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Stability problems of steel structures in the presence of stochastic and fuzzy uncertainty

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Abstract

General ideas and problems of probability approach and its utilization in the verification of structural design procedures of EUROCODES are mentioned. The paper is aimed at the probability study of the ultimate limit state of a steel compressed member designed economically according to EUROCODE 3. The theoretical failure probability (reliability index) vs. ratio of permanent to variable load action is calculated by means of the Monte Carlo simulation method. The misalignment of the failure probability according to EN1990 is analysed. Initial imperfections are generally considered as random variables and random fields. The non-linear beam FEM is used. The influence of initial curvature shape and size variability of the member axis on the variability of load-carrying capacity is investigated. The probabilistic analysis is supplemented with the fuzzy analysis of the influence of uncertainties on the failure probability.

Keywords: EUROCODE; Stability; Imperfections; Structural steel; Stochastic yield strength; Structural reliability; Fuzzy sets; Fuzzy reliability

1. Introduction

This paper deals with the probabilistic verification of reliability of steel bar structures designed according to the EUROCODE standard. Design procedures of EUROCODE standards, utilized in dimensioning of steel members, stems from the limit state methodology. The reliability of design is secured by partial safety factors.

Unified European standards EUROCODE ensure a satisfactory level of reliability provided that the required corresponding quality of production of metallurgical products in individual EU countries is met.

In the Czech Republic, material and geometric characteristics of steel products are controlled both by manufacturers and at independent scientific workplaces [1]. The greatest attention is paid to the monitoring and analysis of the random variable values of yield strength, material strength and ductility. It has been proved by means of comparison studies that statistical characteristics of yield strength, material strength and ductility of Czech and Austrian steel are in good concordance [2].

In stability problems, initial geometric imperfections of member cross-section and axis have a great influence on the load-carrying capacity of slender members under compression. With increasing member slenderness the influence of the variability of yield strength on the variability of load-carrying capacity decreases [3] and the influence of flexural rigidity EI (the product of Young's modulus E and the second moment of area I), which prevents buckling, increases [3]. The variability of load-carrying capacity of the bar is the most sensitive to the variability of initial imperfection of axial bar curvature with non-dimensional slenderness $\bar{\lambda} = 1.0$ [3].

The paper is aimed at the probabilistic analysis of the ultimate limit state of a compressed member of profile IPE 220 with $\bar{\lambda} = 1.0$. With aim at an accurate description of the influence of initial curvature of strut axis on the failure probability, size and shape imperfections of strut axis are modelled utilizing random fields [4] (see Fig. 1). Computer-based FEM modelling and simulation are required for the stochastic analysis.

The problem involves both aleatory and epistemic uncertainties. During structural design, an information on statistical characteristics of eventual loading is absent. Imprecision (fuzziness) of information on the random

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initial imperfections and their correlations presents a further source of uncertainty.

Newer mathematical approaches, which extend or depart from the probability theory, are available, e.g. in Refs. [5,6]. In order to obtain realistic results from stochastic inference, imprecision (fuzziness) of data has to be modelled quantitatively. This is possible by applying the fuzzy sets theory [7,8].

2. Initial curvature of the axis

In general, the axis of a real member is a curve; an ideally straight member is practically never concerned. Let us consider a long member with initial plane axial curvature (see Fig. 2) with unit weight g acting at the cross-sectional centroid, which is constant per unit length of curved axis. Next let the main central axis x_c of the curve about which the second moment of area of the curved axis of the member is minimal be sought.

An orthogonal coordinate system y_c vs. x_c is considered (see Fig. 2). The curve axis is divided into n equidistant members in the direction of axis x_c, resulting in a random sample of n members. Each member $i = 0, 1, \ldots, n$ is subjected to axial compressive loading passing through the first and the last node of the strut. The force line determines the local strut axis x_i, which is at angle α_i with respect to axis x_c.

