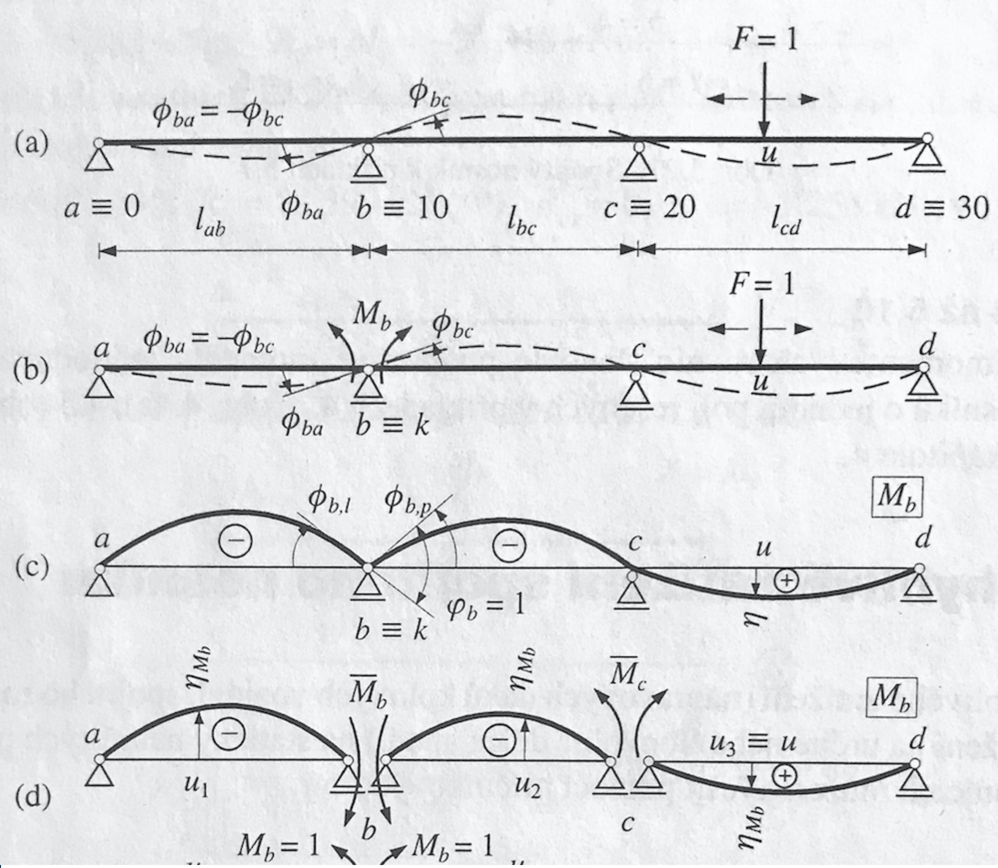
Influence lines

Continuous beam



Influence lines of bending moments above supports

M_b

Kinematic method – proof using Betti's theorem:

$$-M_b(\Phi_{ba}+\Phi_{bc})+F\eta=0$$

$$\text{Because } \Phi_{ba}+\Phi_{bc}=1,\ F=1$$

$$\eta=M_b$$

Three - moment equation :

$$M_a\beta_{ba}+M_b(\alpha_{ba}+\alpha_{bc})+M_c\beta_{bc}+\Phi_{ba}+\Phi_{bc}=0$$

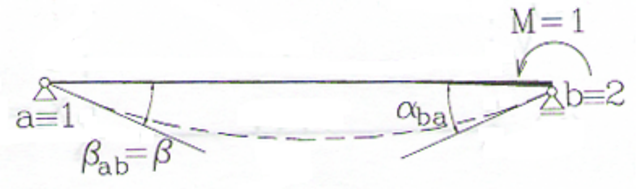
$$M_b\beta_{cb}+M_c(\alpha_{cb}+\alpha_{cd})+M_d\beta_{cd}=0$$

Influence lines

Continuous beam

Because $M_a = 0,\ M_d = 0,\ \Phi_{ba} + \Phi_{bc} = 1,$

$$\alpha = \frac{l}{3EI},\ \ \beta = \frac{l}{6EI} \quad \cdots \text{basic angles}$$



