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# Sensitivity analysis of the stability problems of thin-walled structures

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**Abstract**

Analysing the system behaviour in relation to the input quantities it is often necessary to find out what quantities have the greatest effect on the studied output. The article shows the essential methods of applied sensitivity analysis. The objective of the paper is to analyse the influence of initial imperfections on the resistance of a member under axial compression. The analysis uses the Latin Hypercube Sampling simulation method (LHS) [Novák D, Teplý B, Shiraishi N. Sensitivity analysis of structures. In: Proc. of the fifth int. conference on civil and structural engineering computing. 1993. p. 201–07; Novák D, Lawanwisut W, Bucher C. Simulation of random fields based on orthogonal transformation of covariance matrix and Latin hypercube sampling. In: Proc. of int. conference on Monte Carlo simulation. 2000. p. 129–36] together with advanced models based on the nonlinear beam finite element method. The histograms of initial imperfections obtained by measurement [Melcher J, Kala Z, Holický M, Fajkus M, Rozlívka L. Design characteristics of structural steels based on statistical analysis of metallurgical products. Journal of Constructional Steel Research 2004;60:795–808] were considered.



*Keywords:* Sensitivity analysis; Simulation method; Imperfections; Yield strength; Buckling

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## 1. Introduction

Solving the problems of stability, we are usually, besides the final result (stress and deformation, load-carrying capacity, failure probability, etc.), interested in the fact of how much the input parameters affect the result, or in other words what is the sensitivity of the response to the change of the input parameter. The use of the sensitivity analysis enables us to determine the dominant quantities that must be paid special attention. The sensitivity analysis can be generally divided into two fields:

*The deterministic sensitivity analysis* (or also the design sensitivity) is quite a well known and commonly used means for designing a structure. It is the component part of a design procedure, which uses a computational model enabling a successive change of values of one input quantity and uses parametric study to investigate the effect of the change on the output quantity. Even though these studies are very valuable and provide a quick overview on the model behaviour, they do not usually enable satisfactory conception of the whole spectrum of the possible cases that can occur on the real structure. In this connection we normally use a parametric study (sometimes called "what-if-study").

*The stochastic sensitivity analysis* provides more complex (and quantified!) information on the parameter's influence. The procedure of determining the sensitivity is to a certain extent similar to the deterministic sensitivity analysis. We also change the value parameter and observe how it is reflected in the output quantity. The change of the input quantity respects also the frequency of the occurrence, i.e. the realizations of the input random quantities are simulated as if they were received by measuring. The simulation usually indicates a phase of experimental work using a representation of a computational model. The objective of the simulation is to analyse the behaviour of the system in dependence on the input quantities and values of parameters.

In recent years, many various stochastic sensitivity analysis methods have been developed [12,13] and a number of possibilities for their practical applications has been presented [5,8]. Together with the development of new reliability analysis concepts (see, e.g., [6,9,17]), these methods can contribute to qualitative improvement of structure reliability analysis methods.

## 2. Basic sensitivity analysis methods

The sensitivity analysis is the analysis of the input quantity variability influence on the output quantity variability. In other words, it is the phenomenon of how the random variability of an input quantity influences (in comparison with the others) the structure response variability and how it takes part in the resulting failure probability. The sensitivity analysis answers thus the question of which quantities are dominant, and therefore they must be paid increased attention at (i) the preparation of input; (ii) the considerations and decision making concerning the improvement of technology procedures; (iii) the conception and organization of controlling activities. In cases (ii) and (iii), also economic criteria are usually included. Further on, it is possible to recognize by means of the sensitivity analysis which quantities show only little influence, and therefore they can be considered, as the case may be, only deterministically (as non-random ones) in further analyses. It can contribute to simplification and acceleration of calculations and modelling. Such an approach is applied abroad but only rather marginally [1,15].

The contemporary models have been mostly elaborated, based on FEM (geometrically and materially nonlinear), and processed by simulation technologies of the Monte Carlo type. A small number of simulations runs can be used for acceptable accuracy using some stratified sampling; one often used alternative is the Latin Hypercube Sampling (LHS) technique. This technique belongs to the category of advanced simulation methods [10,3]. The distribution-free approach to how to consider also correlated random variables was suggested by [4,14]. Using a small number of simulations, LHS results in satisfactory good estimates of basic statistical parameters of response (especially mean and standard deviation). A superiority of the LHS technique compared to the simple random sampling Monte Carlo was first shown in [10]. The LHS method can be also used for sensitivity analysis.