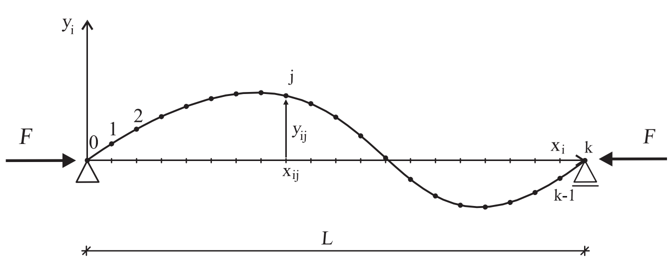

Fig. 1. Random field of strut axis curvature.

Each ith member is subdivided into k adjacent equidistant elements $k+1$ (see Fig. 1). The angle α_i of the local coordinate system y_i vs. x_i is dependent on the position of the initial y_{ci0} and final y_{cik} node of the member and may be determined from the relation

$$\tan(\alpha_i) = \frac{\Delta y_i}{\Delta x_i} = \frac{y_{cik} - y_{ci0}}{x_{cik} - x_{ci0}}. \tag{1}$$

Coordinates of the jth node in the coordinate system y_c vs. x_c are transformed into the local coordinate system y_i vs. x_i according to the relations

$$y_{ij} = (y_{cij} - y_{ci0})\cos(\alpha_i) - x_{cij}\sin(\alpha_i), \tag{2}$$

$$x_{ij} = (y_{cij} - y_{ci0})\sin(\alpha_i) + x_{cij}\cos(\alpha_i). \tag{3}$$

For $\Delta y_i \ll \Delta x_i$ and $(y_{cij} - y_{ci0}) \ll 1$ it holds approximately that $x_{ij} \approx x_{ij}$, $x_{ij} \approx x_{cij}$, which is a frequent case practically.

The initial deformation of the jth node in the coordinate system y_c vs. x_c is a random variable, which will be denoted as y_{cj}. When the number of struts n is a sufficiently large number it holds that the mean value m_{ycj} of random variable y_{cj} of the jth node is approximately zero:

$$m_{ycj} = \frac{1}{n}\sum_{i=1}^{n} y_{cij} \approx 0 \quad \text{for } j = 0, 1, \ldots, k. \tag{4}$$

The shape of initial curvature of the strut axis depicted in Fig. 1 represents the ith random observation from n struts. Let us denote the random deviation of the jth node in the local coordinate system y vs. x as y_j (see Fig. 3). It can be illustrated (e.g. by Monte Carlo simulation) that the mean value m_{yj} of random deviation y_j is equal to zero:

$$m_{yj} = \frac{1}{n}\sum_{i=1}^{n} y_{ij} \approx 0 \quad \text{for } j = 0, 1, \ldots, k, \tag{5}$$

where y_{ij} was evaluated according to Eq. (2). The shape of the random curvature is given by the correlation of variables y_j amongst $k-1$ nodes with $y_0 = y_k = 0$. The correlations amongst variables y_{cj} are approximately the

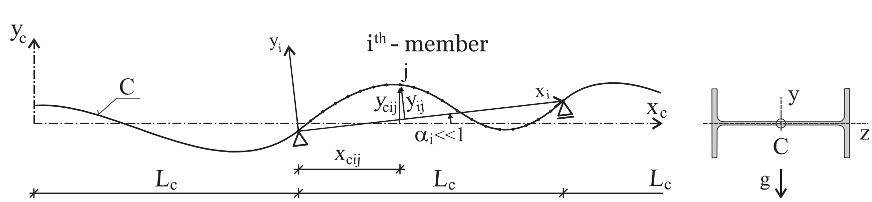

Fig. 2. Global and local coordinate system of axial curvature.

