 40 \cdot 10^6 \cdot 0{,}12 = 4{,}8 \cdot 10^6\ kPa \cdot m^2$

$\Delta t_0 = \frac{1}{2}(5 + 10) = 7{,}5\ °C$

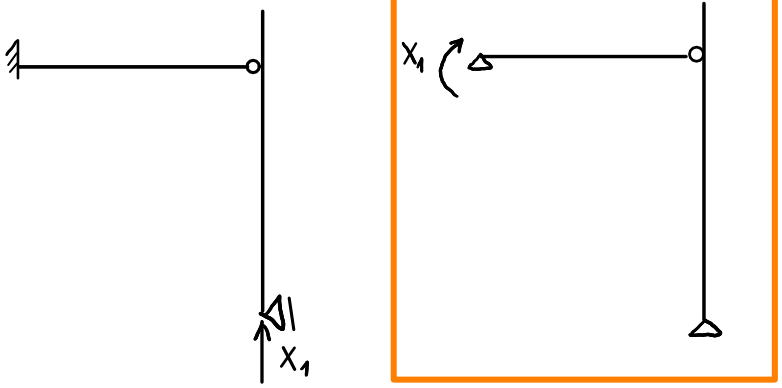
$\Delta t_1 = t_d - t_h = 10 - 5 = 5\ °C$

→ **ŘEŠENÍ SILOVOU METODOU** (ZANEDBEJTE VLIV POSOUVAJÍCÍCH SIL)

→ 1) STUPEŇ STAT. NEURČITOSTI, ZÁKLADNÍ SOUSTAVA

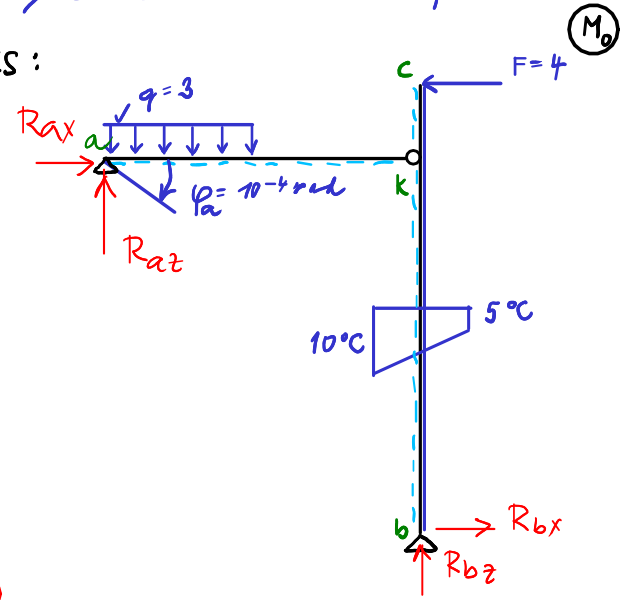
$n_s = 1$

Volba ZSUS:

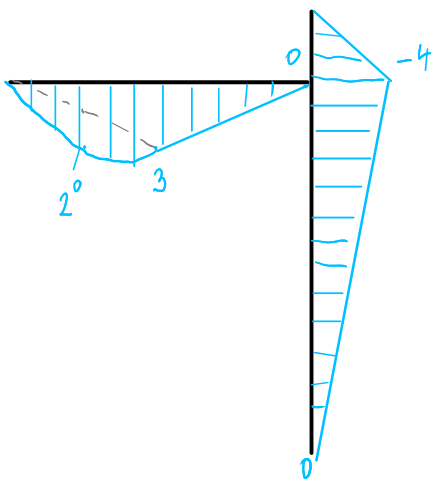


→ 2) ZATĚŽOVACÍ STAVY, VYKRESLENÍ N, V, M

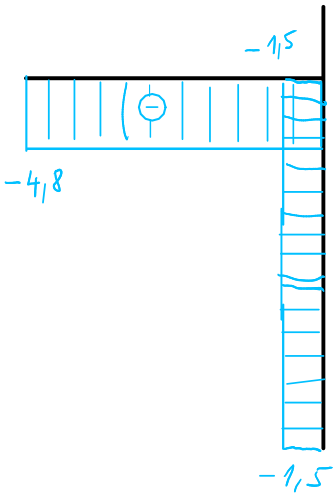
0.ZS:



M_0



N_0



⊕↺ $\Sigma M_{ik}^P = 0: \quad F \cdot 1 + R_{bx} \cdot 5 = 0 \rightarrow R_{bx} = -0{,}8\ kN\ (\leftarrow)$

