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Sensitivity analysis of stability problems of steel plane frames

Zdeněk Kala *

Brno University of Technology, Faculty of Civil Engineering, Department of Structural Mechanics, Veveří 95, 602 00 Brno, Czech Republic

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ABSTRACT

The objective of the paper is to analyse the influence of initial imperfections on the load-carrying capacity of a single storey steel plane frame comprised of two columns loaded in compression. The influence of the variance of initial imperfections on the variance of the load-carrying capacity was calculated by means of Sobol' sensitivity analysis. Monte Carlo based procedures were used for computing full sets of first order and second order sensitivity indices of the model. The geometrical nonlinear finite element solution, which provides numerical results per run, was employed. The mutual dependence of sensitivity indices and column non-dimensional slenderness is analysed. The derivation of the statistical characteristics of system imperfections of the initial inclination of columns is described in the introduction of the present work. Material and geometrical characteristics of hot-rolled IPE members were considered to be random quantities with histograms obtained from experiments. The Sobol sensitivity analysis is used to identify the crucial input random imperfections and their higher order interaction effects.

1. Introduction

The reliability analysis of structural systems in the limit state methods is aimed at an assessment of safety and serviceability. Steel structures are composed of thin members and hence the problem of stability can prove to be one of the most important constituents of the safety. The stability loss is caused by the change of geometry of steel structures or structural components. Thus, to assess the structural stability, the equilibrium equations must be described under deformed geometry. This implies that the consideration of geometrical nonlinearity is inevitable for ultimate limit state analysis.

The behaviour of compressed members in the loading process leading to the ultimate limit state is influenced by initial imperfections generally divided into three categories: geometrical imperfections, material imperfections and structural imperfections [1,2]. The limit state theory for individual struts has been worked out and corroborated with experiments. However, isolated struts occur rarely in real structures. Generally, each structure is a system of members, which mutually influence each other by their behaviour. This interaction is most significant in structures with rigid joints (frame systems). On the contrary, this mutual interaction is small in truss structures and is generally neglected.

Within the division of structures into members and frame systems, we can accept the further division of initial imperfections into local (member) and global (frame systems) ones [1]. The local imperfections include: initial straightness deviation of member axis, deviation from the theoretical layout of the hot-rolled cross section, load eccentricity, dispersion of the mechanical material properties, residual stress, etc. Global imperfections include initial inclination of any column in systems and imperfections in the realization of joints, connections, anchorage and other structural details, which are apparent in comparison with the theoretical assumptions introduced in the solution of idealized system.

One of the challenging issues in modern civil engineering analysis is the typically large number of random quantities defining the input and system parameters [3]. Most building structures are atypical and hence a higher number of measurements are conceivable from the statistical point of view just for local imperfections of mass produced members. The basic indicators of production quality include the yield strength, tensile strength, ductility and geometrical characteristics of hot-rolled IPE profiles which have been under long term statistical evaluation within the framework of non-commercially aimed research programmes, see e.g. [4,6]. Relatively sufficient statistical information on material and geometrical characteristics of mass produced members of steel structures is available in comparison to other building structures.

The frame depicted in Fig. 1 represents a typical stability problem of a system comprised of more members. The fundamental question in terms of safety of a structure is how significant is the effect of inevitable initial imperfections on the load-carrying capacity. An approach to make such problems tractable is to identify the most important sources of uncertainty and to focus attention primarily on those uncertainties of the input space. Such a method using the Sobol decomposition [7], global sensitivity analysis method, is proposed here. The Sobol decomposition is used to decompose the

* Tel.:+420 541147382; fax: +420 541240994.
E-mail address: kala.z@fce.vutbr.cz

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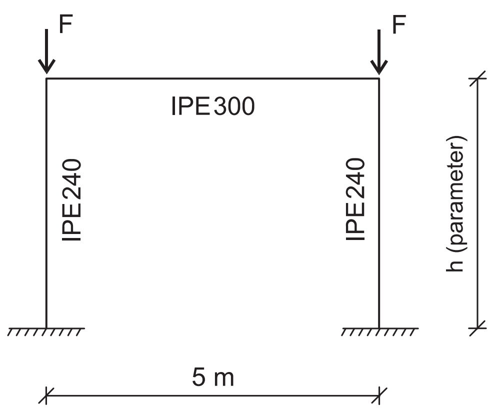

Fig. 1. IPE-section symmetric portal frame.

