BD004
Structural Analysis 2

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BD004 – Structural Analysis 2

Main topics:

- Moving loads and influence lines of statically indeterminate beams, load combinations on continuous beam.
- The slope deflection method (displacement method) for analysis of indeterminate structures
  - Principle of the method, numerical model, degree of kinematic indeterminacy
  - Analyses of simple beams, continuous beams, frames and trusses.
  - Temperature loading, support settlement, beam with variable cross section.

BD004 – Structural Analysis 2

Course organization:

- 4 hours a week =
  - 2-hour lecture (David Lehký) +
  - 2-hour exercise (Jan Eliáš)

Course completion: credit + exam

Credit – more details will provide Jan Eliáš

Exam – calculation of practical tasks + theoretical questions,
– 2 terms (a regular and a correction term) during examination period at the end of the winter semester

Moving loads

- Civil and bridge engineering structures (bridges, crane tracks) – moving loads are important (cars, trucks, trains, cranes, etc.)

Load is moving along the structure

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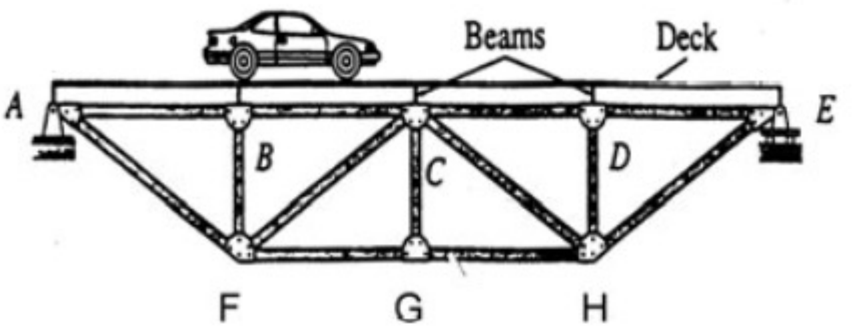
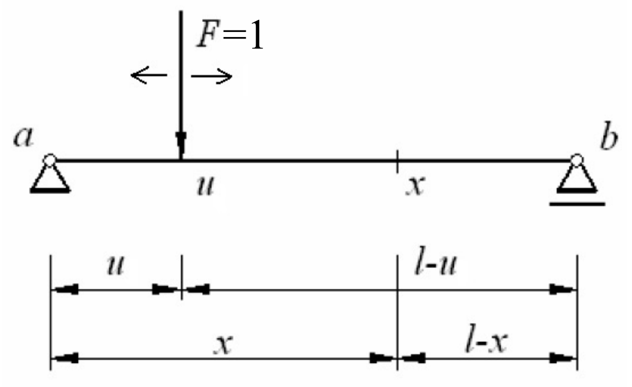
Changes of internal forces and bending moments



- In previous course (Structural Analysis 1), we have dealt with moving loads on statically determinate beams. Let's focus on statically indeterminate beams now.

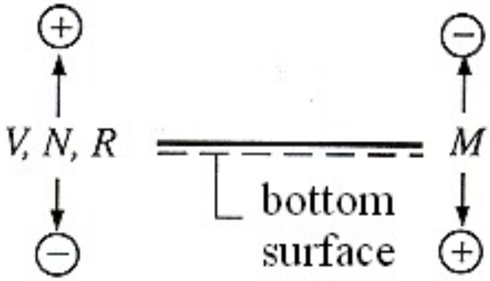
Influence lines

- Internal forces and bending moments caused by moving loads can be solved with the help of influence lines.
- Influence line is created using unit load (force) F=1 which moves along the structure.



Influence line S_x graphs the variation of a quantity S at a specific point x on a beam or truss caused by a unit load F=1 placed at any point u along the structure.

