OptiTendon
Optimization of Prestressing Tendon Paths

Technical Software Documentation

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OptiTendon is a modular Python framework for the automatic optimization of prestressing tendon paths in prestressed concrete bridge decks using genetic algorithms. The implementation models construction stages, time-dependent effects, and all prestress force losses required by Eurocode standards, with the primary objective of finding the most efficient tendon layouts (paths) and the minimum required prestressing force while meeting structural and code constraints.

1 Mechanical model description

1.1 Concept of prestressing

Prestressing is a special state of stress and deformation that is actively introduced into a structure to improve its structural behavior. It allows to design lighter and more slender structures in a straightforward manner. Long span reinforced concrete girders and bridges can't be realized technically and economically without prestressing, among other things due to large displacements and difficult reinforcement arrangements. Prestressing was originally designed to balance the effect of permanent loads and partially also variable loads – partial prestressing. Initiation of cracks was allowed since they close immediately after the variable load is removed, assuming a compressive reserve in the entire structure in long term. However, according to EN 1992-1-1 the decompression limit for frequent load combination requires that no tensile stresses occur anywhere within a certain distance from the tendon or its duct. It requires higher prestress level compared to the former approaches (prior to EN); design of an optimal level of prestress may be challenging for statically indeterminate structures. Optimal prestressing is not primarily based on the ultimate limit states but on a stress concept, where prestressing is sized for "design approaches" according to serviceability limit states with respect to the service life of the structure. Design of prestressing is based on:

- State of stress
- Resistance of cross section (flexural, shear)
- Deformations (during construction and throughout service life)
- Method of construction, prestressing techniques
- Durability of the structure
- Economy

1.2 Prestressing force and level of prestressing

Prestressing actively counterbalances external actions of permanent and variable loads. In the ideal case without any loss of prestress, the compressing force acts on cross sections of the beam as uniform stress from end to end. However, in the real design an engineer must consider variation of prestressing force due to losses from prestressing operations and time dependent losses due to creep and shrinkage of concrete and relaxation of steel. The purpose of optimal prestressed structure design is to find optimal magnitude of prestressing force and optimal geometry of the tendon. Prestressing force in tendons and its layout should counterbalance external load and guarantee state of stress for minimal as well as for maximal loads. The compressive margin before the action of variable loads minimizes the tension in the final state of stress. Moreover, compression stresses must be below a certain threshold to avoid development of longitudinal cracks in concrete. No tensile stresses should be allowed under permanent load, although crack can develop under variable load. To prevent cracking in the surroundings of tendons, the tensile stress under frequent load combination is prohibited within a certain distance neighborhood of the tendon or its duct (recommended to be 100 mm for bridges). The exact conditions imposed on stress are stated in EN 1992-1-1, chapter 7.2; conditions imposed on cracks are in chapter 7.3. A higher level of prestressing should reduce the area of reinforcing steel since overall resistance of cross section is function of combination of prestressing and reinforcing steel. However too high level of prestressing can cause damaging due to dangerous cracking (during prestressing operation and also in service life) and inadmissible upward deflections of slender sections. High level of prestressing also causes large compression stresses which result into extensive creep. Thus, it is obvious from the above that design of prestressing requires a comprehensive analysis.

1.3 Mechanical model

The mechanical model uses simple Euler-Bernoulli beam theory to compute deflections, internal forces, and stresses within the structure. Degrees of freedom are displacement and rotations at nodes. In every time step an inelastic steady-state analysis is performed by iterative search for global balance using elastic stiffness matrix of the structure. The necessity for inelastic analysis is given by time-dependent nature of the problem, i.e., by presence of creep, relaxation, shrinkage and aging of the structural parts.

1.4 Time dependent analysis (TDA)

Two dimensional structural models composed of beams elements are analyzed in a fully time-dependent manner. The structural behavior is evaluated in time steps with exponentially growing size. Another discretization is applied spatially along the beam central lines, where internal forces and deflections are evaluated. The Euler-Bernoulli beam theory with linear stress and strain along the cross section is assumed. The integral approach to the time dependent analysis is implemented, the creep strain at every spatial, x, and time, t, point is evaluated:

$$\varepsilon(x,t) = \int_0^t \dot{\sigma}(x,t')J(t,t')\,\mathrm{d}t' \tag{1}$$

where $\dot{\sigma}$ is time derivative of normal stress, t' is dummy time variable, and J is the compliance function with aging. The compliance function is evaluated according to EN 1992-2. Besides the creep strain, the shrinkage strain according to the same code is added. Both short- and long-term losses of prestress force are implemented. The former group involves loss due to friction and anchor slip; the latter one takes into account relaxation of steel and elastic and inelastic deformation of concrete. The aging of concrete is considered via increase of its elastic modulus in time – see Fig. 1.