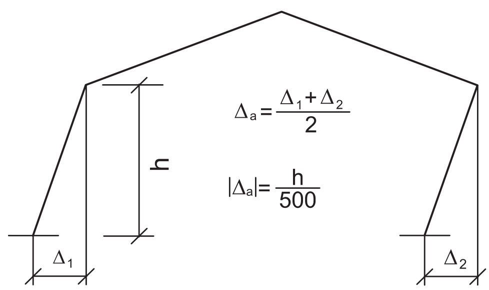

Fig. 2. Inclination of a single storey portal frame building.

variance of the load-carrying capacity into contributions of the individual input variables (initial imperfections).

The Sobol sensitivity analysis quantifies the relative importance of input imperfections in determining the value of load-carrying capacity. The crucial imperfections, which should be paid greater attention both in the modeling phase and in the interpretation of model results, are identified using sensitivity analysis. One of the advantages of the Sobol sensitivity analysis is that it enables the identification of interaction effects between input quantities on the monitored output. With the development of new concepts of the reliability analysis, these procedures can contribute to a qualitative improvement of the reliability analysis of structures.

2. Input random imperfections

2.1. Initial inclination of columns

Permitted inclination deviations of columns of a single storey portal frame are listed in the standard EN1090-2. The permitted deviation of mean inclination Δ_a of all columns in the same frame is $h/500$, see Fig. 2. This value is listed in the EN1090-2 for both Class 1 and Class 2.

The essential (normative) tolerances of inclination of each column have no specified limit; however, supplementary (informative) tolerances list the permitted deviations $|\Delta_b| = h/150$ for Class 1 and $|\Delta_b| = h/300$ for Class 2, see EN1090-2 and Fig. 3. Execution classes pertain to the production category and service category, in conjunction with the effect classes listed in Appendix B in the EN 1990:2002.

Inclination of each column is generally a random variable. In concordance with Figs. 2 and 3, let us denote the inclination of the left column as e_1 and that of the right column as e_2. Let us assume that measurements of e_1 and e_2 were performed on a large number of real frames, with respect to the permitted deviations according to the EN1090-2. Let us assume that imperfections e_1 and e_2 are

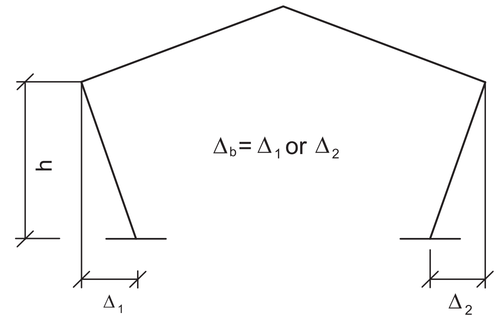

Fig. 3. Inclination of each column.

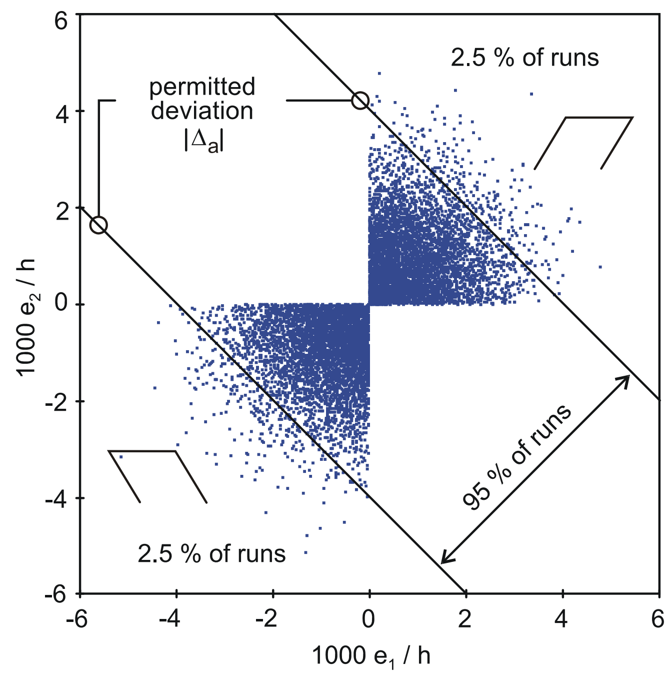

Fig. 4. MC runs, Case 1, $\sigma_{e1} = \sigma_{e2} = h/790$.