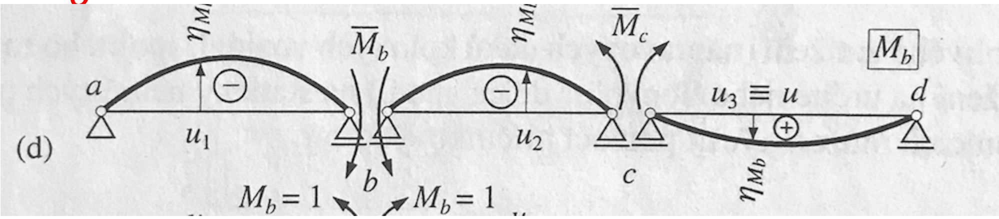
then

$$M_b\left(\frac{l_{ab}}{3EI_{ab}} + \frac{l_{bc}}{3EI_{bc}}\right) + M_c\frac{l_{bc}}{6EI_{bc}} = -1$$

$$M_b\frac{l_{bc}}{6EI_{bc}} + M_c\left(\frac{l_{bc}}{3EI_{bc}} + \frac{l_{cd}}{3EI_{cd}}\right) = 0$$

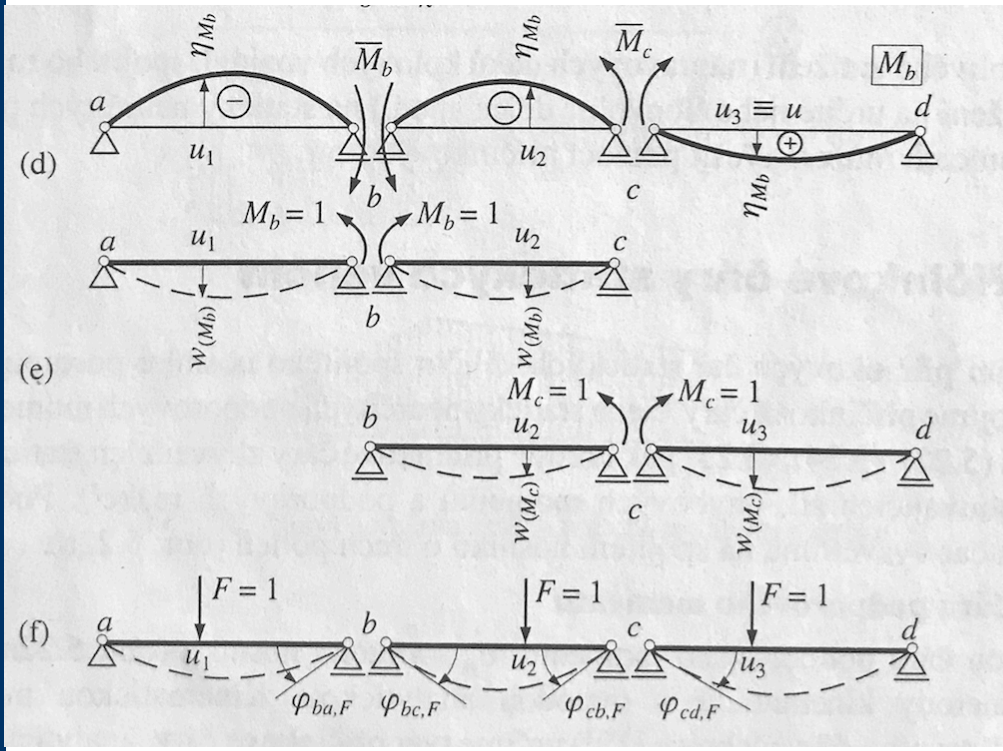
=> M_b , M_c

Influence line is deflection line of simply supported beam loaded by moments M_b *and* M_c.



Influence lines

Continuous beam



Deflection can be calculated e.g. using unit dummy force method.

$$w(u) = \int_l \frac{M(u)\overline{M}(u)}{EI(u)} du$$

(e.g. tables)