At each time step, an iterative procedure updating mechanical, creep and shrinkage strains and stresses in concrete as well as stresses in cables takes place. The iterative loop starts with computing the concrete variables using the prestressing force from the last time step, change in structural boundary condition, aging, creep and shrinkage. All these calculations are done on

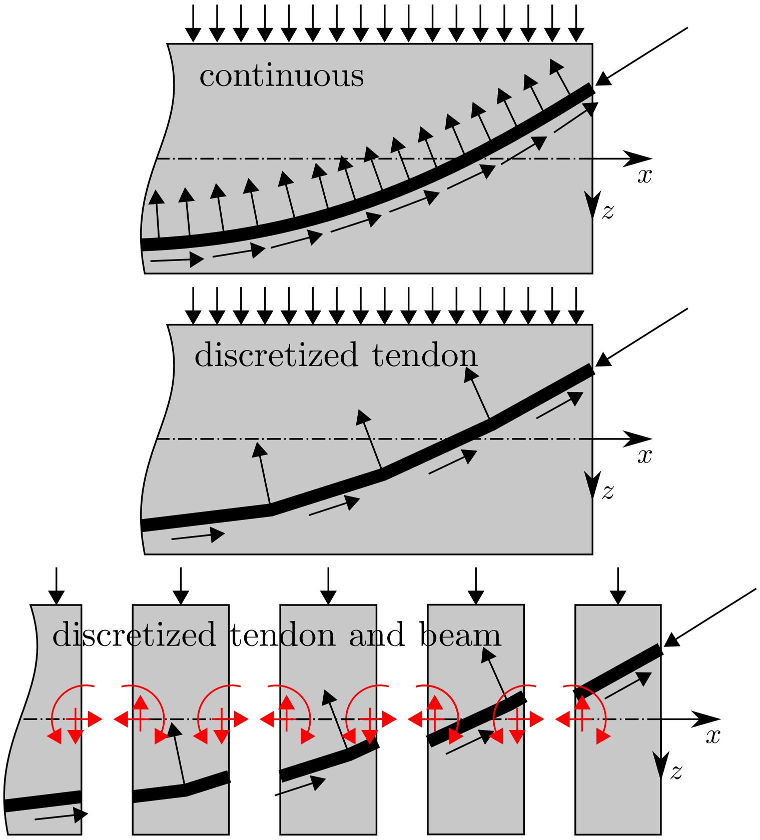

Figure 1: Actions on the beam due to external loads and tendon: continuous (top) and discretized idealizations (center and bottom). The red color indicates internal forces that balance all the external actions in black color.

the upper and bottom fibers of the beams. Then, the tendon force is updated based on relaxation and concrete normal strain. The concrete strain at the tendon location is linearly interpolated from the normal strain at the top and bottom fibers. The new tendon force then serves as an input for the next iteration. Typically, around 5 iterations suffice to reach practically zero difference between the input and output tendon forces.

1.5 Discretization

An idealization of the tendon into the curve line allows to compute its actions (friction and radial forces) on the concrete beam. The radial forces are computed from the second differentiation of the tendon z coordinate with respect to x. The tendon is actually discretized into straight sections – see Fig. 2, the radial forces are then simply computed from the balance of these sections.

An independent discretization of the beam is performed in parallel. The external actions within each beam section are included in the balance equations leading to internal forces (in red in Fig. 2).

1.6 Optimization

A genetic algorithm (GA) is a search and optimization technique inspired by the process of natural selection. It simulates evolution by iteratively improving a population of candidate solutions. Each solution, encoded as a chromosome, is evaluated based on a fitness function. Through genetic operators like selection (choosing the fittest individuals), crossover (combining traits of parent solutions), and mutation (randomly altering traits), the algorithm explores the solution space, gradually converging toward an optimal or near-optimal solution.

