

Tabulka 8.1 Primární lokální vektory koncových sil prutu $\{R_{a,b}^*\}$ (pokračování) - úprava Vlk 2018

	(8.1c(1))	(8.1c(2))	(8.1c(3))	(8.1c(4))	(8.1c(5))	(8.1c(6))	(8.1c(7))	(8.1c(8))	(8.1c(9))	(8.1c(10))	
(c)											
	$X_{a,b}^*$ $Z_{a,b}^*$ $M_{a,b}^*$ $X_{b,a}^*$ $Z_{b,a}^*$ $M_{b,a}^*$	$-\frac{b}{l}F_x$ $-\frac{3l-b}{2l^3}F_z$ 0 $-\frac{a}{l}F_x$ $\frac{3l^2-a^2}{2l^3}F_z$ $\frac{l+a}{2l^2}F_z$	$-\frac{1}{2}F_x$ $-\frac{5}{16}F_z$ 0 $-\frac{1}{2}F_x$ $-\frac{11}{16}F_z$ $-\frac{3}{16}F_z$	0 $-\frac{l^2-a^2}{3 \cdot 2l^3}M$ 0 0 $-\frac{l^2-a^2}{3 \cdot 2l^3}M$ $\frac{l^2-3a^2}{2l^2}M$	0 $\frac{9M}{8l}$ 0 0 $\frac{9M}{8l}$ $\frac{1}{8}M$	0 $\frac{3M}{2l}$ 0 0 $\frac{3M}{2l}$ $\frac{1}{2}M$	$-\frac{1}{2}nl$ $-\frac{3}{8}ql$ 0 $-\frac{1}{2}nl$ $-\frac{5}{8}ql$ $-\frac{1}{8}ql^2$	$-\frac{2al-a^2}{2l}n$ $8al^3-6a^2l^2+a^4$ $8l^3$ $-\frac{a^2}{2l}n$ $6a^2l^2-a^4$ $8l^3$ $2a^2l^2-a^4$ $8l^2$	$-\frac{1}{8}nl$ $-\frac{7}{128}ql$ 0 $-\frac{3}{8}nl$ $-\frac{57}{128}ql$ $-\frac{9}{128}ql^2$	$-\frac{2n_a+n_b}{6}l$ $\frac{11q_a+4q_b}{40}l$ 0 $-\frac{n_a+2n_b}{6}l$ $\frac{9q_a+16q_b}{40}l$ $-\frac{7q_a+8q_b}{120}l^2$	$EA\alpha_T\Delta T_0$ $\frac{3EI}{2hl}\alpha_T\Delta T_1$ 0 $-EA\alpha_T\Delta T_0$ $\frac{3EI}{2hl}\alpha_T\Delta T_1$ $-\frac{3EI}{2h}\alpha_T\Delta T_1$
(d)											
	$X_{a,b}^*$ $Z_{a,b}^*$ $M_{a,b}^*$ $X_{b,a}^*$ $Z_{b,a}^*$ $M_{b,a}^*$	$-\frac{b}{l}F_x$ $-\frac{b}{l}F_z$ 0 $-\frac{a}{l}F_x$ $-\frac{a}{l}F_z$ 0	$-\frac{1}{2}F_x$ $-\frac{1}{2}F_z$ 0 $-\frac{1}{2}F_x$ $-\frac{1}{2}F_z$ 0	0 $\frac{M}{l}$ 0 0 $\frac{M}{l}$ 0	0 $\frac{M}{l}$ 0 0 $\frac{M}{l}$ 0	0 $\frac{M_1+M_2}{l}$ 0 0 $\frac{M_1+M_2}{l}$ 0	$\frac{1}{2}nl$ $\frac{1}{2}ql$ 0 $\frac{1}{2}nl$ $\frac{1}{2}ql$ 0	$\frac{2al-a^2}{2l}n$ $2al-a^2$ $2l$ $-\frac{a^2}{2l}n$ $-\frac{a^2}{2l}n$ 0	$-\frac{1}{8}nl$ $-\frac{1}{8}ql$ 0 $-\frac{3}{8}nl$ $-\frac{3}{8}ql$ 0	$-\frac{2n_a+n_b}{6}l$ $\frac{2q_a+q_b}{6}l$ 0 $-\frac{n_a+2n_b}{6}l$ $\frac{q_a+2q_b}{6}l$ 0	$EA\alpha_T\Delta T_0$ 0 0 $-EA\alpha_T\Delta T_0$ 0 0