statistically independent random variables with mean values equal to zero, i.e., $\mu_{e1} = \mu_{e2} = 0$ (perfectly vertical columns). Let us further assume that 95% of realizations of e_1 and e_2 remain within the tolerance limits of the standard EN1090-2 and that both variables have a Gauss probability density function. Based on these assumptions, the first half of measured frames will have variables e_1 and e_2 of the same sign (Case 1), the second half will have variables e_1 and e_2 with opposite signs (Case 2). Analogously, let us introduce, for the permitted deviations Δ_a, Δ_b, random variables $e_a = (e_1 + e_2)/2$ and $e_b = e_1$ or e_2. The problem was analysed using 20 000 runs of the Monte Carlo (MC) method, see Figs. 4 and 5.

Case 1: If we introduce $\sigma_{e1} = \sigma_{e2} = h/790$, then it holds with 95% probability that $|e_a| \le |\Delta_a|$, see Fig. 4. At the same time, it holds for both classes that $|e_b| \le |\Delta_b|$ with a probability which is higher than 95%.

Case 2: If we introduce $\sigma_{e1} = \sigma_{e2} = h/670$, then 95% of realizations of e_b will be found within the tolerance limit $\pm h/300$(Class 2), see Fig. 5. Similarly, it holds that if we introduce $\sigma_{e1} = \sigma_{e2} = h/335$, then 95% of realizations of e_b will be found within the tolerance limit $\pm h/150$ (Class 1). The fulfillment of the condition $|e_a| \le |\Delta_a|$ together with 95% probability leads to $\sigma_{e1} = \sigma_{e2} = h/430$, see Fig. 6. In order to secure reliability from the point of view of the limit states, it is necessary to consider a smaller (safer) standard deviation for Class 1. In practice, this means that for Class 1, we shall introduce $\sigma_{e1} = \sigma_{e2} = h/430$ and for Class 2, $\sigma_{e1} = \sigma_{e2} = h/670$.

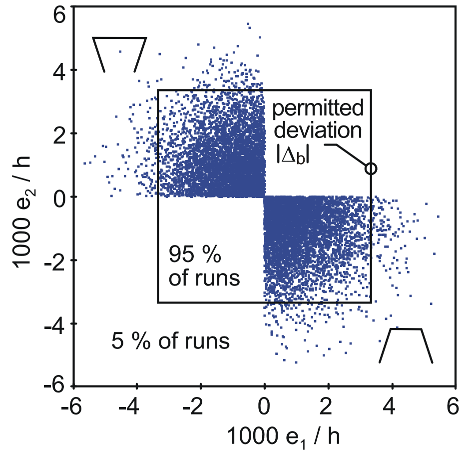

Fig. 5. MC runs, Case 2, Class 2, $\sigma_{e1}=\sigma_{e2}=h/670$.

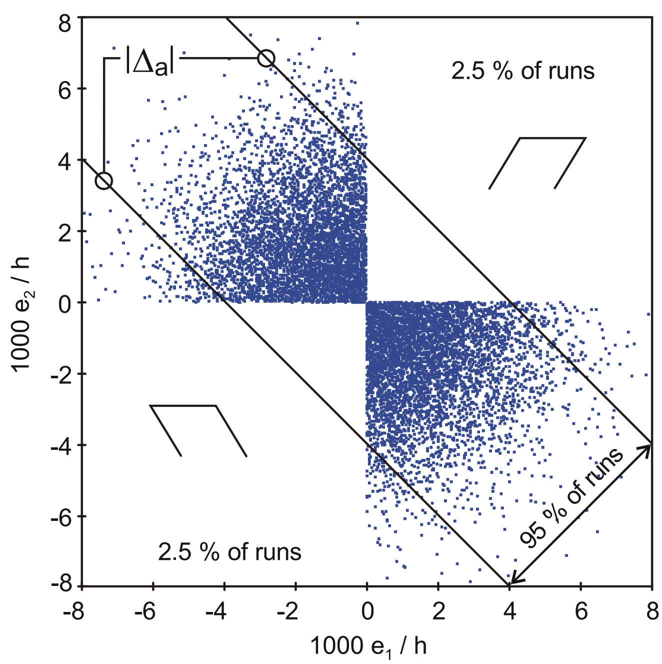

Fig. 6. MC runs, Case 2, Class 1, $\sigma_{e1}=\sigma_{e2}=h/430$.