1.7 Definition of cable geometry

To implement the developed optimization algorithm, the cable's profile (geometry) must be parameterized. For this, a quadratic Bézier curve is used for each cable region to represent the tendon's central line. The Bézier curve is convenient for shape optimization thanks to its straightforward shape control and versatility. For continuous beams, the connection between curves is achieved by incorporating an arch curve with matching tangential direction to the Bézier curves on both sides. The quadratic Bézier curve (which corresponds to a parabolic segment) is defined by three control points, each specified by its x and y coordinates. Any point on the Bézier curve (tendon) can be calculated using the parameter t, where . The position, , of any point on the curve is determined using the parameter through the following equation:

$$B(t) = (1-t)^2\mathbf{C}_1 + 2(1-t)t\,\mathbf{C}_2 + t^2\mathbf{C}_3 \tag{2}$$

For each individual in population, the vector of state variables (chromosome) is designed with the minimum possible number of variables to enable optimization without introducing an unnecessary complexity. For this reason, the horizontal coordinates of the control points can be set as fixed. Since the geometry is defined by one Bézier curve per span of the beam, multiple curves are used in the case of a continuous beam, and only vertical coordinates of their control points are left free for optimization. The last remaining state variable is the prestressing force, . The form of each individual's chromosome is therefore:

$$\mathbf{a} = \{C_z^1, C_z^2, \ldots, C_z^i, a_P\} \tag{3}$$

1.8 Fitness function

The fitness, , of each individual is composed of the objective function, , and penalties for violating imposed constraints.

$$f_P(a) = f(a) + \sum_{i=1}^{n} q_i \max\{0, |h_j(a) - \epsilon|\} \tag{4}$$

For the purposes of this optimization, only inequality constraints, , are considered. It is worth noting that when the conditions are satisfied, the penalty function is equal to the objective function. The objective function simply equals to the number of prestressing cables (the lower the better) expressed as the prestressing force divided by the reference force per one cable. The constraints ensure that the concrete threshold stress values are not exceeded and that the cable lies within the structure. The first two of the following equations address the minimum (maximum compression) and maximum (maximum tension) stresses in the beam, while the last two conditions ensure that the tendon geometry remains within the beam by constraining the geometry geometry:

$$\begin{aligned}
h_1 &: \int_0^l |\sigma_{x,\max}(x) - \sigma_{\max}| \;\; [\sigma_{x,\max}(x) > \sigma_{\max}]\, dx \\
h_2 &: \int_0^l |\sigma_{x,\min}(x) - \sigma_{\min}| \;\; [\sigma_{x,\min}(x) < \sigma_{\min}]\, dx \\
h_3 &: \int_0^l |z_B(x) - z_{\max}| \;\; [z_B(x) > z_{\max}]\, dx \\
h_4 &: \int_0^l |z_B(x) - z_{\min}| \;\; [z_B(x) < z_{\min}]\, dx
\end{aligned} \tag{5}$$

An exterior penalty-based method known as the Automatic Dynamic Penalization (ADP) method was implemented to set up penalties. The goal of this method is to allow the algorithm to explore the boundaries of non-feasibility. The evaluation of a non-feasible individual, with respect to each condition, is treated the same as the evaluation of the best individual in the generation. This approach is particularly useful for tendon geometry optimization, as the optimal solutions are expected to lie on the boundary of the infeasible region. To facilitate this search, non-feasible points are not eliminated during evaluation and are allowed to remain in the population, serving as attraction points. According to ADP, the scaling coefficient for constraint violation, qi, are determined separately for each generation based on the individuals that violate the i-th constraint:

$$q_i = \max_{\mathbf{a} \in \mathbf{a}_{\mathrm{NF}i}} \left| \frac{f(\mathbf{a}_F^{\mathrm{BEST}}) - f(\mathbf{a})}{h_i(\mathbf{a})} \right| \tag{6}$$

Here, represents the best feasible individual in the generation. If no feasible individual is present in the generation, its value is assigned as zero. Every time a new individual is assessed, an analysis of internal forces and stresses is conducted throughout the entire lifespan of the structure. A time dependent analysis (TDA) is performed, addressing creep, shrinkage, and prestress losses according to EN-1992 – see the previous Section 2. During the time dependent analysis, the maximum and minimum normal stresses for all times during the lifespan are recorded, and the envelope of extreme normal stresses throughout the lifespan is then used for evaluation of fitness of each individual.