Using Maxwell's theorem :

field $a-b$

$$w_{(M_b=1)} = \varphi_{ba,F=1}$$

field $b-c$

$$w_{(M_b=1)} = \varphi_{bc,F=1}, \quad w_{(M_c=1)} = \varphi_{cb,F=1}$$

field $c-d$

$$w_{(M_c=1)} = \varphi_{cd,F=1}$$

Influence lines

Continuous beam

Then, influence line of bending moment M_b is calculated as:

field $a-b$

$$\eta_{M_b} = w_{(M_b=1)} \cdot M_b = \varphi_{ba,F=1} \cdot M_b$$

field $b-c$

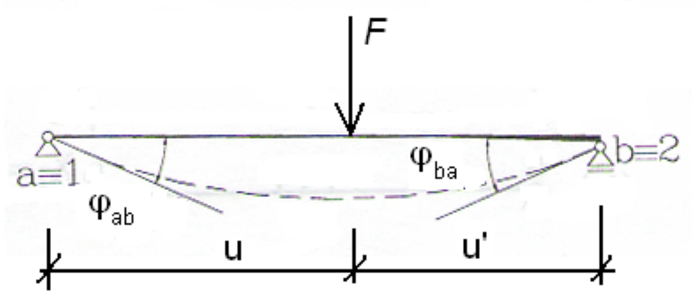
$$\eta_{M_b} = w_{(M_b=1)} \cdot M_b + w_{(M_c=1)} \cdot M_c = \varphi_{bc,F=1} \cdot M_b + \varphi_{cb,F=1} \cdot M_c$$

field $c-d$

$$\eta_{M_b} = w_{(M_c=1)} \cdot M_c = \varphi_{cd,F=1} \cdot M_c$$

From tables:

$$\varphi_{ab} = \frac{Fuu'}{6EIl}\left(l+u'\right), \quad \varphi_{ba} = \frac{Fuu'}{6EIl}\left(l+u\right)$$

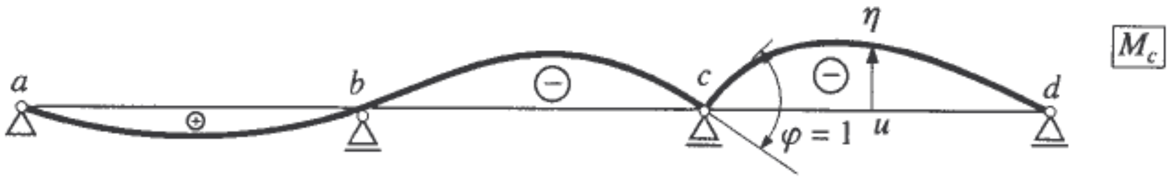


Influence lines

Continuous beam

M_c

Influence line of bending moment M_c is calculated the same way as M_b but now the hinge and unit rotation is applied at cross-section *c*.



Three - moment equation :

$$M_b\left(\frac{l_{ab}}{3EI_{ab}}+\frac{l_{bc}}{3EI_{bc}}\right)+M_c\frac{l_{bc}}{6EI_{bc}}=0$$

$$M_b\frac{l_{bc}}{6EI_{bc}}+M_c\left(\frac{l_{bc}}{3EI_{bc}}+\frac{l_{cd}}{3EI_{cd}}\right)=-1$$