Practical analysis based on the MC simulation method may be performed in the following manner. Let us introduce imperfections e_1 and e_2 as statistically independent random variables with Gauss probability density functions with $\mu_{e1}=\mu_{e2}=0$ and $\sigma_{e1}=\sigma_{e2}=h/790$. In the case that the random realization of the inclinations of the first column and the second one have an opposite sign, we shall multiply the inclination of each column with the coefficient 79/43 for Class 1 and 79/67 for Class 2. Let us note that the multiplications described in the preceding sentence are carried out in the form of a computation model, i.e., the condition that the input random quantities e_1 and e_2 are uncorrelated is fulfilled.

2.2. Material and geometrical characteristics

Traditionally, material yield strength, tensile strength and ductility have been studied among the mechanical characteristics of steel structures. Strength characteristics in the Czech Republic have been statistically studied for a long time within the framework of non-commercially aimed research [4], and results have been compared with results of similar studies performed abroad [5]. For dimensioning of structures, the yield strength is most important above all. Statistical characteristics of yield strength of steel grade S235 of the IPE profile used in the hereby presented study were published in [4]. For non-measured quantities (Young's modulus), the study was based on data obtained from technical literature; for example, statistical characteristics of Young's modulus are given in [8,9]. Geometrical characteristics of profiles IPE were considered according to results of experimental research [6]. With the aim of focusing attention on the influence of global sway imperfections e_1 and e_2 (and their higher order interaction effects), local bow imperfections of the columns were neglected. The influence of the random size and shape of strut axis on the ultimate limit state was studied in [10]. The influence of residual stress was not taken into consideration in the numerical study. All the input characteristics, given synoptically in Table 1, are statistically independent of one another.

Table 1
Input random quantities.

No.	Member	Symbol		Mean value	Std. deviation
1	Left column	h_1	*	240.22 mm	1.0616 mm
2		b_1	*	121.67 mm	1.1842 mm
3		t_{w1}	*	6.53 mm	0.2421 mm
4		t_{f1}	*	9.73 mm	0.4494 mm
5		E_1	**	210 GPa	10.5 GPa
6		f_{y1}	*	297.3 MPa	16.8 MPa
7	Cross-beam	h_0	*	300.27 mm	1.327 mm
8		b_0	*	152.08 mm	1.4802 mm
9		t_{w0}	*	7.48 mm	0.2773 mm
10		t_{f0}	*	10.62 mm	0.4907 mm
11		E_0	**	210 GPa	10.5 GPa
12		f_{y0}	*	297.3 MPa	16.8 MPa
13	Right column	h_2	*	240.22 mm	1.0616 mm
14		b_2	*	121.67 mm	1.1842 mm
15		t_{w2}	*	6.53 mm	0.2421 mm
16		t_{f2}	*	9.73 mm	0.4494 mm
17		E_2	**	210 GPa	10.5 GPa
18		f_{y2}	*	297.3 MPa	16.8 MPa
19	System	e_1	**	0	Section 2.1
20		e_2	**	0	Section 2.1

* Histogram.
** Gauss pdf.

3. Computation model

The elastic resistance was calculated by finding a stress distribution which equilibrates the ultimate internal forces and moments without exceeding the yield strength. The frame geometry was meshed using beam elements. Each column was meshed using ten beam elements. Internal forces and bending moments were calculated by geometric nonlinear solution using linear stress–strain laws. The geometric nonlinear solution was elaborated and programmed by the author of the present paper [11]. The step-by-step Euler Newton–Raphson iterative procedures simulate real experiments. The first criterion (i.e., strength condition) for the load-carrying capacity is given by loading at which plasticization of the flange is initiated. The second criterion (i.e., stability condition) for the load-carrying capacity is represented by loading corresponding to a decrease of the determinant of stiffness matrix to zero (this phenomenon occurs at high yield strength values with a small initial system imperfection). The ultimate one-parametric loading is defined as the lowest value from the strength and stability criterion of load-carrying capacity.