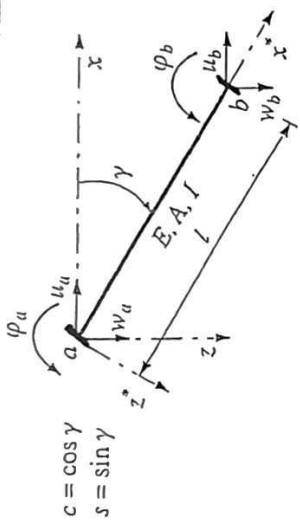
Obdélník, průřez:
 $\Delta T_0 = (\Delta T^d + \Delta T^h)/2$;
 $\Delta T_1 = \Delta T^d - \Delta T^h$

Obdélník, průřez:
 $\Delta T_0 = (\Delta T^d + \Delta T^h)/2$;
 $\Delta T_1 = \Delta T^d - \Delta T^h$

	(a)	(8.1a/1)	(8.1a/2)	(8.1a/3)	(8.1a/4)	(8.1a/5)	(8.1a/6)	(8.1a/7)	(8.1a/8)	(8.1a/9)	(8.1a/10)
		$\begin{Bmatrix} X_{ab}^* \\ Z_{ab}^* \\ M_{ab}^* \end{Bmatrix}$	$\begin{Bmatrix} -\frac{b}{l}F_x \\ -\frac{1}{2}F_x \\ \frac{1}{8}F_z l \end{Bmatrix}$	$\begin{Bmatrix} -\frac{1}{2}F_x \\ -\frac{1}{2}F_x \\ \frac{1}{8}F_z l \end{Bmatrix}$	$\begin{Bmatrix} 0 \\ -\frac{6}{l^3}abM \\ \frac{2l-3b}{l^2}M \end{Bmatrix}$	$\begin{Bmatrix} 0 \\ \frac{3}{2}M \\ \frac{1}{4}M \end{Bmatrix}$	$\begin{Bmatrix} 0 \\ \frac{3}{2}M \\ \frac{1}{4}M \end{Bmatrix}$	$\begin{Bmatrix} -\frac{1}{2}nl \\ -\frac{1}{2}ql \\ \frac{1}{12}ql^2 \end{Bmatrix}$	$\begin{Bmatrix} -\frac{1}{8}nl \\ -\frac{3}{32}ql \\ \frac{5}{192}ql^2 \end{Bmatrix}$	$\begin{Bmatrix} -\frac{2n_a+n_b}{6}l \\ \frac{7q_a+3q_b}{20}l \\ \frac{3q_a+2q_b}{60}l^2 \end{Bmatrix}$	$\begin{Bmatrix} EA\alpha_T\Delta T_0 \\ 0 \\ \frac{EI}{h}\alpha_T\Delta T_1 \end{Bmatrix}$
		$\begin{Bmatrix} X_{ba}^* \\ Z_{ba}^* \\ M_{ba}^* \end{Bmatrix}$	$\begin{Bmatrix} -\frac{a}{l}F_x \\ -\frac{1}{2}F_x \\ \frac{1}{8}F_z l \end{Bmatrix}$	$\begin{Bmatrix} -\frac{1}{2}F_x \\ -\frac{1}{2}F_x \\ \frac{1}{8}F_z l \end{Bmatrix}$	$\begin{Bmatrix} 0 \\ -\frac{6}{l^3}abM \\ \frac{2l-3a}{l^2}M \end{Bmatrix}$	$\begin{Bmatrix} 0 \\ \frac{3}{2}M \\ \frac{1}{4}M \end{Bmatrix}$	$\begin{Bmatrix} -\frac{1}{2}nl \\ -\frac{1}{2}ql \\ \frac{1}{12}ql^2 \end{Bmatrix}$	$\begin{Bmatrix} -\frac{2al-a^2}{21}n \\ -\frac{2al^3-2a^3l+a^4}{2l^3}q \\ \frac{a^2}{12l^2} \end{Bmatrix}$	$\begin{Bmatrix} -\frac{1}{8}nl \\ -\frac{13}{32}ql \\ \frac{11}{192}ql^2 \end{Bmatrix}$	$\begin{Bmatrix} -\frac{2n_a+n_b}{6}l \\ \frac{7q_a+3q_b}{20}l \\ \frac{3q_a+2q_b}{60}l^2 \end{Bmatrix}$	$\begin{Bmatrix} EA\alpha_T\Delta T_0 \\ 0 \\ -\frac{EI}{h}\alpha_T\Delta T_1 \end{Bmatrix}$