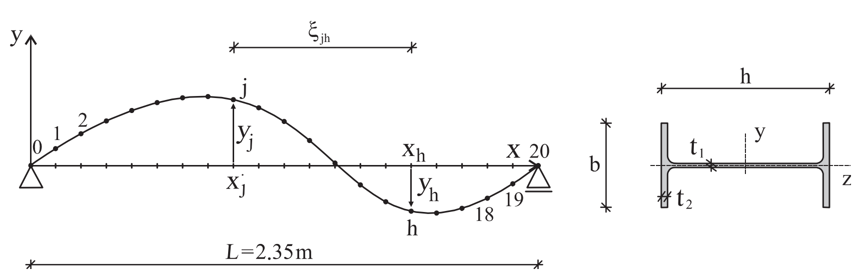

Fig. 3. Geometrical imperfections of strut axis curvature and cross-section dimensions.

Table 1
Input random imperfections

Symbol	Value	Density	Mean	Standard deviation
h	Cross-section height	Histogram	220.22 mm	0.975 mm
b	Flange width	Histogram	111.49 mm	1.093 mm
t_1	Web thickness	Histogram	6.225 mm	0.247 mm
t_2	Flange thickness	Histogram	9.136 mm	0.421 mm
y_j	Initial imperfections	Gauss	0 m	S_{yj} according to Eq. (6)
E	Modulus of elasticity	Gauss	210 GPa	12.6 GPa
f_y	Yield strength	Histogram	297.3 MPa	16.8 MPa

same as amongst variables y_j. This may be proven utilizing the Monte Carlo simulation and Spearman's rank-order correlation. It can be assumed that the correlation decreases with increasing distance between nodes and that the standard deviation S_{yj} is nil at the end nodes $j = 0$, $j = k$ and maximal at the centre of the strut.

The standard deviation along the length of the strut is approximately given by the sine function (this is corroborated for e.g. by Monte Carlo simulation). In the case that the number of nodes k is even, the standard deviation S_{yj} of the jth node can be expressed by the standard deviation S_{ya} of the central node with index $a = k/2$:

$$S_{yj} \approx S_{ya} \sin\left(\frac{\pi x_j}{L}\right). \tag{6}$$

It may be summarized that the initial random deviations y_j for $j = 0, 1, \ldots, k$ have a mean value of $m_{yj} = 0$ and the standard deviation S_{yj} along the length of the strut is given by a sine function.

Statistical characteristics of other random variables and their correlations may be obtained only through elaborated statistical analysis based on experimental studies. The real curvature of steel members is given by the technology of rolling, transportation, welding and other production processes. It would be interesting to determine what relation exists between the correlations of adjacent nodes and the amplitude of the standard deviation S_{ya}.

The maximal deformations of real members are limited by the tolerance standard [9] and are checked through output production inspections. The standard deviation S_{ya} may be theoretically evaluated provided that 95% of observations occur within the tolerance limits of standard [9], which tolerates a maximal initial curvature of 0.15% of member length L in the case of profile IPE 220. The Gaussian probability density function is considered for y_i in the subsequent numerical study.

3. Input random imperfections

During the simulation of the deviation of nodes y_i, e.g. by Monte Carlo simulation, the correlation amongst deviations of nodes y_i eliminate the unreal shapes of initial axial curvature. Let us consider the coordinate system y vs. x (see Fig. 3). The degree of correlation amongst deviations y_i is most frequently represented by the Gauss autocorrelation function [4], which defines the correlation amongst y_j and y_h:

$$\rho_{jh} = \mathrm{e}^{-(\xi_{jh}/L_{\mathrm{cor}})^2}, \tag{7}$$

where L_{cor} represents the correlation length of the random field. The Gauss density function of y_j is considered. The correlation length $L_{\mathrm{cor}} = 1.2\,\mathrm{m}$ was obtained through virtual simulation [10]; Monte Carlo simulations of the realizations of random fields of axial curvature were performed, such that the virtual results corresponded to experimentally obtained results published in Ref. [11]. Ninety-five percent of observations of y_a occur within the tolerance limits [9] (-3.525 mm; 3.525 mm) for $m_{ya} = 0$ and $S_{ya} = 1.8$ mm.