Realizations of input variables were computed using the MC method, by means of which repetitions of experiments were simulated. The obtained output value is that of the random load-carrying capacity. The load-carrying capacity was determined with 0.1% accuracy in each simulation run.

4. Sensitivity analysis

The sensitivity analysis is a study of how uncertainty in the model output can be apportioned to different sources of uncertainty in the model input factors [7]. Within the scope of modeling, the notion "sensitivity analysis" has different meaning for different people, see e.g. [11–16]. The information on problems and applications of the sensitivity analysis of thin-walled members is presented, e.g. in [17–22]. The sensitivity analysis contributes to model development, model calibration, model validation, reliability and robustness analysis, decision-making under uncertainty, quality-assurance, and model reduction. The basic division of sensitivity analysis is deterministic sensitivity analysis (sometimes referred to as the "what-if study") and stochastic sensitivity analysis.

We can generally distinguish two types of sensitivity analysis: regression-based methods and variance-based methods [3]. Regression-based methods use a regression of the output on the input vector, and variance-based methods decompose the variance of the output as a sum of contributions of each input variable. The variance-based techniques are sometimes called ANOVA techniques, i.e., ANalysis Of VAriance. Variance-based methods have a long history in sensitivity analysis, starting with the Fourier implementation in the seventies [23], and have a milestone in the work of Sobol [24].

4.1. Sobol sensitivity analysis

The sensitivity analysis concept enabling the analysis of the influence of arbitrary subgroups of input quantities on the monitored output was worked out by the Russian mathematician Ilja M. Sobol [24]. The concept of Sobol sensitivity analysis was worked out in detail and published along with a number of examples from a number of scientific fields, e.g. in [7]. The case with statistically independent input random variables X_i (input imperfections) was studied. Sobol first order sensitivity indices may be written in the following form:

$$S_i = \frac{V(E(Y|X_i))}{V(Y)} \tag{1}$$

S_i measures the first order (e.g. additive) effect (so-called the main effect) of X_i on the model output Y. The second order sensitivity index S_{ij} is the interaction term (3) between factors X_i, X_j. It captures that part of the response of Y to X_i, X_j which cannot be written as a superposition of effects separately due to X_i and X_j:

$$S_{ij} = \frac{V(E(Y|X_i,X_j))}{V(Y)} - S_i - S_j \tag{2}$$

Other Sobol sensitivity indices enabling the quantification of higher order interactions may be expressed similarly:

$$\sum_i S_i + \sum_i \sum_{j>i} S_{ij} + \sum_i \sum_{j>i} \sum_{k>j} S_{ijk} + \cdots + S_{123\ldots M} = 1 \tag{3}$$

The number of members in (3) is $2^M - 1$, i.e., for $M=3$ we obtain 7 sensitivity indices S_1, S_2, S_3, S_{12}, S_{23}, S_{13}, S_{123}; for $M=20$ we obtain more than one million of sensitivity indices; it is excessively large for practical usage. The main limitation in the determination of all members of (3) is the computational demandingness.

The sensitivity indices S_i were evaluated applying the MC method. The conditional random arithmetical mean $E(Y|X_i)$ was evaluated for 10 000 simulation runs; the variance $V(E(Y|X_i))$ was calculated for 10 000 simulation runs, as well. The variance $V(Y)$ of load-carrying capacity is calculated under the assumption that all the input imperfections are considered to be random ones; 100 000 runs were applied. The second-order sensitivity indices S_{ij} were calculated analogously.

5. Sensitivity analysis results

The sensitivity analysis is applied to the study of the influence of initial imperfections (input values from Table 1) on the load-carrying capacity (output value). Sobol sensitivity indices, S_i and S_{ij}, were determined in dependence on the non-dimensional slenderness of columns. The non-dimensional slenderness $\overline{\lambda}$ is given in EUROCODE 3 by

$$\overline{\lambda} = \sqrt{\frac{A f_{y,c}}{F_{cr}}} \tag{4}$$

where A is the cross-sectional area of the column, $f_{y,c}$=235 MPa is the characteristic value of yield strength and F_{cr} is the buckling load (Euler critical force) of steel plane frame. The buckling load of the steel plane frame (with ideally vertical columns) was calculated using the beam finite element method including the second-order effect. The stability condition for the buckling load is represented by loading corresponding to a decrease of the determinant of stiffness matrix to zero (eigenvalue problem). The load-carrying capacity for $\overline{\lambda}=0$ was evaluated for a short perfectly vertical column without consideration of the buckling effect.