=> M_b , M_c

Influence lines

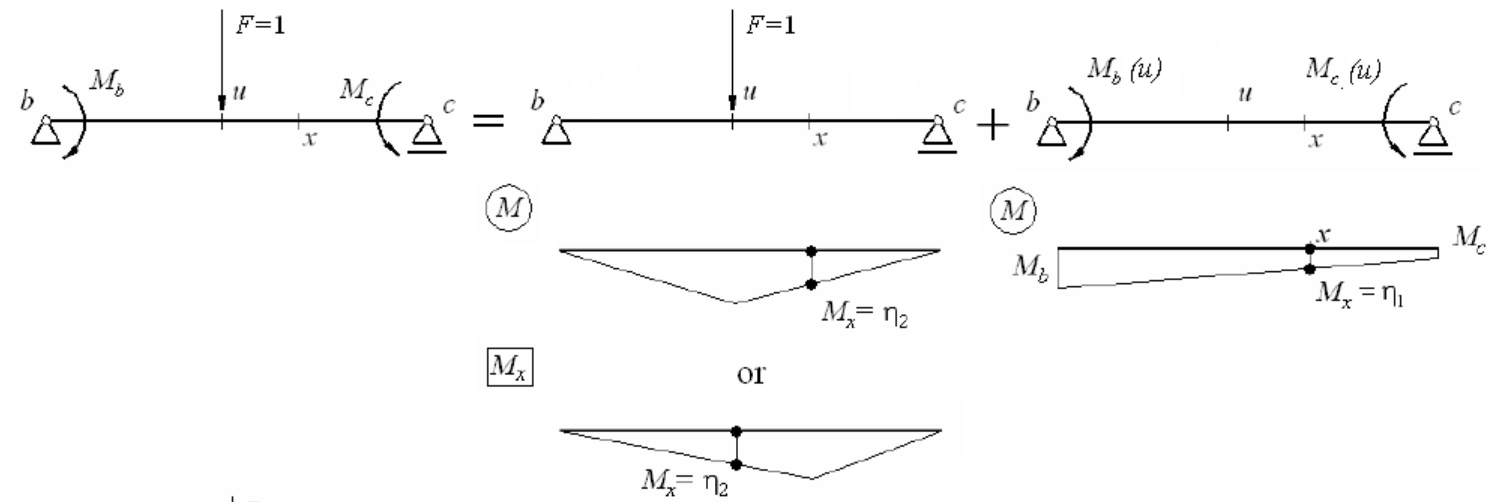
Continuous beam

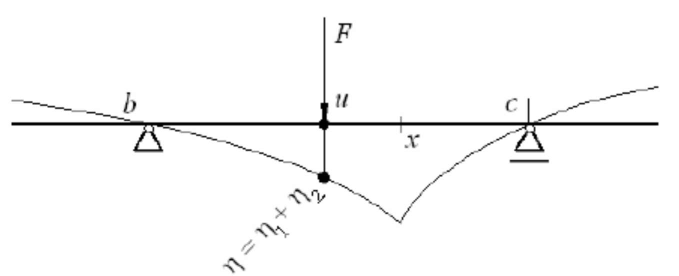
Influence line of bending moment at point *x*

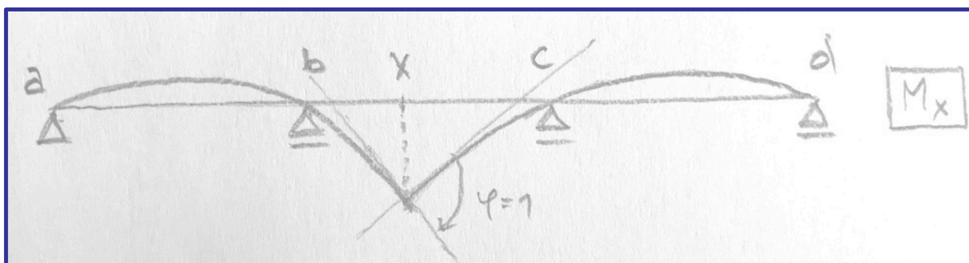
M_x

$$M_x = \eta = M_{x,F} + M_{x,Mb} + M_{x,Mc} = M_{x,0} + M_b \frac{x'}{l} + M_c \frac{x}{l}$$

$M_{x,0} \neq 0$ only in field with x







Influence lines

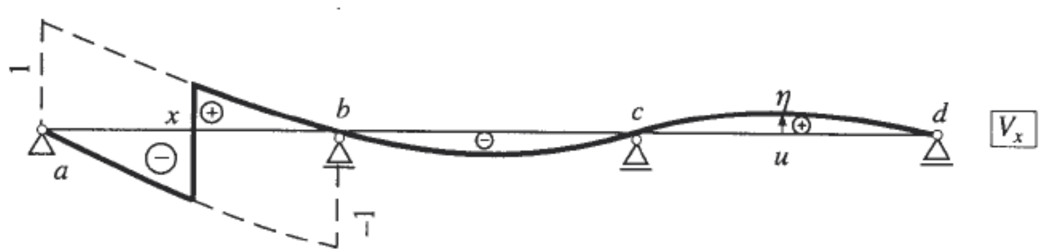
Continuous beam

Influence line of shear force at point *x*

V_x

$$V_x = \eta = V_{x,F} + V_{x,Ma} + V_{x,Mb} = V_{x,0} - \frac{M_a}{l} + \frac{M_b}{l}$$

$V_{x,0} \neq 0$ only in field with x

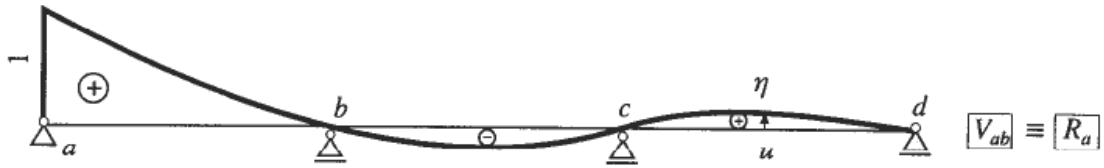


Influence line of reactions

R_a

$$R_a = \eta = V_a = V_{x=0} = V_{a,F} + V_{a,Ma} + V_{a,Mb} = V_{a,0} - \frac{M_a}{l} + \frac{M_b}{l}$$