		(8.1b/1)	(8.1b/2)	(8.1b/3)	(8.1b/4)	(8.1b/5)	(8.1b/6)	(8.1b/7)	(8.1b/8)	(8.1b/9)	(8.1b/10)
		$\begin{Bmatrix} -\frac{b}{l}F_x \\ -\frac{1}{2}F_x \\ \frac{1}{8}F_z l \end{Bmatrix}$	$\begin{Bmatrix} -\frac{1}{2}F_x \\ -\frac{1}{2}F_x \\ \frac{1}{8}F_z l \end{Bmatrix}$	$\begin{Bmatrix} 0 \\ -\frac{3}{2l^3}l^2-b^2M \\ \frac{l^2-3b^2}{2l^2}M \end{Bmatrix}$	$\begin{Bmatrix} 0 \\ \frac{9}{8}M \\ \frac{1}{8}M \end{Bmatrix}$	$\begin{Bmatrix} 0 \\ \frac{3}{2}M \\ \frac{1}{2}M \end{Bmatrix}$	$\begin{Bmatrix} -\frac{1}{2}nl \\ -\frac{5}{8}ql \\ \frac{1}{8}ql^2 \end{Bmatrix}$	$\begin{Bmatrix} -\frac{2al-a^2}{21}n \\ -\frac{8al^3-4a^3l+a^4}{8l^3}q \\ \frac{a^2}{8l^2} \end{Bmatrix}$	$\begin{Bmatrix} -\frac{1}{8}nl \\ -\frac{23}{128}ql \\ \frac{7}{128}ql^2 \end{Bmatrix}$	$\begin{Bmatrix} -\frac{2n_a+n_b}{6}l \\ \frac{16q_a+9q_b}{40}l \\ \frac{8q_a+7q_b}{120}l^2 \end{Bmatrix}$	$\begin{Bmatrix} EA\alpha_T\Delta T_0 \\ -\frac{3EI}{2hl}\alpha_T\Delta T_1 \\ \frac{3EI}{2h}\alpha_T\Delta T_1 \end{Bmatrix}$
		$\begin{Bmatrix} -\frac{a}{l}F_x \\ -\frac{1}{2}F_x \\ \frac{1}{8}F_z l \end{Bmatrix}$	$\begin{Bmatrix} -\frac{1}{2}F_x \\ -\frac{1}{2}F_x \\ \frac{1}{8}F_z l \end{Bmatrix}$	$\begin{Bmatrix} 0 \\ -\frac{3}{2l^3}l^2-a^2M \\ \frac{l^2-3a^2}{2l^2}M \end{Bmatrix}$	$\begin{Bmatrix} 0 \\ \frac{9}{8}M \\ \frac{1}{8}M \end{Bmatrix}$	$\begin{Bmatrix} 0 \\ \frac{3}{2}M \\ \frac{1}{2}M \end{Bmatrix}$	$\begin{Bmatrix} -\frac{1}{2}nl \\ -\frac{5}{8}ql \\ \frac{1}{8}ql^2 \end{Bmatrix}$	$\begin{Bmatrix} -\frac{2al-a^2}{21}n \\ -\frac{8al^3-4a^3l+a^4}{8l^3}q \\ \frac{a^2}{8l^2} \end{Bmatrix}$	$\begin{Bmatrix} -\frac{1}{8}nl \\ -\frac{23}{128}ql \\ \frac{7}{128}ql^2 \end{Bmatrix}$	$\begin{Bmatrix} -\frac{2n_a+n_b}{6}l \\ \frac{16q_a+9q_b}{40}l \\ \frac{8q_a+7q_b}{120}l^2 \end{Bmatrix}$	$\begin{Bmatrix} EA\alpha_T\Delta T_0 \\ -\frac{3EI}{2hl}\alpha_T\Delta T_1 \\ \frac{3EI}{2h}\alpha_T\Delta T_1 \end{Bmatrix}$

Obdélník průřez:
 $\Delta T_0 = (\Delta T^d + \Delta T^h)/2$;
 $\Delta T_1 = \Delta T^d - \Delta T^h$