The graphs of sensitivity indices S_i and S_{ij} are depicted in Fig. 7. The study shows several trends in the effect of initial imperfections upon the load-carrying capacity. Very small values of sensitivity indices are not depicted. Due to the fact that the frame, load action and boundary conditions are symmetrical, the values of sensitivity indices of both left and right columns are the identical, i.e., they are depicted by a single curve only.

5.1. Influence of imperfections on the load-carrying capacity

Imperfections e_1, e_2: Maximum values of the first order sensitivity indices S_{19}=S_{20}=0.18 (Class 1) and S_{19}=S_{20}=0.15 (Class 2) of imperfections e_1 and e_2 were calculated for $\overline{\lambda}=0.95$ (buckling length L_{cr}=8.9 m). Maximum values of the second order

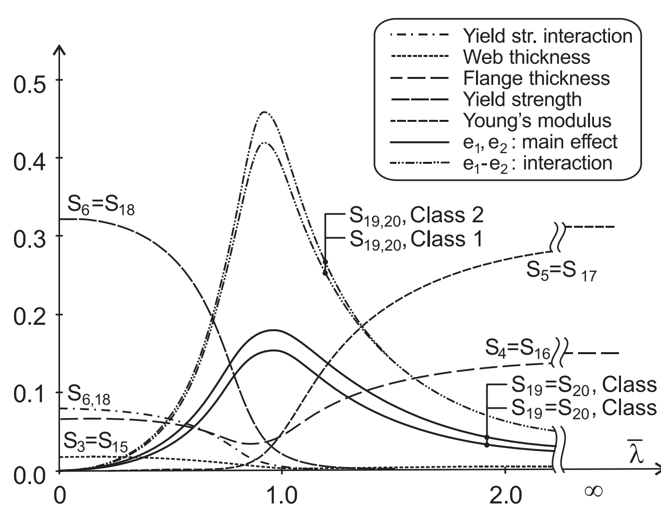

Fig. 7. Sobol sensitivity indices vs. non-dimensional slenderness.

sensitivity indices $S_{19,20}=0.42$ (Class 1) and $S_{19,20}=0.46$ (Class 2) describing interactions between e_1 and e_2 were calculated for $\bar{\lambda}=0.93$ (buckling length $L_{cr}=8.7$ m). Differences in results for Class 1 and Class 2 are apparent, see Fig. 7. The first order sensitivity indices of Class 1 are higher than the first order sensitivity indices of Class 2. The second order sensitivity indices of Class 1 are lower than the first order sensitivity indices of Class 2.

Yield strength f_{y1}, f_{y2}: The influence of the yield strength on the load-carrying capacity decreases with increase in $\bar{\lambda}$, see Fig. 7. The load-carrying capacity is most influenced by the yield strengths of the left or right column for $\bar{\lambda}<0.7$. For $\bar{\lambda}=0$, the first order sensitivity indices S_6 (f_{y1}) and S_{18} (f_{y2}) are dominant, and the second order effect $S_{6,18}$ between f_{y1} and f_{y2} has the maximum effect on the sum of all second order sensitivity indices, see Fig. 8. For $\bar{\lambda}=0$ there is no difference between the results for Class 1 and Class 2, differences are very small for $\bar{\lambda}>0$. For $\bar{\lambda}>1.2$, the load-carrying capacity is almost insensitive to changes in the yield stress in both columns. For high slenderness, the stress in the structure in limit state is significantly lower than the yield

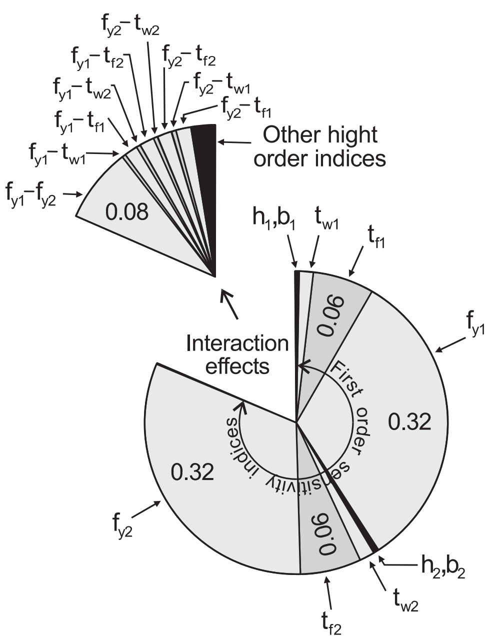

Fig. 8. Sensitivity analysis for $\bar{\lambda}=0$.