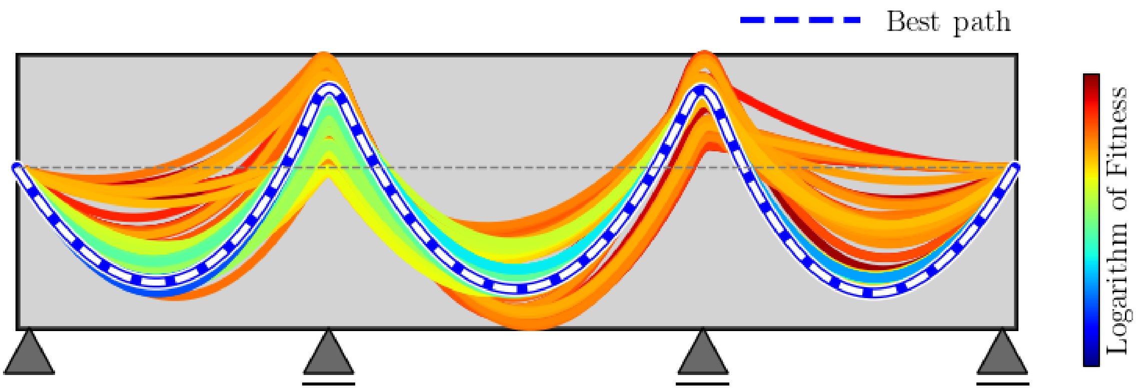

Figure 2: cables

1.9 Genetic operators

The selection process for individuals undergoing genetic operators differs between the first generation and subsequent generations. In the first generation, the total number of individuals is set to individuals for a more robust screening of the configuration space. Selection is based solely on fitness evaluation, and 500 individuals are chosen to undergo crossover and mutation, proceeding to the second generation. From the second generation onward, the number of individuals remains constant at. Selection is performed according to the following criteria: elitism is applied to feasible individuals according to each of constraints. This results in the number of elite individuals being equal to . In this work, we set the number of elite individuals for each constraint, . The remaining individuals are selected randomly. Elite individuals are included in the selection pool, meaning they may also be subjected to genetic operators. In such cases, both the original elite individual and its modified version are carried forward to the next generation.

2 Software usage

2.1 Installation

```
git clone ...
cd OptiTendon
```

2.2 Example Usage

```
python optimizer.py continuous_bezier
```

2.3 Standards and References

- EN 1992-1 (Eurocode 2: Concrete structures)
- EN 1992-2 (Eurocode 2: Concrete bridges)

The progress of the optimization can be monitored throughout the computation, see Figure 3 top. Upon completion of the optimization, the resulting internal forces and structural deformations can be examined via the graphical user interface for selected times during the service life, see Figure 3 bottom. At the bottom panel, results can be exported in text format as well as PDF. The cable path coordinates are available for export as well.