$V_{a,0} \neq 0$ only in field with x

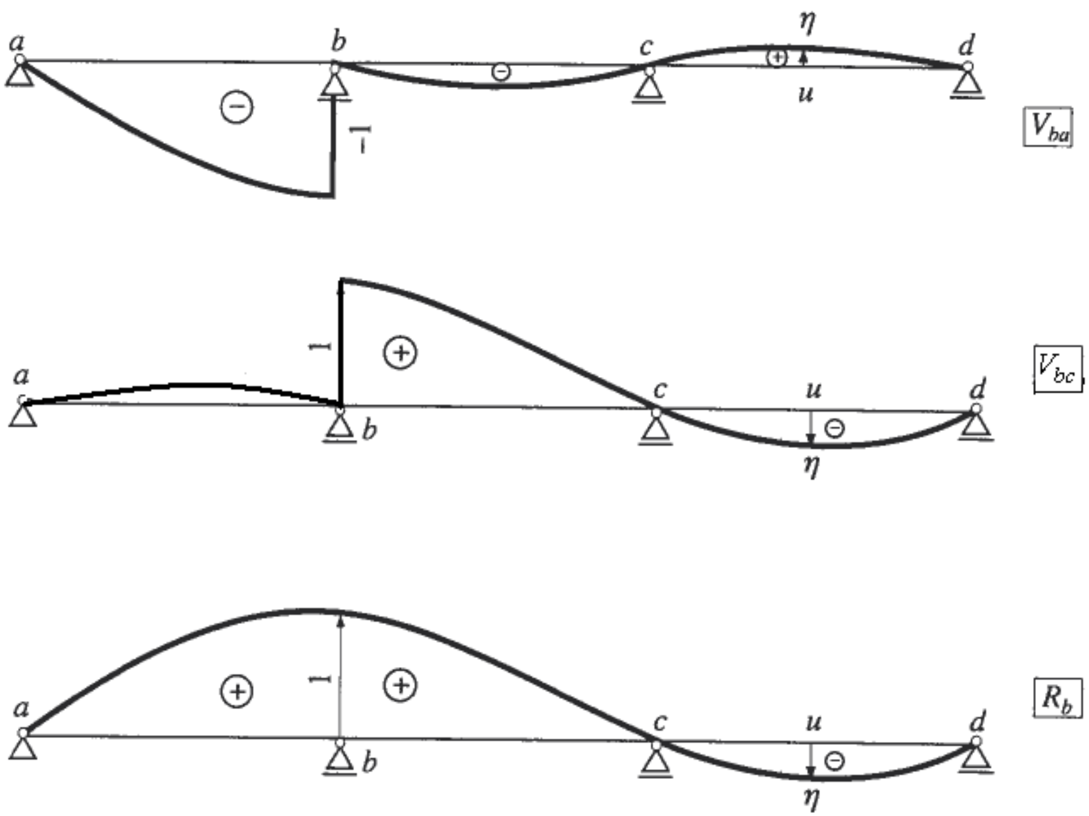


Influence lines

Continuous beam

R_b

$$R_b = \eta = -V_{b,l} + V_{b,r} = -V_{ba} + V_{bc}$$



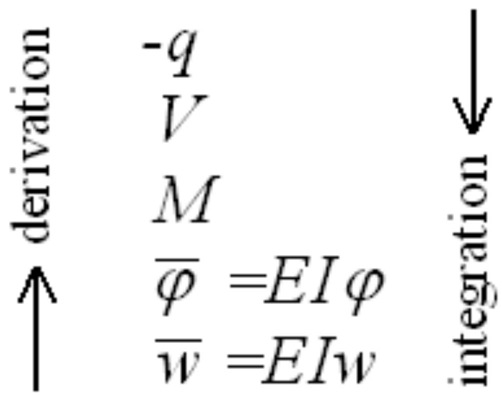
Influence lines

Continuous beam

Functions of influence lines for continuous beam are cubic parabolas.

Why?