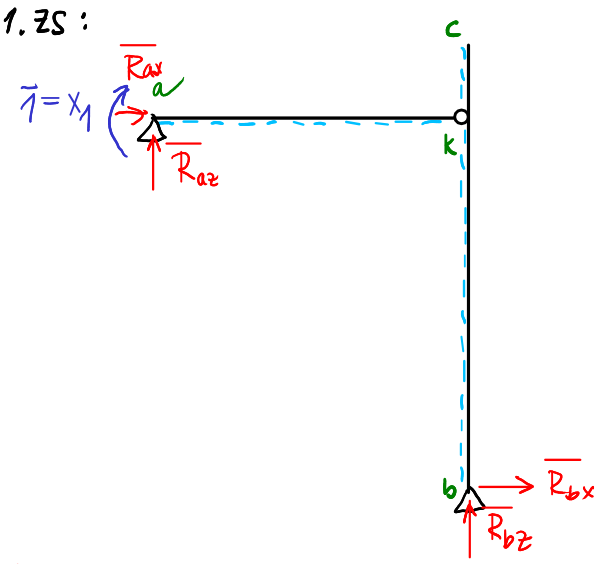
⊕→ $\Sigma F_{ix} = 0: \quad R_{ax} + R_{bx} - F = 0 \rightarrow R_{ax} = 4{,}8\ kN\ (\rightarrow)$

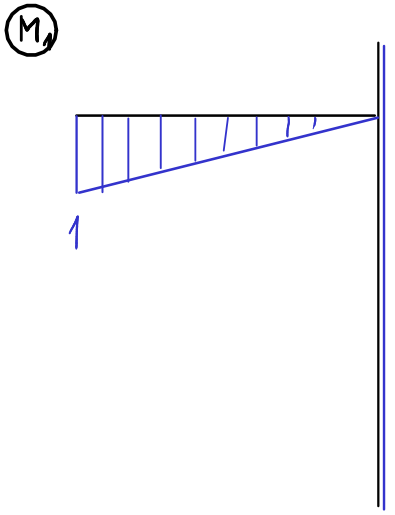
⊕↺ $\Sigma M_{ik}^L = 0: \quad q \cdot 2 \cdot 3 - R_{az} \cdot 4 = 0 \rightarrow R_{az} = 4{,}5\ kN\ (\uparrow)$

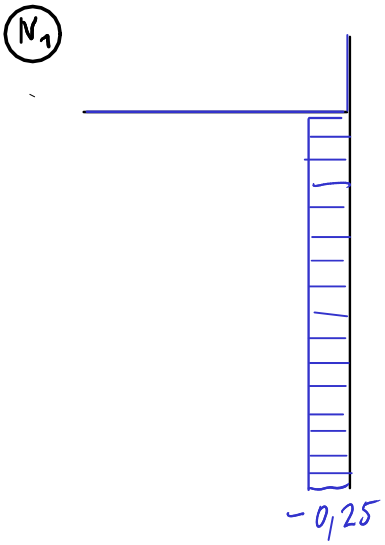
⊕↑ $\Sigma F_{iz} = 0: \quad R_{az} - q \cdot 2 + R_{bz} = 0 \rightarrow R_{bz} = 1{,}5\ kN\ (\uparrow)$

KO: $\Sigma M_{ib} = 0: \quad q \cdot 2 \cdot 3 - R_{az} \cdot 4 - R_{ax} \cdot 5 + F \cdot 6 = 0$ ✓

1.ZS:







$\oplus$ $\Sigma M_{ik}^{P} = 0 : \overline{R_{bx}} \cdot 5 = 0 \rightarrow \overline{R_{bx}} = 0$

$\oplus$ $\Sigma F_{ix} = 0 : \overline{R_{ax}} + \overline{R_{bx}} = 0 \rightarrow \overline{R_{ax}} = 0$

$\oplus$ $\Sigma M_{ia} = 0 : -X_1 + \overline{R_{bx}} \cdot 5 + \overline{R_{bz}} \cdot 4 = 0 \rightarrow \overline{R_{bz}} = 0,25 \ (\uparrow)$

$\oplus$ $\Sigma M_{ik}^{L} = 0 : -X_1 - \overline{R_{az}} \cdot 4 = 0 \rightarrow \overline{R_{az}} = -0,25 \ (\downarrow)$