(a) Prut oboustranně monoliticky připojený

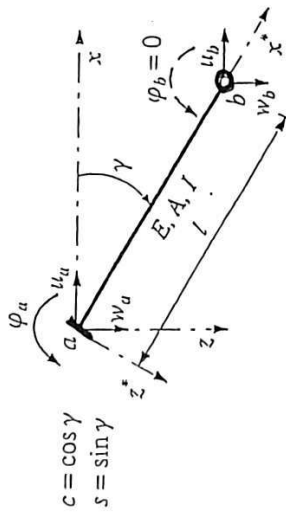


$$c = \cos \gamma$$

$$s = \sin \gamma$$

$$k_{ab} = \begin{bmatrix} \frac{EA}{l} c^2 + \frac{12EI}{l^3} s^2 & \left(\frac{EA}{l} - \frac{12EI}{l^3}\right) cs & \frac{6EI}{l^2} s & -\left(\frac{EA}{l} c^2 + \frac{12EI}{l^3} s^2\right) & \left(\frac{EA}{l} - \frac{12EI}{l^3}\right) cs & \frac{6EI}{l^2} s \\ \left(\frac{EA}{l} - \frac{12EI}{l^3}\right) cs & \frac{EA}{l} s^2 + \frac{12EI}{l^3} c^2 & -\frac{6EI}{l^2} c & -\left(\frac{EA}{l} - \frac{12EI}{l^3}\right) cs & \frac{EA}{l} s^2 + \frac{12EI}{l^3} c^2 & -\frac{6EI}{l^2} c \\ \frac{6EI}{l^2} s & -\frac{6EI}{l^2} c & \frac{4EI}{l} & -\frac{6EI}{l^2} s & -\frac{6EI}{l^2} c & \frac{4EI}{l} \\ -\left(\frac{EA}{l} c^2 + \frac{12EI}{l^3} s^2\right) & \left(\frac{EA}{l} - \frac{12EI}{l^3}\right) cs & -\frac{6EI}{l^2} s & \frac{EA}{l} c^2 + \frac{12EI}{l^3} s^2 & \left(\frac{EA}{l} - \frac{12EI}{l^3}\right) cs & -\frac{6EI}{l^2} s \\ \left(\frac{EA}{l} - \frac{12EI}{l^3}\right) cs & \frac{EA}{l} s^2 + \frac{12EI}{l^3} c^2 & -\frac{6EI}{l^2} c & -\left(\frac{EA}{l} - \frac{12EI}{l^3}\right) cs & \frac{EA}{l} s^2 + \frac{12EI}{l^3} c^2 & -\frac{6EI}{l^2} c \\ -\frac{6EI}{l^2} s & \frac{6EI}{l^2} c & \frac{4EI}{l} & \frac{6EI}{l^2} s & \frac{6EI}{l^2} c & \frac{4EI}{l} \end{bmatrix}$$

(b) Prut pravostranně kloubově připojený

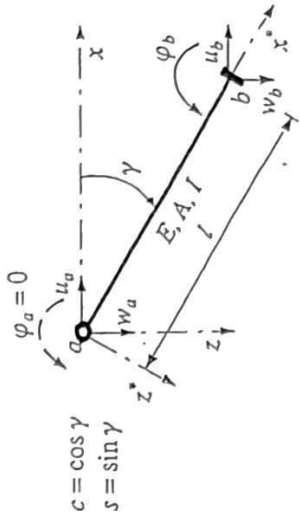


$$c = \cos \gamma$$

$$s = \sin \gamma$$

$$k_{ab} = \begin{bmatrix} \frac{EA}{l} c^2 + \frac{3EI}{l^3} s^2 & \left(\frac{EA}{l} - \frac{3EI}{l^3}\right) cs & \frac{3EI}{l^2} s & -\left(\frac{EA}{l} c^2 + \frac{3EI}{l^3} s^2\right) & \left(\frac{EA}{l} - \frac{3EI}{l^3}\right) cs & \frac{3EI}{l^2} s \\ \left(\frac{EA}{l} - \frac{3EI}{l^3}\right) cs & \frac{EA}{l} s^2 + \frac{3EI}{l^3} c^2 & -\frac{3EI}{l^2} c & -\left(\frac{EA}{l} - \frac{3EI}{l^3}\right) cs & \frac{EA}{l} s^2 + \frac{3EI}{l^3} c^2 & -\frac{3EI}{l^2} c \\ \frac{3EI}{l^2} s & -\frac{3EI}{l^2} c & \frac{EI}{l} & -\frac{3EI}{l^2} s & -\frac{3EI}{l^2} c & \frac{EI}{l} \\ -\left(\frac{EA}{l} c^2 + \frac{3EI}{l^3} s^2\right) & \left(\frac{EA}{l} - \frac{3EI}{l^3}\right) cs & -\frac{3EI}{l^2} s & \frac{EA}{l} c^2 + \frac{3EI}{l^3} s^2 & \left(\frac{EA}{l} - \frac{3EI}{l^3}\right) cs & -\frac{3EI}{l^2} s \\ \left(\frac{EA}{l} - \frac{3EI}{l^3}\right) cs & \frac{EA}{l} s^2 + \frac{3EI}{l^3} c^2 & -\frac{3EI}{l^2} c & -\left(\frac{EA}{l} - \frac{3EI}{l^3}\right) cs & \frac{EA}{l} s^2 + \frac{3EI}{l^3} c^2 & -\frac{3EI}{l^2} c \\ -\frac{3EI}{l^2} s & \frac{3EI}{l^2} c & \frac{EI}{l} & \frac{3EI}{l^2} s & \frac{3EI}{l^2} c & \frac{EI}{l} \end{bmatrix}$$