Other statistical material and geometrical characteristics of profile IPE 220 were considered according to results of experimental research [1]. The influence of residual stress was not taken into consideration in the numerical study. Statistical characteristics of Young's modulus E are considered according to two independently performed experimental researches [11,12] (see Table 1).

4. Probabilistic verification of structural stability design procedures

For an illustration of the probabilistic evaluation of the specified design procedure, the problem of compressed member of profile IPE 220 is studied. The design load-carrying capacity of the strut according to the unified European concept of EUROCODE 3 [13] is given as

$$R_{\mathrm{d}} = \frac{\chi f_{\mathrm{y}} A_{\mathrm{n}}}{\gamma_{\mathrm{M1}}} = \frac{0.597 \times 3.34 \times 10^{-3} \times 235 \times 10^{6}}{1.0} = 468.6\,\mathrm{kN}. \tag{8}$$

A compressed member loaded by permanent action combined with single variable action is considered. The design load action can be expressed by

$$F_{\mathrm{d}} = \gamma_{\mathrm{G}} G_{\mathrm{k}} + \gamma_{\mathrm{Q}} Q_{\mathrm{k}}. \tag{9}$$

It is assumed that the structure is designed for maximum exploitation (economic design), i.e., $F_{\mathrm{d}} = R_{\mathrm{d}}$. The partial safety factors $\gamma_{\mathrm{G}} = 1.35$ and $\gamma_{\mathrm{Q}} = 1.5$ are considered according to Ref. [14]. The characteristic values G_{k} and Q_{k} can be determined according to Eq. (9) in dependence on the chosen ratio δ given by $\delta = Q_k/(G_k + Q_k)$. For permanent action G, the Gaussian density function [15] with mean value $m_G = G_k$ and variation coefficient 0.1 is assumed [10]. For variable action Q, Gumbel-max density function [15] with mean value $m_Q = 0.6Q_k$ and standard deviation $S_Q = 0.21Q_k$ is considered. The analysis of the member reliability is based on the condition

$$K_R R > K_F(G + Q). \tag{10}$$

Load-carrying capacity R is calculated for input imperfections from Table 1. The geometrical non-linear FEM utilizing Euler Newton–Raphson method is used [3]. The member is meshed by 20 beam elements (see Fig. 1 and/or Fig. 3). R is evaluated from the condition that the normal stress in the most stressed strut section is equal to yield strength f_y. The probabilistic analysis results are obtained utilizing the Monte Carlo simulation for 5 millions runs. The probability that Eq. (10) is not fulfilled is evaluated.

K_R and K_F represent coefficients of model epistemic uncertainties of the load-carrying capacity R and the loading effect F in relation (10). The aim of further studies is not the elaborated analysis of the origin of model uncertainties, but rather the theoretical quantification of their influence on the behaviour of failure probability P_f in dependence on parameter δ. For this purpose coefficients K_R, K_F are chosen as fuzzy numbers with linear triangular

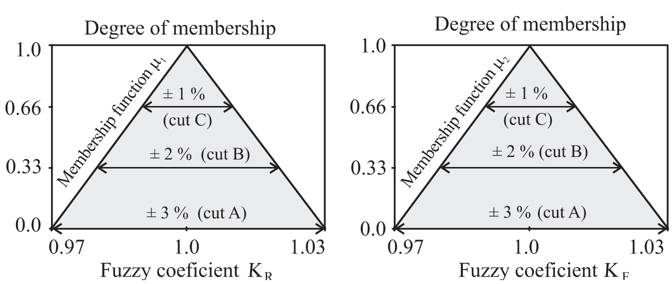

Fig. 4. Fuzzy numbers K_R and K_F of uncertainties in the determination of resistance and load action.

symmetrical membership functions (see Fig. 4). The fuzzy analysis of failure probability P_f is evaluated according to the general extension principle for 10 α-cuts [7] (see Fig. 5)

$$\mu_{P_f}(K_R, K_F) = \underset{P_f}{\vee}(\mu_1(K_R) \wedge \mu_2(K_F)). \tag{11}$$

