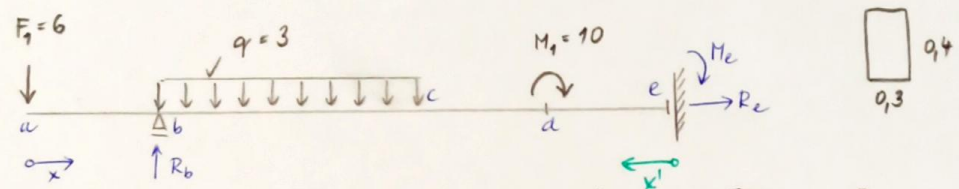
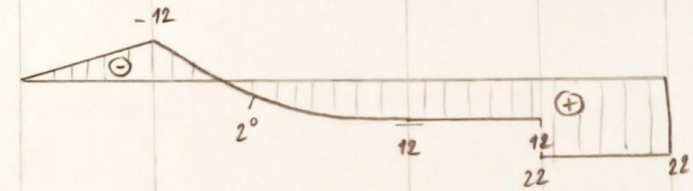
Př.: Určete φ_a, w_a, w_c

$E = 25$ GPa





$\Sigma F_{ix} = \emptyset$:

$R_e = \emptyset$

$\oplus\circlearrowleft \Sigma M_{ib} = \emptyset$:

$F_1 2 - q \cdot 4 \cdot 2 - M_1 - M_e = \emptyset$

$M_e = 6 \cdot 2 - 3 \cdot 4 \cdot 2 - 10$

$M_e = -22$ kNm

$\oplus\circlearrowleft \Sigma M_{ie} = \emptyset$:

$F_1 10 - R_b \cdot 8 + q \cdot 4 \cdot 6 - M_1 - M_e = \emptyset$

$R_b = \frac{1}{8}\left(6 \cdot 10 + 3 \cdot 4 \cdot 6 - 10 - (-22)\right)$

$R_b = 18$ kN

$\oplus\downarrow \Sigma F_{iz} = \emptyset$

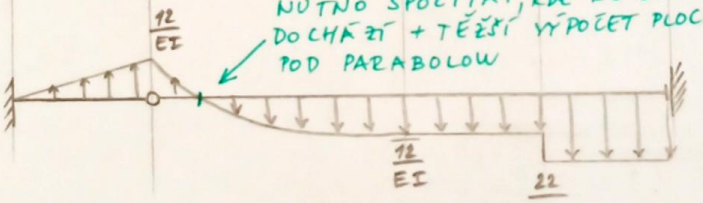
$F_1 - R_b + q \cdot 4 = \emptyset$

$6 - 18 + 3 \cdot 4 = \emptyset$ ✓

MOHROVA ANALOGIE

Duální nosník:

ZDE NENÍ VHODNÁ Z DŮVODU PŘECHODU PARABOLY ZE ZÁPORNÉHO DO KLADNÉHO ZATÍŽENÍ → NUTNO SPOČÍTAT, KDE K ZMĚNĚ DOCHÁZÍ + TĚŽŠÍ VÝPOČET PLOCHY POD PARABOLOU



$EI = 25 \cdot 10^6 \cdot \frac{1}{12} \cdot 0{,}3 \cdot 0{,}4^3$

$EI = 40\,000$ kPa $\cdot m^4 = 4 \cdot 10^4$ kPa m^4

CLEBSCHOVA METODA: (na statickém nosníku)

$$M(x) = -F_1 \cdot x \Big|_{x>2} + R_b (x-2) - q \frac{(x-2)^2}{2} \Big|_{x>6} + q \frac{(x-6)^2}{2} \Big|_{x>8} + M_1 (x-8)^0$$

$$\varphi(x) = -\frac{1}{EI}\left[C_1 - 6\frac{x^2}{2} \Big|_{x>2} + 18\frac{(x-2)^2}{2} - 3\frac{(x-2)^3}{6} \Big|_{x>6} + 3\frac{(x-6)^3}{6} \Big|_{x>8} + 10(x-8) \right]$$

$$w(x) = -\frac{1}{EI}\left[C_2 + C_1 x - 6\frac{x^3}{6} \Big|_{x>2} + 18\frac{(x-2)^3}{6} - 3\frac{(x-2)^4}{24} \Big|_{x>6} + 3\frac{(x-6)^4}{24} \Big|_{x>8} + 10\frac{(x-8)^2}{2} \right]$$

okrajové podmínky: $w_b = 8w(x=2) = \emptyset \Rightarrow C_2 + C_1 \cdot 2 - 6\frac{2^3}{6} = \emptyset$

$\varphi_e = \varphi(x=10) = \emptyset \Rightarrow C_1 - 6\frac{10^2}{2} + 18\frac{8^2}{2} - 3\frac{8^3}{6} + 3\frac{4^3}{6} + 10 \cdot 2 = \emptyset$