(c) Prut levostranně kloubově připojený

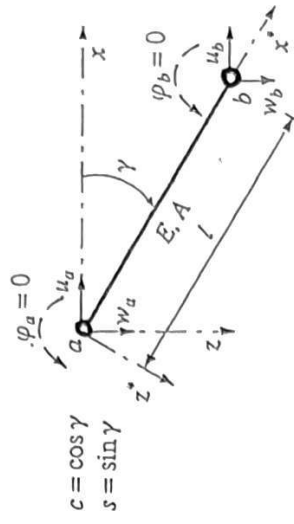


$$c = \cos \gamma$$

$$s = \sin \gamma$$

$$k_{ab} = \begin{bmatrix} \frac{EA}{l} c^2 + \frac{3EI}{l^3} s^2 & \left(\frac{EA}{l} - \frac{3EI}{l^3}\right) cs & \frac{3EI}{l^2} s & -\left(\frac{EA}{l} c^2 + \frac{3EI}{l^3} s^2\right) & \left(\frac{EA}{l} - \frac{3EI}{l^3}\right) cs & \frac{3EI}{l^2} s \\ \left(\frac{EA}{l} - \frac{3EI}{l^3}\right) cs & \frac{EA}{l} s^2 + \frac{3EI}{l^3} c^2 & -\frac{3EI}{l^2} c & -\left(\frac{EA}{l} - \frac{3EI}{l^3}\right) cs & \frac{EA}{l} s^2 + \frac{3EI}{l^3} c^2 & -\frac{3EI}{l^2} c \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -\left(\frac{EA}{l} c^2 + \frac{3EI}{l^3} s^2\right) & \left(\frac{EA}{l} - \frac{3EI}{l^3}\right) cs & \frac{3EI}{l^2} s & \frac{EA}{l} c^2 + \frac{3EI}{l^3} s^2 & \left(\frac{EA}{l} - \frac{3EI}{l^3}\right) cs & -\frac{3EI}{l^2} s \\ \left(\frac{EA}{l} - \frac{3EI}{l^3}\right) cs & \frac{EA}{l} s^2 + \frac{3EI}{l^3} c^2 & -\frac{3EI}{l^2} c & -\left(\frac{EA}{l} - \frac{3EI}{l^3}\right) cs & \frac{EA}{l} s^2 + \frac{3EI}{l^3} c^2 & -\frac{3EI}{l^2} c \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(d) Prut oboustranně kloubově připojený



$$c = \cos \gamma$$

$$s = \sin \gamma$$

$$k_{ab} = \begin{bmatrix} c^2 & cs & 0 & -c^2 & -cs & 0 \\ cs & s^2 & 0 & -cs & -s^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -c^2 & -cs & 0 & c^2 & cs & 0 \\ -cs & -s^2 & 0 & cs & s^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

K výpočtu lokálních koncových sil z globálních parametrů deformace:

$$\vec{R}_{ab} = k_{ab} \cdot \vec{r}_{ab} = \frac{EA}{l} \begin{bmatrix} c & s & 0 & -c & -s & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -c & -s & 0 & c & s & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \vec{r}_{ab}$$