A comprehensive review of various sensitivity analysis methods is given in [13,12]. For our research, the methods which can be applied for the sensitivity analysis evaluation can be divided as follows:

(1) The method based on the correlation coefficient study.
(2) The method based on the study of variation coefficient ratios.
(3) Special methods for the probability analysis.

The first method can be applied practically for all numerical simulation methods of the Monte Carlo type. The method described is based on the assumption that there will be a higher correlation degree with the output in the case of the quantities relatively more sensitive to the output. The so-called Spearman rank-order correlation $r_i$ is frequently applied within the framework of a simulation method [4]. The Spearman rank-order correlation can be defined as:

$$r_i = 1 - \frac{6\sum_j (m_{ji} - n_j)^2}{N(N^2-1)}, \qquad r_i \in [-1, 1] \tag{1}$$

where $r_i$ is the order representing the value of random variable $X_i$ in an ordered sample among $N$ simulated values applied in the $j$th simulation (the order $m_i$ equals the permutation at LHS), and $n_j$ is the order of an ordered sample of the resulting variable for the $j$th run of the simulation process ($m_{ji} - n_j$ is the difference between the ranks of two samples). If the coefficient $r_i$ had a value near to 1 or $-1$, it would suggest a very strong dependence of the output on the input. Opposite to this, the coefficient with its value near to zero will signalise a low influence.

The second method is based on the comparison of sensitivity coefficients $k_i$, defined on behalf of variation coefficients by the relation:

$$k_i = 100\frac{v_{yi}^2}{v_y^2}[\%]. \tag{2}$$

$v_{yi}$ is the variation coefficient of the output quantity, assuming that all the input quantities except the $i$th one ($i = 1, 2, \ldots, M$; where $M$ is number of input variables) are considered to be *deterministic* (during the simulation, they are equal to the mean value);

$v_y$ is the variation coefficient of the output quantity, assuming that all the input quantities are considered to be random ones.

It is worthy of notice that, when calculating the second power of the variation coefficient $v_{yi}$, the input quantities with the exception of the $i$th one are equal to the mean value. It can cause numerical problems in the very complicated calculation model where the mean value of an input quantity represents a limit case of the corresponding physical process. One of the possible solutions is the transformation of relation (2) to the following relation:

$$k_i = 100\frac{v_y^2 - \hat{v}_{yi}^2}{v_y^2}[\%]. \tag{3}$$

$\hat{v}_{yi}$ is the variation coefficient of an output quantity when supposing that all the input quantities with the exception of the $i$th one are considered to be *random* (within the simulation framework, the $i$th input quantity is left to equal the mean value);

$v_y$ is the variation coefficient of an output quantity when supposing that all the input quantities are considered to be random ones.

Relation (3) is only a modification of relation (2) and it should offer the same numerical values of coefficients $k_i$ in common well defined problems. The mathematic derivation of relation (2) and its application to a very complicated calculation model of steel plane frame stability problem were presented in [7].

## 3. Input random quantities

The member under axial compression made of a hot rolled steel profile IPE 180 was analysed. At the first end of plane member the horizontal and vertical displacements were fixed. At the second end of the member the horizontal (axial) displacement was free and the perpendicular displacement was fixed. The non-dimensional slenderness is taken as the decisive parameter. Three members with non-dimensional slenderness $\overline{\lambda} = 0.6; 1.0; 1.4$ were considered. The values of non-dimensional slenderness were chosen to correspond to real slenderness in practice design according to EC3 [18]. Corresponding strut lengths calculated by [18] are $L = 1.2$ m, $L = 2.0$ m and $L = 2.8$ m, respectively.

The yield strength $f_y$ variability of the steel grade S235 was taken into consideration by the experimentally determined histogram; see Fig. 4 in [11] and Table 1. The cross-section height $h$, the flange thickness $t_2$, the flange width $b$ and the web thickness $t_1$ were considered as histograms; see [11] and Table 1. A biaxially symmetrical cross-section was assumed, i.e., for the quantities $b$, $t_2$, the histograms obtained by geometry measurement of only one flange were considered.