$C_1 = -72$

$C_2 = 152$

úprava rovnic:

$$\varphi(x) = -\frac{1}{4 \cdot 10^4}\left[-72 - 3x^2 \Big|_{x>2} + 9(x-2)^2 - 0{,}5(x-2)^3 \Big|_{x>6} + 0{,}5(x-6)^3 \Big|_{x>8} + 10(x-8) \right]$$

$$w(x) = -\frac{1}{4 \cdot 10^4}\left[152 - 72 \cdot x - x^3 \Big|_{x>2} + 3(x-2)^3 - \frac{(x-2)^4}{8} \Big|_{x>6} + \frac{(x-6)^4}{8} \Big|_{x>8} + 5(x-8)^2 \right]$$

výpočet přemístění:

$$\varphi_a = \varphi(x=0) = -\frac{1}{4\cdot 10^4}\left[-72\right] = \underline{1{,}8\cdot 10^{-3}\ rad}$$

$$w_a = w(x=0) = -\frac{1}{4\cdot 10^4}\left[152\right] = \underline{-3{,}8\cdot 10^{-3}\ m}$$

$$w_c = w(x=6) = -\frac{1}{4\cdot 10^4}\left[152 - 72\cdot 6 - 6^3 + 3\cdot 4^3 - \frac{4^4}{8}\right] = \underline{8{,}4\cdot 10^{-3}\ m}$$

CLEBSCHOVOU METODOU (POKUD VE TO OPODSTATNĚNÉ) LZE ŘEŠIT NOSNÍK I PRO X KÓTOVANÉ ZPRAVA → ZDE OZNAČENO JAKO x'
→ POTOM W VYJDE STEJNĚ A φ S OPAČNÝM ZNAMÉNKEM

$$M(x') = -M_e\Big|_{x'>2} - M_1(x'-2)^0\Big|_{x'>4} - q\frac{(x'-4)^2}{2}\Big|_{x'>8} + q\frac{(x'-8)^2}{2} + R_b(x'-8)$$

$$\varphi(x') = -\frac{1}{EI}\left[C_1 - (-22)\cdot x'\Big|_{x'>2} - 10(x'-2)\Big|_{x'>4} - 3\frac{(x'-4)^3}{6}\Big|_{x'>8} + 3\frac{(x'-8)^3}{6} + 18\frac{(x'-8)^2}{2}\right.$$

$$w(x') = -\frac{1}{EI}\left[C_2 + C_1 x' + 22\frac{x'^2}{2}\Big|_{x'>2} - 10\frac{(x'-2)^2}{2}\Big|_{x'>4} - 3\frac{(x'-4)^4}{24}\Big|_{x'>8} + 3\frac{(x'-8)^4}{24} + 18\frac{(x'-8)^3}{6}\right.$$

okrajové podmínky:

$$w_b = w(x'=8) = 0 \Rightarrow C_2 + C_1\cdot 8 + \frac{22\cdot 8^2}{2} - 10\cdot\frac{6^2}{2} - 3\cdot\frac{4^4}{24} - 0 \Rightarrow \underline{C_2 = -492}$$

$$\varphi_e = \varphi(x'=0) = 0 \Rightarrow \underline{C_1 = 0}$$

úprava rovnic:

$$\varphi(x') = -\frac{1}{4\cdot 10^4}\left[22x'\Big|_{x'>2} - 10(x'-2)\Big|_{x'>4} - 0{,}5(x'-4)^3\Big|_{x'>8} + 0{,}5(x'-8)^3 + 9(x'-8)^2\right.$$

$$w(x') = -\frac{1}{4\cdot 10^4}\left[-492 + 11x'^2\Big|_{x'>2} - 5(x'-2)^2\Big|_{x'>4} - \frac{(x'-4)^4}{8}\Big|_{x'>8} + \frac{(x'-8)^4}{8} + 3(x'-8)^3\right.$$

výpočet přetvoření:

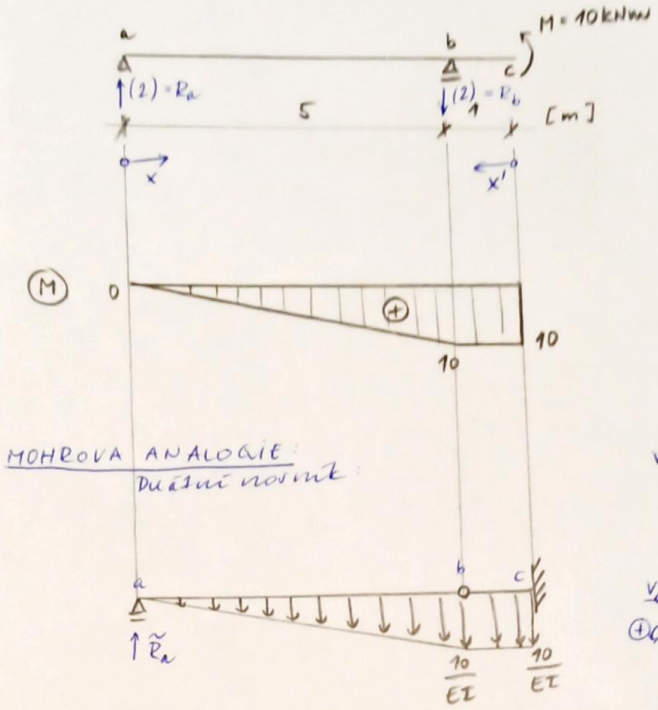
$$\varphi_a = \varphi(x'=10) = -\frac{1}{4\cdot 10^4}\left[22\cdot 10 - 10\cdot 8 - 0{,}5\cdot 6^3 + 0{,}5\cdot 2^3 + 9\cdot 2^2\right] = \underline{-1{,}8\cdot 10^{-3}\ rad}$$