KO: $\Sigma F_{iz} = 0 : \overline{R_{az}} + \overline{R_{bz}} = 0$ ✓

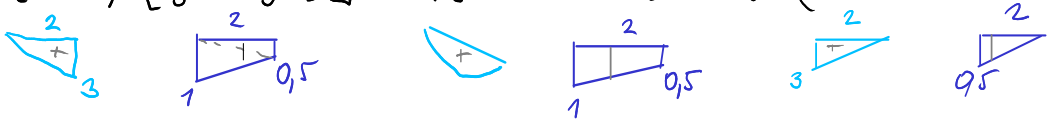
3) SOUSTAVA PŘETVÁRNÝCH ROVNIC, VYČÍSLENÍ PŘETVÁRNÝCH KOEFICIENTŮ

1) X_1 2) φ_a

$$\delta_{1,0} + \delta_{1,1} \cdot X_1 = 10^{-4} \qquad / \ \varphi_a = 10^{-4} /$$

$$\delta_{1,0} = \int \frac{M_1 M_0}{EI} dx + \int \frac{N_1 N_0}{EA} dx + \int M_1 \alpha_T \Delta t_1 \, dx + \int N_1 \alpha_T \Delta t_0 \, dx - \Sigma \overline{R} \cdot \delta$$

$$\delta_{1,0} = \frac{1}{64 \cdot 10^3} \cdot \left\{ \left(\tfrac{1}{2} \cdot 3 \cdot 2\right) \cdot \left[\tfrac{1}{3} \cdot 1 + \tfrac{2}{3} \cdot \tfrac{1}{2}\right] + \left(\tfrac{2}{3} \cdot \tfrac{1}{8} \cdot 3 \cdot 2^2 \cdot 2\right) \cdot [0,75] + \left(\tfrac{1}{2} \cdot 2 \cdot 3\right) \cdot \left[\tfrac{2}{3} \cdot \tfrac{1}{2}\right] \right\} +$$



$$+ \frac{1}{4,8 \cdot 10^6} \cdot \left\{ (-1,5 \cdot 5 \cdot 1) \cdot [-0,25] \right\} + \phi + 10^{-5} \cdot 7,5 \cdot (-0,25) \cdot 5 - \phi$$

$$\delta_{1,0} = \frac{45}{64 \cdot 10^3} + \frac{1,875}{4,8 \cdot 10^6} + \phi - 9,375 \cdot 10^{-5} - \phi$$

$$\underline{\delta_{1,0} = -2,30469 \cdot 10^{-5}}$$

$$\delta_{1,1} = \int \frac{M_1 M_1}{EI} dx + \int \frac{N_1 N_1}{EA} dx$$

$$\delta_{1,1} = \frac{1}{64 \cdot 10^3} \left\{ \left(\tfrac{1}{2} \cdot 1 \cdot 4\right) \cdot \left[\tfrac{2}{3} \cdot 1\right] \right\} + \frac{1}{4,8 \cdot 10^6} \cdot \left\{ (-0,25 \cdot 5) \cdot [-0,25] \right\} = \frac{4}{3 \cdot 64 \cdot 10^3} + \frac{0,3125}{4,8 \cdot 10^6}$$

$$\underline{\delta_{1,1} = 2,08984 \cdot 10^{-5}}$$

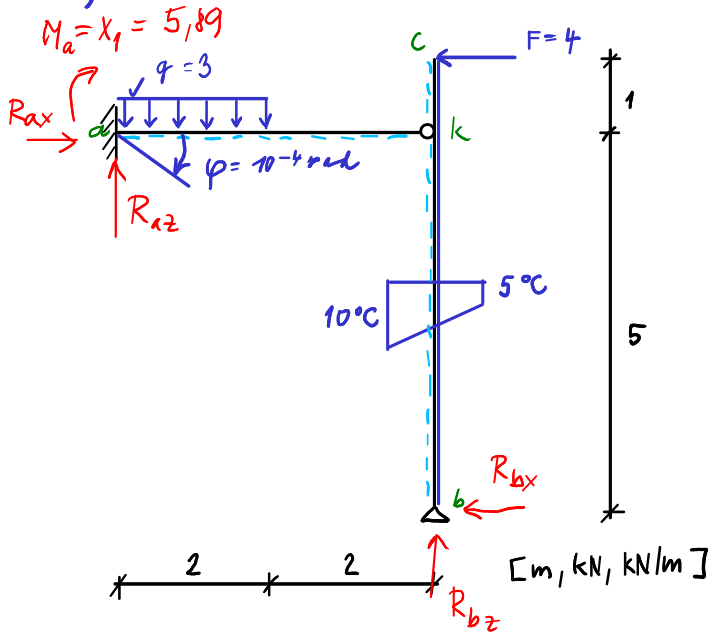
4) ŘEŠENÍ SOUSTAVY PŘETVÁRNÝCH ROVNIC

$$\delta_{1,0} + \delta_{1,1} \cdot X_1 = 10^{-4}$$

$$-2{,}30469 \cdot 10^{-5} + 2{,}08984 \cdot 10^{-5} \cdot X_1 = 10^{-4}$$

$$X_1 = 5{,}888 \text{ kNm} \quad (\curvearrowright)$$

5) VYKRESLENÍ VNITŘNÍCH SIL:



$\oplus\circlearrowleft \sum M_{ik}^{P} = 0: \quad F \cdot 1 - R_{bx} \cdot 5 = 0 \rightarrow R_{bx} = 0{,}8 \text{ kN} \ (\leftarrow)$

$\oplus\rightarrow \sum F_{ix} = 0: \quad R_{ax} - R_{bx} - F = 0 \rightarrow R_{ax} = 4{,}8 \text{ kN} \ (\rightarrow)$

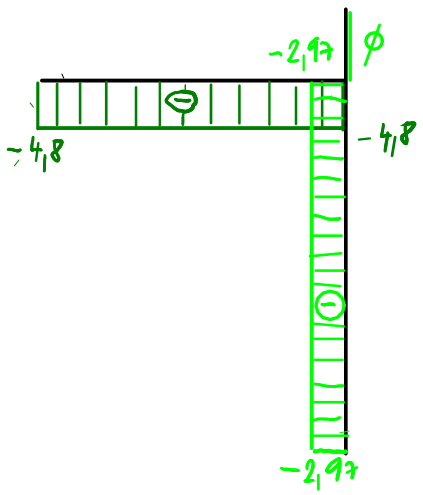
$\oplus\circlearrowleft \sum M_{ia} = 0: \quad -M_a - q \cdot 2 \cdot 1 + F \cdot 1 + R_{bz} \cdot 4 - R_{bx} \cdot 5 = 0$

$\rightarrow R_{bz} = 2{,}97 \text{ kN} \ (\uparrow)$

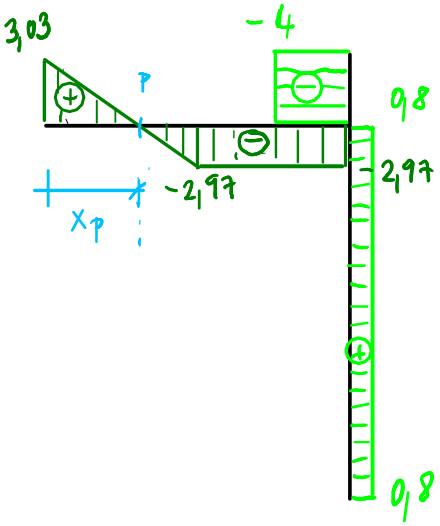
$\oplus\uparrow \sum F_{iz} = 0: \quad R_{az} - q \cdot 2 + R_{bz} = 0 \rightarrow R_{az} = 3{,}03 \text{ kN} \ (\uparrow)$

KO: $\oplus\circlearrowleft \sum M_{ik}^{L} = 0: \quad -M_a - R_{az} \cdot 4 + q \cdot 2 \cdot 3 = 0$ ✓

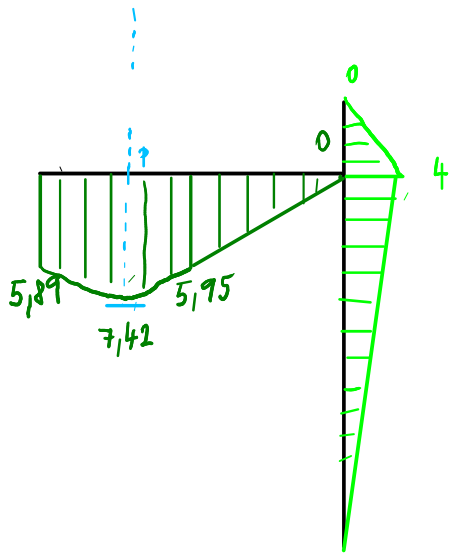
(N)



(V)



(M)



$$x_p = \frac{V_p^L}{q} = \frac{3{,}03}{3} = 1{,}01 \text{ m}$$

$$M_p = 5{,}89 + 3{,}03 \cdot 1{,}01 - 3 \cdot \frac{1{,}01^2}{2} = 7{,}42 \text{ kNm}$$