Buckling in the direction of the axis perpendicular to the web plane was taken into account. The initial curvature of the member axis was introduced as one half-wave of the sine function with random amplitude $e_0$. The Gaussian distribution function of the initiation curvature amplitude $e_0$ was introduced. Its statistical characteristics were calculated so that the frequency of the occurrence of random realizations within the

Table 1
The statistical characteristics of input random quantities

| No. | Quantity | Name of random quantity | Type of distribution | Dimensions | Mean value | Standard deviation |
|---|---|---|---|---|---|---|
| 1 | $h$ | Cross-section height | Histogram | mm | 180.18 | 0.797 |
| 2 | $b$ | Flange width | Histogram | mm | 92.092 | 0.934 |
| 3 | $t_1$ | Web thickness | Histogram | mm | 5.592 | 0.222 |
| 4 | $t_2$ | Flange thickness | Histogram | mm | 7.904 | 0.349 |
| 5 | $e_0$ | Amplitude of curvature | Gauss | m | L/1500 | L/4800 |
| 6 | $f_y$ | Yield strength | Histogram | MPa | 297.3 | 16.8 |
| 7 | $E$ | Young's modulus | Gauss | GPa | 210 | 12.6 |

interval $\langle 0; L/1000\rangle$ was 95%. For the Young modulus $E$, the study was based on the data given in the literature [2,16]. The influence of deviations of physical–mechanical material characteristics (e.g., heterogeneousness), is included in the Young modulus variability.

## 4. Sensitivity analysis of the load-carrying capacity

The geometrical nonlinear solution elaborated and programmed by the author of the present paper was applied in the sensitivity analysis, see, e.g., [6]. The load-carrying capacity was solved by the nonlinear Euler incremental method and combined with the Newton–Raphson method.

The first criterion (i) for the load-carrying capacity is a loading at which plastification of the flange is initiated. The second criterion (ii) for the load-carrying capacity is represented by a loading corresponding to a decrease of the tangential toughness determinant to zero. The ultimate one-parametric loading is defined as the lowest value of load-carrying capacities (i) and (ii). The strut geometry was modelled, in a very minute manner, by means of a mesh of beam elements with initial curvature in form of a parabola of the third degree. The LHS simulation method was applied for 1000 simulation runs. The LHS simulation method and sensitivity analysis methods (1) and (2) were programmed by the author of the present paper too. In each run of the simulation method, the load-carrying capacity was determined, accurate to 0.1%. Altogether 1000 runs of calculations have been performed resulting in a spectrum of ultimate random load-carrying capacity presented in Fig. 1. In this way, it was found out which initial imperfections had the greatest influence on the load-carrying capacity.

## 5. Conclusion

It is evident from Figs. 2 to 4 that the sensitivity coefficients vary in dependence on the member slenderness.

It is presented in Fig. 4 that the load-carrying capacity variability of a member with non-dimensional slenderness [18] $\overline{\lambda} = 1.4$ is highly sensitive to the variability of flange thickness $t_2$ and further, to the variability of flange width $b$ and of Young's modulus $E$ above all. The positive value of the correlation coefficient means that with increasing

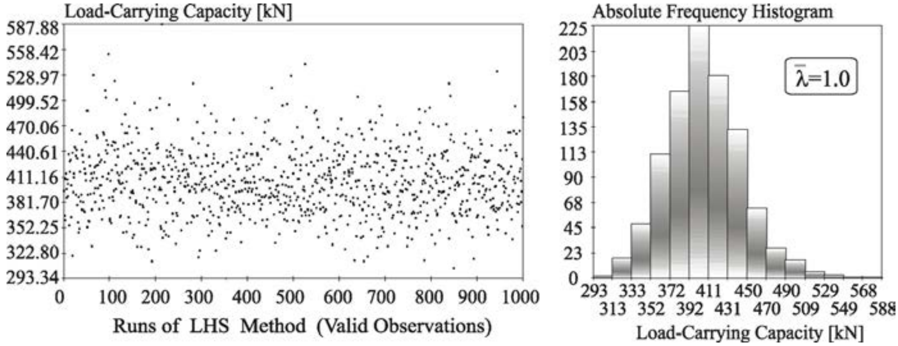


Fig. 1. Values of random load-carrying capacity of strut with length $L = 2.0$ m ($\overline{\lambda} = 1.0$).