$$w_a = w(x'=10) = -\frac{1}{4\cdot 10^4}\left[-492 + 11\cdot 10^2 - 5\cdot 8^2 - \frac{6^4}{8} + \frac{2^4}{8} + 3\cdot 2^3\right] = \underline{-3{,}8\cdot 10^{-3}\ m}$$

$$w_c = w(x'=4) = -\frac{1}{4\cdot 10^4}\left[-492 + 11\cdot 4^2 - 5\cdot 2^2\right] = \underline{8{,}4\cdot 10^{-3}\ m}$$

Př: Určete w_c:

EI = konst.



$$w_c = \tilde{M}_c = \tilde{R}_a \cdot 6 - \frac{1}{2} \cdot 5 \cdot \frac{10}{EI} \cdot \left(1 + \frac{1}{3} \cdot 5\right) - 1 \cdot \frac{10}{EI} \cdot \left(\frac{1}{2} \cdot 1\right)$$

výpočet $\tilde{R}_a$:

$$\oplus \circlearrowleft \sum \tilde{M}_{ib} = 0 : \quad -\tilde{R}_a \cdot 5 + \frac{1}{2} \cdot 5 \cdot \frac{10}{EI} \left(\frac{1}{3} \cdot 5\right) = 0$$

$$\tilde{R}_a = \frac{25}{3EI}$$

dosazení:

$$w_c = \frac{25}{3EI} \cdot 6 - \frac{1}{2} \cdot 5 \cdot \frac{10}{EI} \cdot \frac{5}{3} - \frac{10}{EI} \cdot \frac{1}{2} = -\frac{65}{3EI} \ [m]$$

CLEBSCHOVA METODA:

$$M(x) = R_a \cdot x \Big|_{x>5} - R_b \cdot (x-5)$$

$$\varphi(x) = -\frac{1}{EI}\left[C_1 + 2\frac{x^2}{2}\Big|_{x>5} - 2\frac{(x-5)^2}{2}\right]$$

$$w(x) = -\frac{1}{EI}\left[C_2 + C_1 x + 2\frac{x^3}{6}\Big|_{x>5} - 2\frac{(x-5)^3}{6}\right]$$

okrajové podmínky:

$$w_a = w(x=0) = 0 \Rightarrow \quad C_2 = 0$$

$$w_b = w(x=5) = 0 \Rightarrow \quad C_2 + C_1 \cdot 5 + 2 \cdot \frac{5^3}{6} = 0$$

$$C_1 = -\frac{25}{3}$$

rovnice:

$$\varphi(x) = -\frac{1}{EI}\left[-\frac{25}{3} + x^2\Big|_{x>5} - (x-5)^2\right]$$

$$w(x) = -\frac{1}{EI}\left[-\frac{25}{3}x + \frac{x^3}{3}\Big|_{x>5} - \frac{(x-5)^3}{3}\right]$$

výpočet w_c:

$$w_c = w(x=6) = -\frac{1}{EI}\left[-\frac{25}{3} \cdot 6 + \frac{6^3}{3} - \frac{1^3}{3}\right]$$

$$w_c = -\frac{65}{3EI}$$

CLEBSCHOVA METODA ZPRAVA (x'):

$$M(x') = M\Big|_{x>1} - R_b(x'-1)$$

$$\varphi(x') = -\frac{1}{EI}\left[C_1 + M \cdot x'\Big|_{x'>1} - 2\frac{(x'-1)^2}{2}\right]$$

$$w(x') = -\frac{1}{EI}\left[C_2 + C_1 x' + 10\frac{x'^2}{2}\Big|_{x'>1} - 2\frac{(x'-1)^3}{6}\right]$$

okrajové podmínky:

$$w_a = w(x'=6) = 0 \Rightarrow \quad C_2 + C_1 \cdot 6 + 10 \cdot \frac{6^2}{2} - 2 \cdot \frac{5^3}{6} = 0$$

$$w_b = w(x'=1) = 0 \Rightarrow \quad C_2 + C_1 \cdot 1 + 10 \cdot \frac{1^2}{2} = 0$$

$$C_1 = -\frac{80}{3}$$

$$C_2 = \frac{65}{3}$$

rovnice:

$$\varphi(x') = -\frac{1}{EI}\left[-\frac{80}{3} + 10x'\Big|_{x'>1} - (x'-1)^2\right]$$

$$w(x') = -\frac{1}{EI}\left[\frac{65}{3} - \frac{80}{3}x' + 5x'^2\Big|_{x'>1} - \frac{(x'-1)^3}{3}\right]$$

výpočet w_c:

$$w_c = w(x'=0) = -\frac{1}{EI}\left[\frac{65}{3}\right]$$

$$w_c = -\frac{65}{3EI}$$