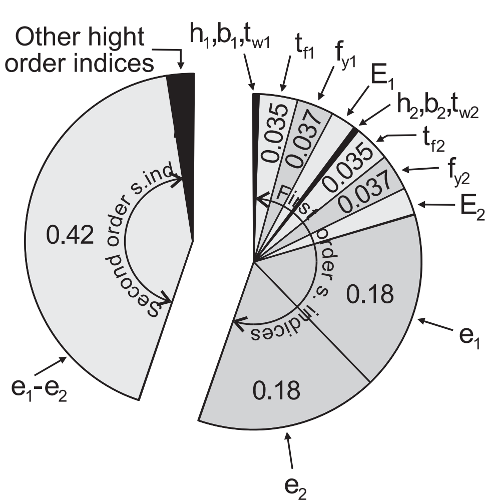

Fig. 9. Sensitivity analysis for $\bar{\lambda}=0.93$, Class 1.

strength and therefore the variance of the yield strength has no influence on the variance of the load-carrying capacity (Fig. 7).

Young's modulus E_1, E_2: With increase in $\bar{\lambda}$, the load-carrying capacity approaches the buckling load, and therefore is more sensitive to changes of Young's modulus values E_1, E_2, see Fig. 7. Interaction effects of variables E_1, E_2 with the other variables are practically negligible. Let us note that variables E_1, E_2, in comparison with the other variables in Table 1, have a relatively small variation coefficient. The influence is apparent only for high slenderness where the influence of variables f_{y1}, f_{y2}, e_1, e_2 is small.

Flange thickness t_{f1}, t_{f2}: Increase in variables t_{f1}, t_{f2} contributes to an increase in the load-carrying capacity by increasing the cross-section area and the second moment of area of the columns. For small column slenderness, the load-carrying capacity is influenced by the cross-section area. Interaction effects between the flange thickness and yield strength of both columns are small, see Fig. 8. For high slenderness, the stability and corresponding load-carrying capacity are influenced by the second moment of area.

Web thickness t_{w1}, t_{w2}: Maximum values of the first order sensitivity indices $S_3=S_{15}=0.017$ were calculated for $\bar{\lambda}=0$. Values of S_3 and S_{15} decrease with increase in slenderness, and in the event that $\bar{\lambda}>0.9$, they are practically equal to zero. Interactions effects are also very small.

The influences of all other characteristics, evident in Table 1, are very small, and therefore their sensitivity indices are not depicted in Fig. 7. Let us note that the behaviour of sensitivity indices depicted in Fig. 7 is analogous to similarly evaluated results of the sensitivity analysis of the load-carrying capacity of a steel strut [2], with the difference being in the manifestation of the influence of interactions amongst input imperfections in the steel frames.

5.2. Interaction effects and reliability

One important distinction between the Sobol sensitivity and the classical one is that the Sobol sensitivity analysis detects interactions of input variables through the second and higher order terms, while classical sensitivity methods give only derivatives with respect to single variables [3]. The imperfections which interact and may thus generate extreme values of load-carrying capacity have been identified. This is important, for example, in structural reliability.

Relatively high values of the second-order sensitivity index $S_{19,20}$ are apparent in Fig. 7. In practice, it means that the influence of imperfections e_1 and e_2 on the load-carrying capacity cannot be expressed by the sum of influences e_1 and e_2. The influence of the interaction between imperfections e_1 and e_2 is most significant for $\bar{\lambda}=0.93$, see Fig. 9. Class 2 has a higher interaction effect than Class 1, because permitted deviations of Class 2 $|\Delta_b|=h/300$ are smaller, (stricter) than these for Class 1 $|\Delta_b|=h/150$. Let us note that, if a positive correlation between variables e_1 and e_2 is introduced, then we will obtain a load-carrying capacity with lower mean value and a higher standard deviation. This can be clearly demonstrated using the MC method. The decrease in load-carrying capacity is influenced namely by runs e_1 and e_2 with the same sign (Case 1). The effect on the load-carrying capacity is negative, being the most negative for $e_1=e_2$ (correlation 1.0). The implementation $e_1=e_2$ is conservative and is commonly used in practical design of steel structures. System imperfections of the frame formally identical to the sway buckling mode may also be considered and studied.