The fuzzy analysis procedure according to Eq. (11) may be explained on the cut A. Let us consider $\delta = 1$. The minimum $P_{f,\min} = 10E{-}5$ is evaluated for $K_F = 0.97$, $K_R = 1.03$ and maximum $P_{f,\max} = 37.7E{-}5$ is estimated for $K_F = 1.03$, $K_R = 0.97$. Fuzzy uncertainty of load-carrying capacity and loading $\pm 3\%$ brings about fuzzy uncertainty P_f quantified by the interval $P_f \in \langle 10E{-}5, 37.7E{-}5 \rangle$. Cuts A (support intervals) are marked by the dashed line (see Fig. 5). The calculation procedure is analogical for cut B, cut C and the other 10 α-cuts.

5. Conclusion

The misalignment of theoretical failure probability P_f (reliability index β) vs. ratio of permanent to variable load action δ is presented in Fig. 5. The purely stochastic solution ($K_R = K_F = 1$) is depicted by the full line in the bottom plane of the 3D graph. Low reliability of design $P_f > 7.2E{-}5$ ($\beta < 3.8$) [14] was obtained for $\delta > 0.74$.

Results of fuzzy analysis quantify the dependence of P_f on the change of coefficients K_R, K_F. The output asymmetric non-linear membership functions vs. triangular symmetric membership functions of coefficients K_R and K_F are obtained. This information is very valuable because it quantifies the non-linear dependence between the coefficients of model uncertainties K_R and K_F and the theoretical failure probability P_f. The defuzzified "crisp" failure probability values are obtained utilizing the centre of gravity method [7]. Due to the non-linear and asymmetrical membership functions of failure probability P_f, the defuzzified values are higher (dot-and-dash line) than the values of the purely stochastic solution (full line) (see Fig. 5).

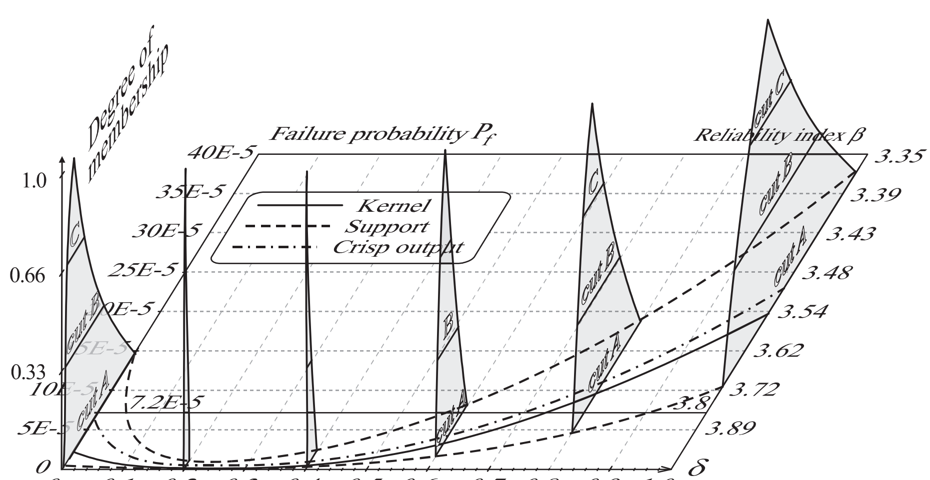

Fig. 5. Fuzzy analysis of the misalignment of failure probability.

Further analytical studies are planned for additional values of partial safety factors γ_G, γ_Q, γ_M. The reliability analysis of the design of steel structures according to the allowable stress design method for $\gamma_G = 1.0$; $\gamma_Q = 1.0$ and $\gamma_M = 1.5$ will be performed. Probabilistic analysis results of the limit state method and the allowable stress method will be compared.

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