Tabulka 8.3 Lokální matice tuhosti prutu $[k_{a,b}^*]$

	<p>(8.3a)</p> $[k_{a,b}^*] = \begin{bmatrix} u_a & w_a & \varphi_a & u_b & w_b & \varphi_b \\ \frac{EA}{l} & 0 & 0 & -\frac{EA}{l} & 0 & 0 \\ 0 & \frac{12EI}{l^3} & \frac{6EI}{l^2} & 0 & \frac{12EI}{l^3} & \frac{6EI}{l^2} \\ 0 & \frac{6EI}{l^2} & \frac{4EI}{l} & 0 & \frac{6EI}{l^2} & \frac{2EI}{l} \\ -\frac{EA}{l} & 0 & 0 & \frac{EA}{l} & 0 & 0 \\ 0 & \frac{12EI}{l^3} & \frac{6EI}{l^2} & 0 & \frac{12EI}{l^3} & \frac{6EI}{l^2} \\ 0 & \frac{6EI}{l^2} & \frac{2EI}{l} & 0 & \frac{6EI}{l^2} & \frac{4EI}{l} \end{bmatrix}$ <p>$\overline{a \quad b}$</p>	<p>(8.3b)</p> $[k_{a,b}^*] = \begin{bmatrix} u_a & w_a & \varphi_a & u_b & w_b & \varphi_b \\ \frac{EA}{l} & 0 & 0 & -\frac{EA}{l} & 0 & 0 \\ 0 & \frac{3EI}{l^3} & \frac{3EI}{l^2} & 0 & -\frac{3EI}{l^3} & 0 \\ 0 & \frac{3EI}{l^2} & \frac{3EI}{l} & 0 & \frac{3EI}{l^2} & 0 \\ -\frac{EA}{l} & 0 & 0 & \frac{EA}{l} & 0 & 0 \\ 0 & \frac{3EI}{l^3} & \frac{3EI}{l^2} & 0 & -\frac{3EI}{l^3} & 0 \\ 0 & \frac{3EI}{l^2} & \frac{3EI}{l} & 0 & \frac{3EI}{l^2} & 0 \end{bmatrix}$ <p>$\overline{a \quad b}$</p>
	<p>(8.3c)</p> $[k_{a,b}^*] = \begin{bmatrix} u_a & w_a & \varphi_a & u_b & w_b & \varphi_b \\ \frac{EA}{l} & 0 & 0 & -\frac{EA}{l} & 0 & 0 \\ 0 & \frac{3EI}{l^3} & 0 & 0 & \frac{3EI}{l^3} & 0 \\ 0 & 0 & 0 & \frac{EA}{l} & 0 & 0 \\ \frac{EA}{l} & 0 & 0 & -\frac{EA}{l} & 0 & 0 \\ 0 & \frac{3EI}{l^3} & 0 & 0 & \frac{3EI}{l^3} & 0 \\ 0 & 0 & 0 & 0 & \frac{3EI}{l^2} & \frac{3EI}{l} \end{bmatrix}$ <p>$\overline{a \quad b}$</p>	<p>(8.3d)</p> $[k_{a,b}^*] = \begin{bmatrix} u_a & w_a & \varphi_a & u_b & w_b & \varphi_b \\ \frac{EA}{l} & 0 & 0 & -\frac{EA}{l} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{EA}{l} & 0 & 0 \\ -\frac{EA}{l} & 0 & 0 & \frac{EA}{l} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ <p>$\overline{a \quad b}$</p>

Transformace vektorů

z lokálních do globálních souřadnic

$$\{\bar{\mathbf{R}}_{a,b}^*\} \rightarrow \{\bar{\mathbf{R}}_{a,b}\}$$

$$\{\bar{\mathbf{R}}_{a,b}\} = \left\{ \begin{array}{l} \bar{X}_a^* \cos \gamma - \bar{Z}_a^* \sin \gamma \\ \bar{X}_a^* \sin \gamma + \bar{Z}_a^* \cos \gamma \\ \hline \bar{M}_a^* \\ \bar{X}_b^* \cos \gamma - \bar{Z}_b^* \sin \gamma \\ \bar{X}_b^* \sin \gamma + \bar{Z}_b^* \cos \gamma \\ \bar{M}_b^* \end{array} \right\}$$

z globálních do lokálních souřadnic

$$\{\mathbf{r}_{a,b}\} \rightarrow \{\mathbf{r}_{a,b}^*\}$$

$$\{\mathbf{r}_{a,b}^*\} = \left\{ \begin{array}{l} u_a \cos \gamma + w_a \sin \gamma \\ -u_a \sin \gamma + w_a \cos \gamma \\ \hline \varphi_a \\ u_b \cos \gamma + w_b \sin \gamma \\ -u_b \sin \gamma + w_b \cos \gamma \\ \varphi_b \end{array} \right\}$$