The interaction between yield strengths both of the left and right columns was determined for small slenderness, see Fig. 8. The load-carrying capacity of the frame increases if we increase the yield strength of both columns. The opposite extreme can be dangerous. Let us note that the yield strength of steel S235 is generally a random variable, statistical characteristics of which were evaluated from a higher number of samples over a longer time period [4]. The variance of yield strength from one production line (with relatively small number of samples from a shorter time period, etc.) can be lower than the variance of the yield strength evaluated over a longer time period. If a positive correlation between the yield strength of the left and right columns is introduced, then the load-carrying capacity will have a high mean value and higher load-carrying capacity. The implementation of this model is debatable and its effect on reliability would have to be studied and proved with other probabilistic studies.

6. Conclusion

The sensitivity analysis was used to detect and rank those imperfections that need to be measured with greater accuracy in order to reduce the variance of load-carrying capacity of the steel plane frame.

System imperfections e_1 and e_2 have a dominant effect on the load-carrying capacity of frequently used column lengths with non-dimensional slenderness close to one. The sensitivity analysis results have shown that interactions between imperfections e_1 and e_2 have approximately twice the effect on the variance of the load-carrying capacity than when considered individually, see Fig. 7. The main advantage of sensitivity analysis is that it provides quantified evaluation of the influence of individual imperfections and their interaction on the ultimate limit state, and results may also be used for probabilistic assessments of reliability and calibration methods and approaches, respectively methods of the verification of tolerance limits Δ_a, Δ_b and other imperfections. The sensitivity analysis results illustrate that the statistical characteristic of system imperfections e_1 and e_2 should be determined with increased accuracy; however, it is difficult or practically impossible under hard service conditions.

The second greatest interaction effect of the second order $S_{6,18}=0.08$ was obtained between the yield strength of the left and right columns for $\overline{\lambda}=0$, see Fig. 8. However, the value of this interaction effect is, in comparison with the main effect S_6 or S_{18}, approximately a quarter $S_{6,18}\approx 0.25\cdot S_6=0.25\cdot S_{18}$ and hence of little significance. Let us note that $S_{6,18}$ is approximately equal to the main effect of flange thickness of the left or right column $S_{6,18}\approx S_4=S_{16}$.

The most important characteristics, checked on mass produced steel IPE members, are the yield strength and geometrical characteristics of the cross-section. For the practice, we can recommend the thorough measurement and check of yield strength and flange thickness, the variances of which have an influence on the variances of the load-carrying capacity of compressed columns and may also be of significance under other types of strain and loading. Let us note that the mean value of flange thickness should be equal to the nominal value. In the case of the yield strength, the member reliability may be increased by decreasing the variance or by increasing the mean value.

The cross-section height of columns, flange width of columns and cross-beam characteristics were identified as the factors which, if left free to vary over their range of uncertainty, make no significant contribution to the variance of the load-carrying capacity. The identified imperfections can be fixed at any given value within their range of variation without affecting the load-carrying capacity. This analysis can be performed on groups of factors, especially for large models, to identify non-influential subset of imperfections.

The load-carrying capacity of very slender members is very sensitive to Young's modulus, the variance of which cannot be influenced in production. The second most significant variable is the flange thickness the variance of which can be favourably influenced in production. No significant higher order interaction effects were found for the hereby analysed problem for high slenderness values.

The initial imperfections are among the basic and frequently most important input data of theoretical studies and hence they should be paid great attention. In this regard, it is generally necessary to point out the publications dedicated to research problems in the field of stability problems [25]. For the load-carrying capacity problems, it is also necessary to study the influence of interaction amongst imperfections formally identical to the buckling modes which cause the instability [26–28], the influence of load action eccentricity [29] and that of joint stiffness [30]. Variance-based methods can be beneficial when studying the first and the higher order interaction effects of imperfections on limit states and reliability of structures.

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