Signs:



Influence lines

Methods for constructing the influence line:

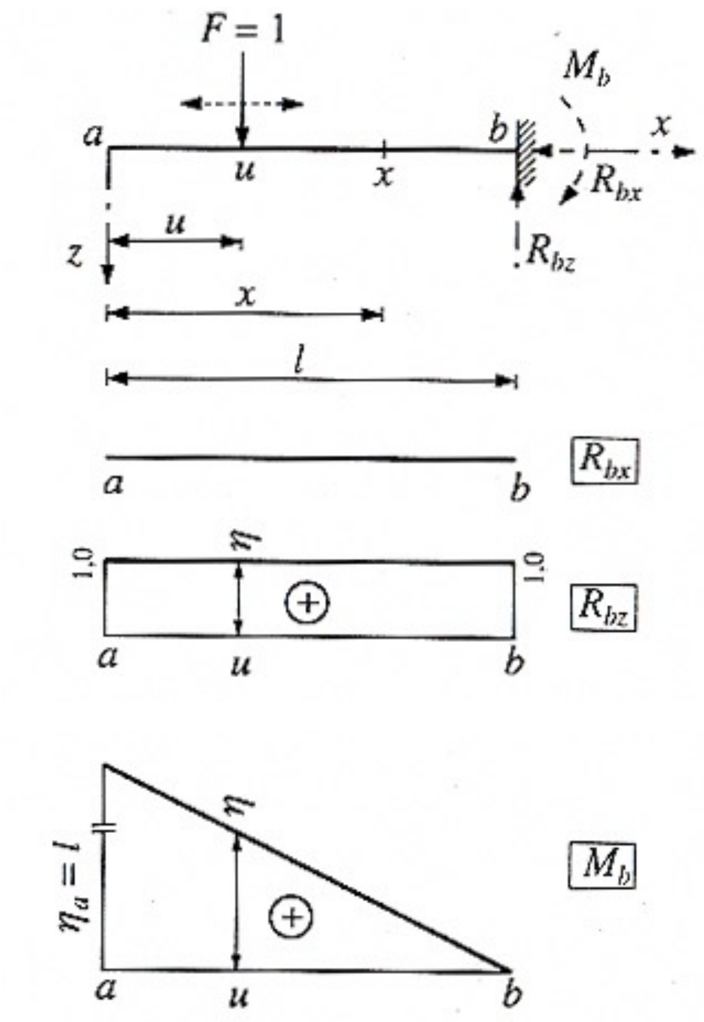
- Analytical method (creating equations using equilibrium conditions)
- Kinematic method (Müller-Breslau Principle; using principal of virtual work)
- Combination of above-mentioned methods

Müller-Breslau principle – The influence line for a force (or moment) response function is given by the deflected shape of the released structure obtained by removing the restraint corresponding to the response function and by giving the released structure a unit displacement (or rotation) at the location and in the direction of the response function, so that only the response function and the unit load perform external work.

Influence lines

Reminder: Cantilever beam – Müller-Breslau principle

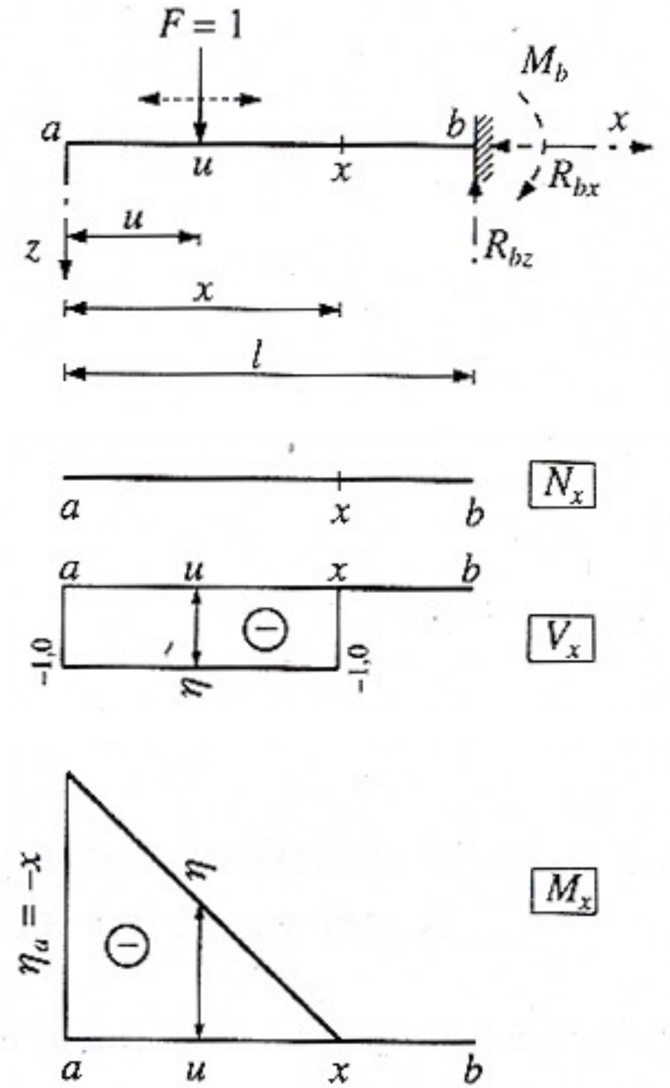
Reactions:



Influence lines

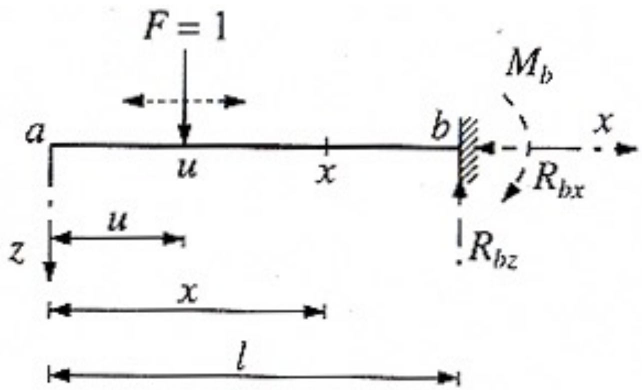
Reminder: Cantilever beam – Müller-Breslau principle

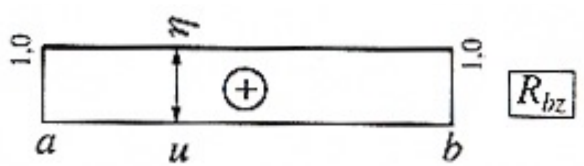
Internal forces:

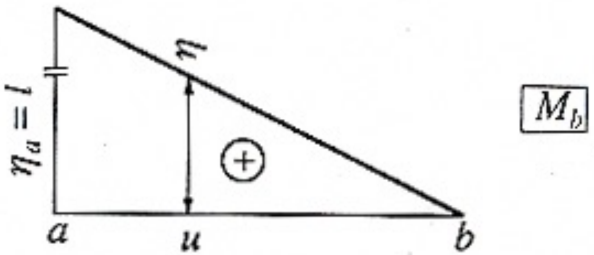


Influence lines

Reminder: Cantilever beam – analytical method







R_{bz}

Equilibrium of vertical forces:

$$\sum F_{iz} = 0:\ 1 - R_{bz} = 0 \rightarrow R_{bz} = \eta = 1$$

M_b

Moment equilibrium to b:

$$\sum M_{ib} = 0:\ 1 \cdot u' - M_b = 0 \rightarrow M_b = \eta =$$

$$= u' = l - u$$

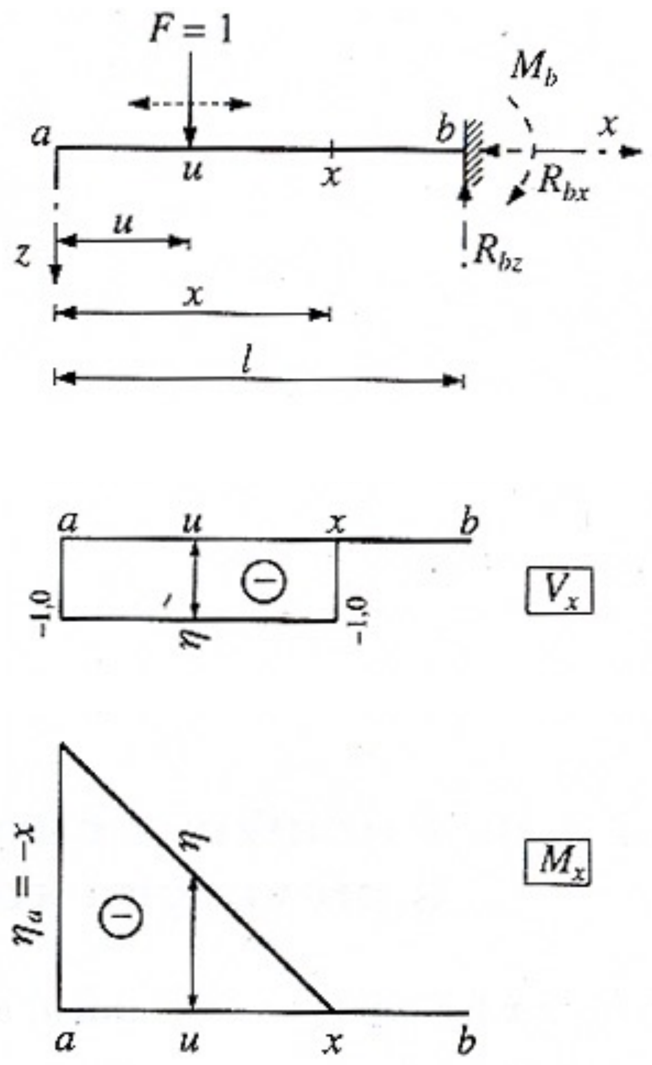
Boundary conditions:

$u = 0 :\ \eta = l$

$u = l :\ \eta = 0$

Influence lines

Reminder: Cantilever beam – analytical method



V_x

Unit force is in the interval $0 \leq u < x$:

$$V_x = \eta = -1$$

Unit force is in the interval $x < u \leq l$:

$$V_x = \eta = 0$$

M_x

Unit force is in the interval $0 \leq u \leq x$:

$$M_x = \eta = -1 \cdot (x - u) = u - x$$

Boundary conditions:

$u = 0 : \ \eta = -x$

$u = x : \ \eta = 0$

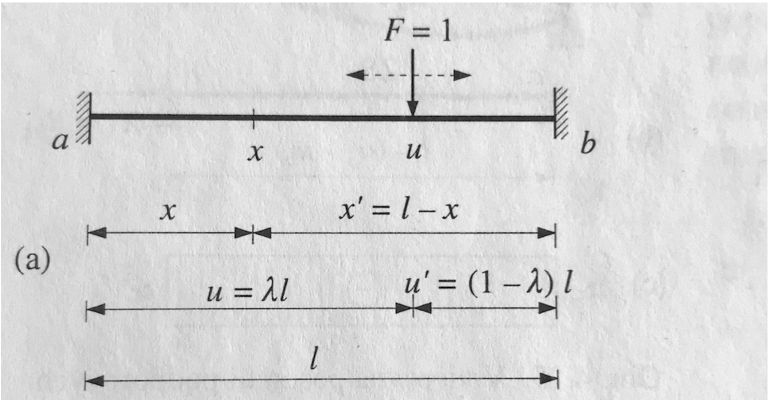
Unit force is in the interval $x \leq u \leq l$:

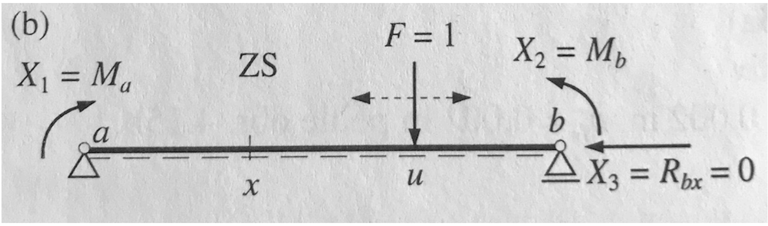
$$M_x = \eta = 0$$

Influence lines

Statically indeterminate beam fixed at both ends – analytical method

M_a , M_b





Moments M_a, M_b using force method:

$$\Phi_{ab} = 0 : \; M_a\alpha_{ab} + M_b\beta_{ab} + \varphi_{ab} = 0$$
$$\Phi_{ba} = 0 : \; M_a\beta_{ba} + M_b\alpha_{ba} + \varphi_{ba} = 0$$