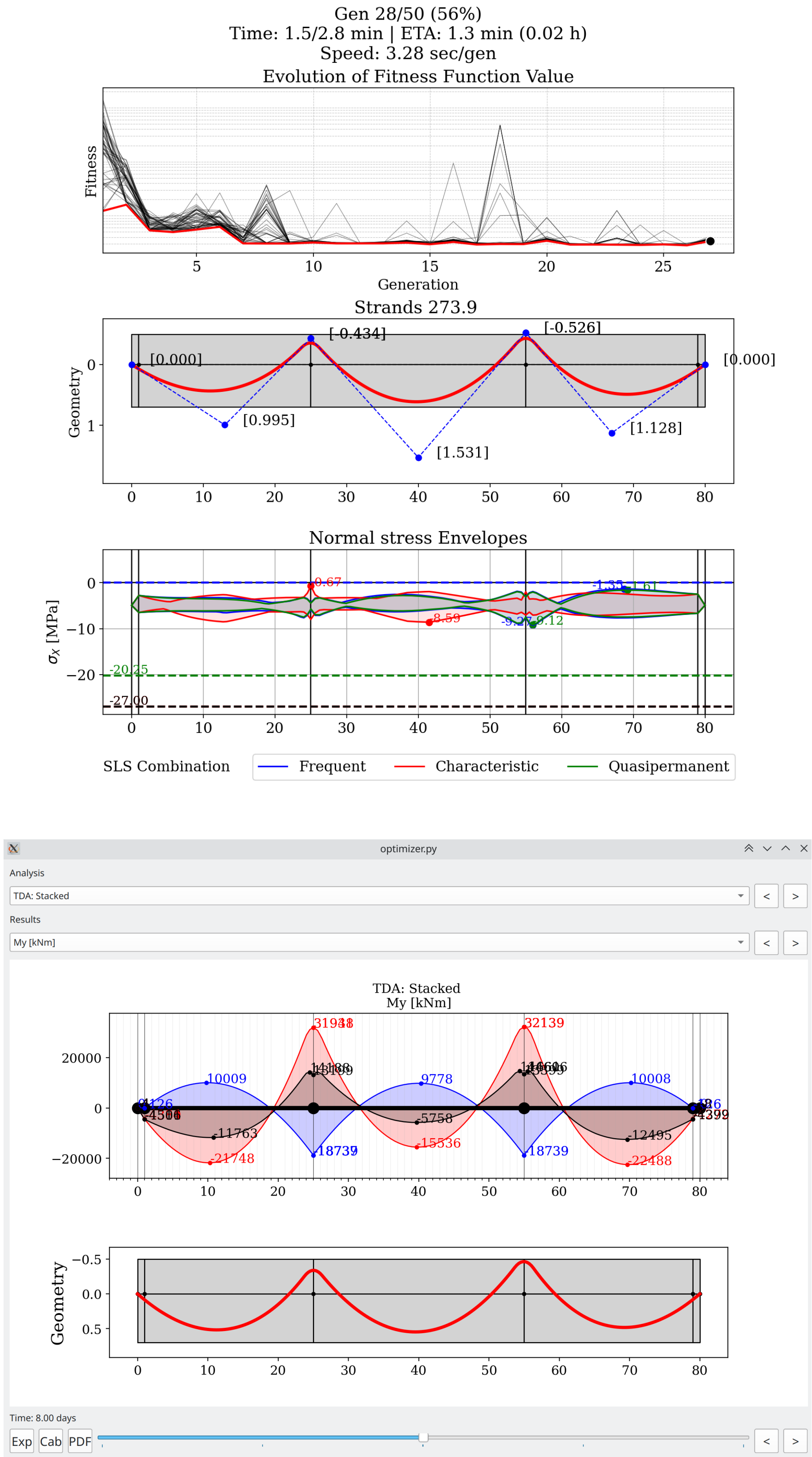

Figure 3: Optimization process overview and optimized internal forces and deformations

3 Model input example

The example numerical model `continuous_bezier` represents a continuous prestressed concrete beam discretized into multiple spans, supports, load cases, and optimization variables.

3.0.1 Geometry and Topology

- The structure is a continuous beam with nodes defined along the longitudinal axis:

$$x_i = \sum_{j=0}^{i-1} L_j, \quad i = 0, \ldots, N$$

  where L_j are individual span lengths.
- Five beam elements connect six nodes sequentially.
- Each beam element is discretized using n internal divisions.

3.0.2 Boundary Conditions

- Node 1 is fully restrained in both longitudinal (x) and vertical (z) directions.
- Nodes 2–4 are vertically restrained.
- All supports are applied at initial time $t = 0$.

3.0.3 Cross-Section Properties

The beam cross-section is defined by:

- Area: A
- Perimeter: u
- Second moment of area: I_y
- Extreme section moduli: $Z_{\max}, Z_{\min}$

3.0.4 Material Properties (Concrete)

Concrete strength and serviceability stress limits are defined for:

- Characteristic compressive strength: f_{ck}
- Allowable compression and tension stresses for:
  - Temporary (TDA)
  - Frequent (FREQ)
  - Characteristic (CHAR)
  - Quasi-permanent (QUASI-P)

3.0.5 Prestressing Cable Definition

- Single cable type with jacking stress σ_{jack}
- Cable area A_c and stressing time t_c
- Stressing sequence with anchor losses
- Friction parameters: curvature coefficient μ and wobble coefficient k
- Number of cables treated as an optimization variable
- Nominal concrete cover of 150 mm

3.0.6 Cable Geometry

- Cable profile defined by 200 divisions
- Geometry composed of alternating:
  - Quadratic Bézier segments
  - Circular arc segments with fixed radius
- Vertical cable ordinates z_i are optimization variables

3.0.7 Load States

The model includes permanent and traffic-induced loads:

- Self-weight as distributed load
- Superimposed dead load
- LM1 traffic model:
  - Distributed loads on individual spans
  - Concentrated axle loads at specified positions
- Reduced permanent load combination $(0.35 \cdot G)$

3.0.8 Load Combinations

Load combinations are defined for:

- Characteristic limit states
- Frequent (serviceability) combinations
- Quasi-permanent combinations

Each combination specifies:

- Active load states
- Corresponding combination coefficients

3.0.9 State Variables (Optimization)

The optimization problem includes:

- Vertical cable ordinates z_i with lower and upper bounds derived from section geometry
- Total number of prestressing cables

3.0.10 Optimization Settings

- Time-dependent effects: creep, shrinkage
- Interaction between prestressing force, creep, shrinkage and prestressing losses iteratively updated
- Genetic algorithm parameters