Derivation-integration scheme:



$$q(x) = 0, F = 1$$

$$V(x) = \text{konst.}$$

$$M(x) \ldots \text{linear function}$$

$$\varphi(x) \ldots \text{quadratic parabola}$$

$$w(x) \ldots \text{cubic parabola}$$

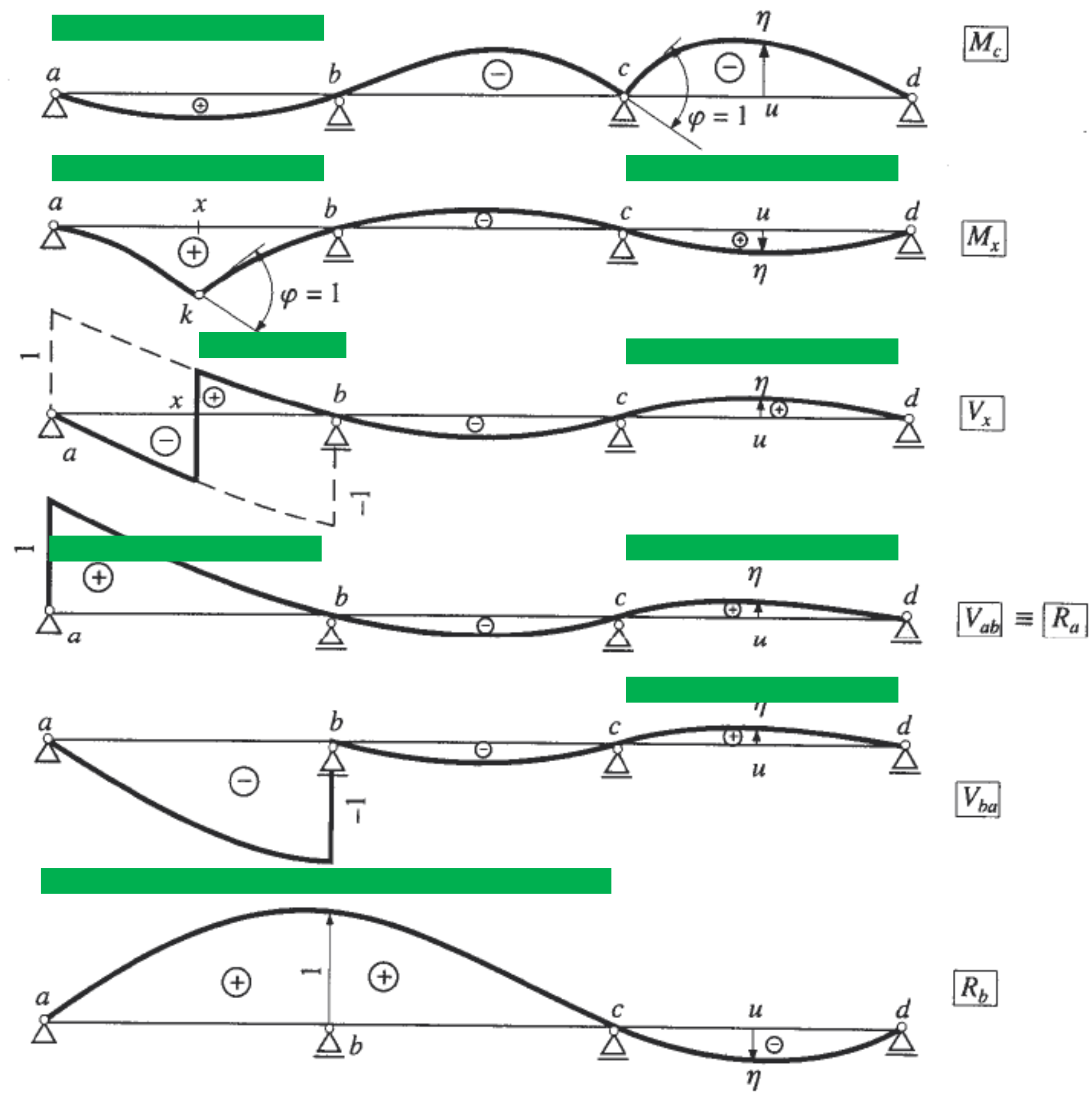
From kinematic definition $\eta(x) = w(x)$

Influence lines

Continuous beam

Extreme positions of live loads

Maximum (+) value of quantity will be obtained when loading parts of the structure with the positive ordinates of corresponding influence line.



Influence lines

Continuous beam

Extreme positions of live loads

Minimum (–) value of quantity will be obtained when loading parts of the structure with the negative ordinates of corresponding influence line.