For constant cross-section:

$$\alpha_{ab} = \alpha_{ba} = \alpha = \frac{l}{3EI}$$
$$\beta_{ab} = \beta_{ba} = \beta = \frac{\alpha}{2} = \frac{l}{6EI}$$

$$M_a = -\frac{2EI}{l}(2\varphi_{ab} - \varphi_{ba})$$
$$M_b = -\frac{2EI}{l}(2\varphi_{ba} - \varphi_{ab})$$

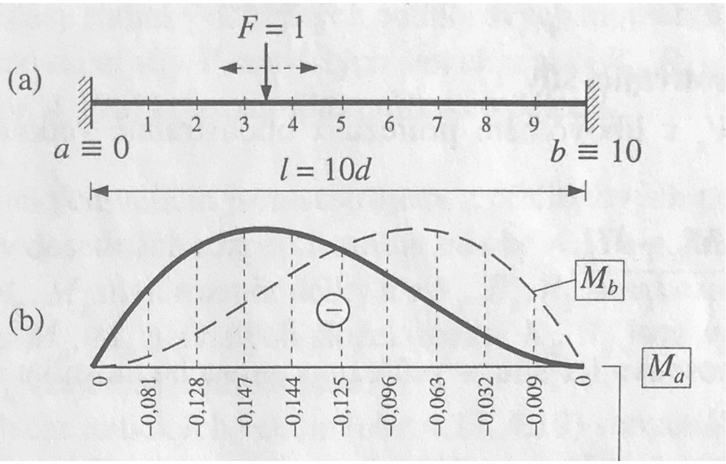
From tables: $\varphi_{ab}^{F=1} = \frac{1 \cdot uu'}{6EIl}(l + u'), \varphi_{ba}^{F=1} = \frac{1 \cdot uu'}{6EIl}(l + u)$

$$M_a = \eta = -\frac{uu'^2}{l^2} = -\frac{u(l-u)^2}{l^2}$$
$$M_b = \eta = -\frac{u^2u'}{l^2} = -\frac{u^2(l-u)}{l^2}$$

Cubic parabola!

Influence lines

Statically indeterminate beam fixed at both ends – analytical method



Selected ordinates:

$$\eta\left(\frac{l}{2}\right) = -\frac{\frac{l}{2}\left(l-\frac{l}{2}\right)^2}{l^2} = -\frac{\frac{l}{2}\cdot\frac{l^2}{4}}{l^2} = -\frac{l}{8} = -0{,}125l$$

$$\eta_{max,min}:\ M_a = -\frac{u(l-u)^2}{l^2} = \frac{-ul^2+2u^2l-u^3}{l^2} =$$

$$= -u + \frac{2u^2}{l} - \frac{u^3}{l^2}$$

$$\frac{dM_a}{du} = -1 + \frac{4}{l}u - \frac{3}{l^2}u^2 = 0$$

$$u_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-\frac{4}{l} \pm \sqrt{\frac{16}{l^2} - \frac{12}{l^2}}}{-\frac{6}{l^2}}$$

$$u_1 = \frac{-\frac{2}{l}}{-\frac{6}{l^2}} = \frac{l}{3} \rightarrow \eta_{min} = -\frac{\frac{l}{3}\left(l-\frac{l}{3}\right)^2}{l^2} = -\frac{4}{27}l = -0{,}148l$$

$$u_2 = \frac{-\frac{6}{l}}{-\frac{6}{l^2}} = l \rightarrow \eta_{max} = 0$$