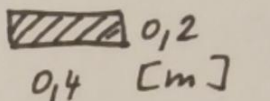
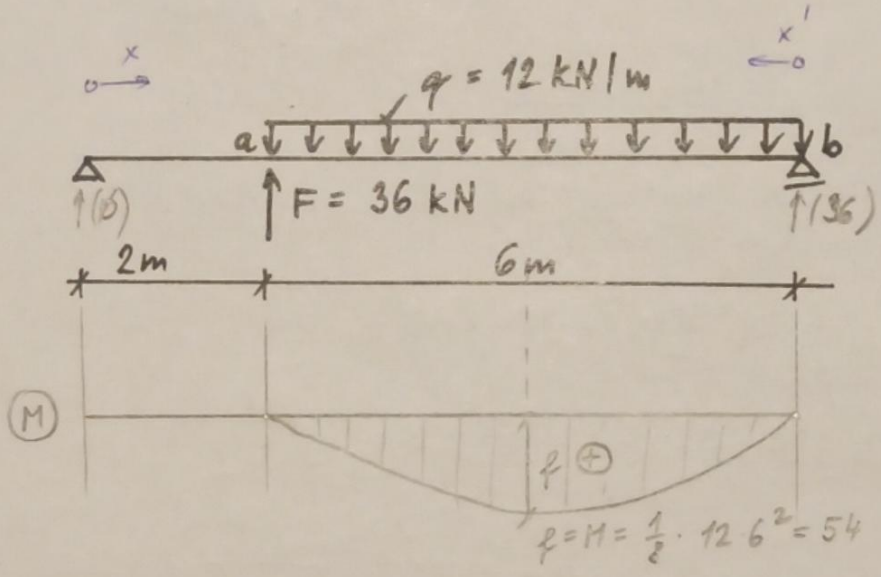
SPOČÍTEJTE PRŮHYB V BODĚ „a" A POOTOČENÍ V BODĚ „b". E = 240 GPa, PRŮŘEZ





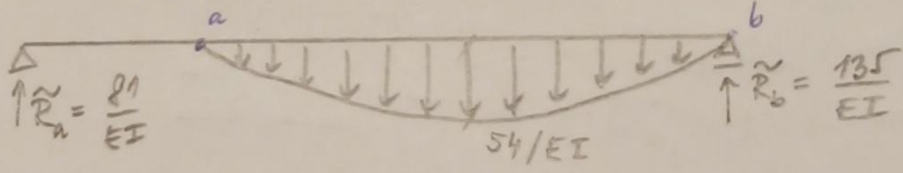
$f = M = \frac{1}{2} \cdot 12 \cdot 6^2 = 54$

$EI = 240 \cdot 10^6 \cdot \frac{1}{12} \cdot 0{,}4 \cdot 0{,}2^3$

$EI = 64000 \text{ kPa} \cdot m^4$

MOHROVA METODA:

Fiktivní nosník



$$w_a = \tilde{M}_a = \tilde{R}_a \cdot 2 = \frac{81}{EI} \cdot 2 = \frac{162}{EI} = \frac{162}{64000} = 2{,}53125 \cdot 10^{-3} m$$

$$\varphi_b = \tilde{V}_b = -\tilde{R}_b = -\frac{135}{EI} = -\frac{135}{64000} = -2{,}109375 \cdot 10^{-3} rad$$

CLEBSCHOVA METODA:

$$M(x) = \Big|_{x>2}^{+} 36(x-2) - 12\frac{(x-2)^2}{2}$$

$$\varphi(x) = -\frac{1}{EI}\left[C_1 \Big|_{x>2}^{+} 36\frac{(x-2)^2}{2} - 6\frac{(x-2)^3}{3} \right]$$

$$w(x) = -\frac{1}{EI}\left[C_2 + C_1 x \Big|_{x>2}^{+} 18\frac{(x-2)^3}{3} - 2\frac{(x-2)^4}{4} \right]$$

okrajové podmínky:

$w(x=0) = 0 \rightarrow C_2 = 0$

$w(x=8) = 0 \rightarrow 0 + 8C_1 + 648 = 0 \rightarrow C_1 = -81$

$$w_a = w(x=2) = -\frac{1}{EI}\left[0 - 81\cdot 2\right] = \frac{162}{EI} = \frac{162}{64000} = 2{,}53125\cdot 10^{-3}\,m$$

$$\varphi_b = \varphi(x=8) = -\frac{1}{EI}\left[-81 + 18\cdot 6^2 - 2\cdot 6^3\right] = \frac{-135}{EI} = \frac{-135}{64000} =$$

$$= -2{,}109375\cdot 10^{-3}\,rad$$

CLEBSCHOVA METODA PRO x' JDOUCÍ ZPRAVA:
(VLOŽITELNÝ)

$$M(x') = 36x' - 12\frac{x'^2}{2}\Big|_{x>6} + 12\frac{(x'-6)^2}{2} + 36(x'-6)$$

$$\varphi(x') = -\frac{1}{EI}\left[C_1 + 36\frac{x'^2}{2} - 6\frac{x'^3}{3}\Big|_{x>6} + 6\frac{(x'-6)^3}{3} + 36\frac{(x'-6)^2}{2}\right]$$

$$w(x') = -\frac{1}{EI}\left[C_2 + C_1x' + 18\frac{x'^3}{3} - 2\cdot\frac{x'^4}{4}\Big|_{x>6} + 2\frac{(x'-6)^4}{4} + 18\frac{(x'-6)^3}{3}\right]$$

okrajové podmínky:

$w(x'=0) = 0 \rightarrow C_2 = 0$

$w(x'=8) = 0 \rightarrow 0 + 8C_1 + 1080 = 0 \rightarrow C_1 = -135$

$$w_a = w(x'=6) = -\frac{1}{EI}\left[0 - 135\cdot 6 + 6\cdot 6^3 - 0{,}5\cdot 6^4\right] =$$

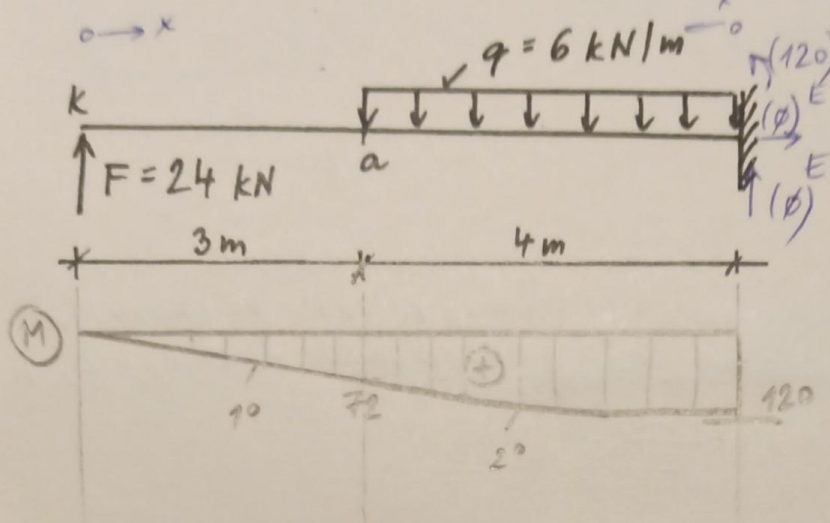
$$= \frac{162}{EI} = 2{,}53125\cdot 10^{-3}\,m$$

$$\varphi_b = \ominus\varphi(x'=0) = -\left(-\frac{1}{EI}\left[-135\right]\right) = \frac{-135}{EI} = -2{,}109375\cdot 10^{-3}\,rad$$

↓

OPAČNÉ ZNAMÉNKO
PRO VÝPOČET ZPRAVA ($\varphi_L = -\varphi_P$)

SPOČÍTEJTE POOTOČENÍ V BODĚ „a" A PRŮHYB V BODĚ „k". E = 60 GPa, PRŮŘEZ

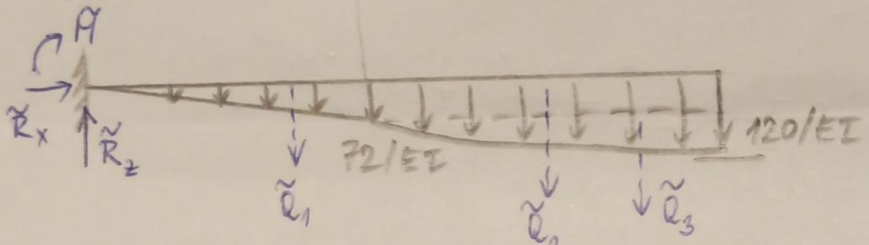


$EI = 60 \cdot 10^6 \cdot \frac{1}{12} \cdot 0{,}2 \cdot 0{,}4^3$

$EI = 64000\ kPa \cdot m^4$

MOHROVA METODA:

Fiktivní nosník



$\varphi_a = \tilde{V}_a = +\tilde{Q}_2 + \tilde{Q}_3 = \frac{1}{EI}\left[72 \cdot 4 + \frac{2}{3} \cdot 4 \cdot 48\right] = \frac{416}{EI} = \frac{416}{64000} = \underline{6{,}5 \cdot 10^{-3}\ rad}$

$w_k = \tilde{M}_k = \tilde{M} = -\tilde{Q}_1 \cdot r_1 - \tilde{Q}_2 \cdot r_2 - \tilde{Q}_3 \cdot r_3 =$

$= -\frac{1}{EI}\left[-\frac{1}{2} \cdot 72 \cdot 3 \cdot 2 - 72 \cdot 4 \cdot 5 - \frac{2}{3} \cdot 4 \cdot 48 \cdot \left(3 + \frac{5}{8} \cdot 4\right)\right] = \frac{-2360}{EI} =$

$= \underline{-0{,}036875\ m}$

CLEBSCHOVA METODA:

$M(x) = 24 \cdot x \Big|^{-6 \cdot \frac{(x-3)^2}{2}}_{x>3}$

$\varphi(x) = -\frac{1}{EI}\left[C_1 + 24\frac{x^2}{2}\Big|^{-3\frac{(x-3)^3}{3}}_{x>3}\right]$

$w(x) = -\frac{1}{EI}\left[C_2 + C_1 x + 12\frac{x^3}{3}\Big|^{-\frac{(x-3)^4}{4}}_{x>3}\right]$

okrajové podmínky:

$w(x=7)=0 \rightarrow C_2 + 7C_1 + 1308 = 0$

$\varphi(x=7)=0 \rightarrow C_1 + 524 = 0 \rightarrow \underline{C_1 = -524}$

$\underline{C_2 = 2360}$

$\underline{\varphi_a} = \varphi(x=3) = -\frac{1}{64000}\left[-524 + 12\cdot 3^2\right] = \frac{416}{64000} = \underline{6{,}5\cdot 10^{-3}\,rad}$

$\underline{w_k} = w(x=0) = -\frac{1}{64000}\left[2360\right] = \frac{-2360}{64000} = \underline{-0{,}036875\,m}$

CLEBSCHOVA METODA PRO X' JDOUCÍ ZPRAVA

(SLOŽITĚJŠÍ, NUTNO ZNÁT REAKCE VE VETKNUTÍ) (ODPADNOU ALE C_1 a C_2)

$$M(x') = +120 - 6\cdot\frac{x'^2}{2}\Big|_{x>4} + 6\,\frac{(x'-4)^2}{2}$$

$$\varphi(x') = \left[C_1 + 120x' - 3\,\frac{x'^3}{3}\Big|_{x>4} + 3\,\frac{(x'-4)^3}{3}\right]\cdot\left(-\frac{1}{EI}\right)$$

$$w(x') = -\frac{1}{EI}\left[C_2 + C_1 x' + 120\frac{x'^2}{2} - \frac{x'^4}{4}\Big|_{x>4} + \frac{(x'-4)^4}{4}\right]$$

okrajové podmínky:

$w(x=0)=0$: $\underline{C_2 = 0}$

$\varphi(x=0)=0$: $\underline{C_1 = 0}$

$\underline{\varphi_a} = -\varphi(x'=4) = +\frac{1}{64000}\left[0 + 120\cdot 4 - 4^3\right] = \frac{416}{64000} = \underline{6{,}5\cdot 10^{-3}\,rad}$

↓ OPAČNÉ ZNAMÉNKO PRO ŘEŠENÍ ZPRAVA ($\varphi_L = -\varphi_P$)

$\underline{w_k} = w(x'=7) = -\frac{1}{64000}\left[0 + 0\cdot 7 + 60\cdot 7^2 - \frac{7^4}{4} + \frac{3^4}{4}\right] =$

$= -\frac{2360}{64000} = \underline{-0{,}036875\,m}$