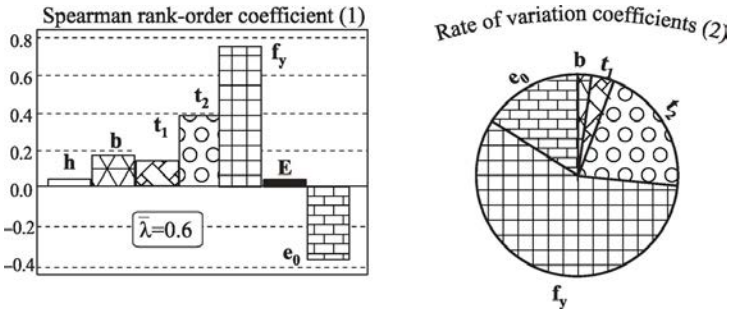


Fig. 2. Results of sensitivity analysis of strut $L = 1.2$ m, $\overline{\lambda} = 0.6$.

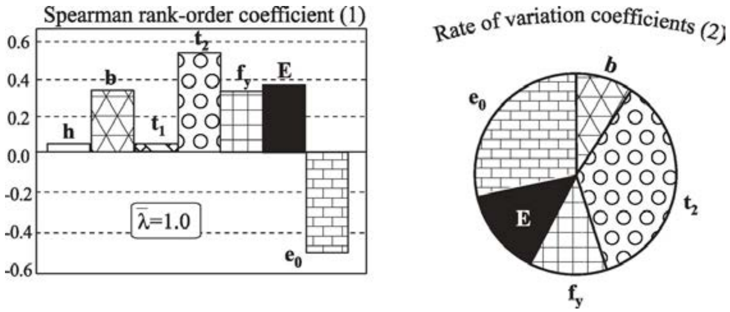


Fig. 3. Results of sensitivity analysis of strut $L = 2.0$ m, $\overline{\lambda} = 1.0$.

value of the given quantity, also the load-carrying capacity increases. The correlation coefficient value of Young's modulus $E$ is comparable with the correlation coefficient of initial curvature $e_0$; the value is, however, negative. The load-carrying capacity is sensitive to the Young modulus variability $f_y$ only very little.

It is obvious from Fig. 2 that, for the member with non-dimensional slenderness $\overline{\lambda} = 0.6$ the yield strength considerably influences the increase in load-carrying capacity.

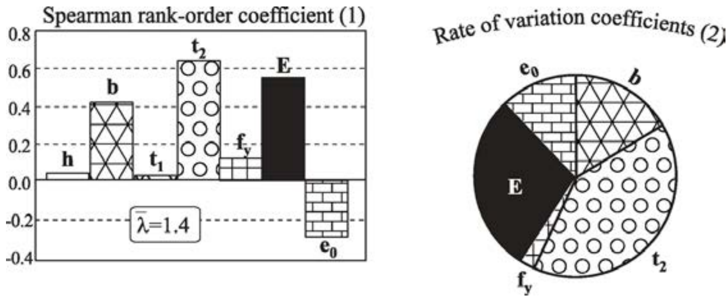


Fig. 4. Results of sensitivity analysis of strut $L = 2.8$ m, $\overline{\lambda} = 1.4$.

The sensitivity coefficients $k_i$ according to (2) refer to the dominant influence of yield strength on load-carrying capacity, as well. As both applied methods (1) and (2) are based on different assumptions the comparison of results is difficult. However, each method has the informative value of the other type. The flange thickness $t_2$ and also the initial curvature $e_0$ are further significant quantities. The variability influence of Young's modulus $E$ on load-carrying capacity is practically negligible.

For the member with non-dimensional slenderness $\overline{\lambda} = 1.0$ it can be seen from Fig. 3 that the influence of yield strength, flange thickness, and also of the other quantities is evidently overlapped by the variability influence of initial curvature $e_0$. However, it is to be noticed that the differentiation of load-carrying capacity of compression members is derived from the effects of residual stresses above all which were neglected in the present study. The buckling curves $a$, $b$, $c$, $d$ of [18] are the most differing and at the same time, they decrease most rapidly for the approximate value of $\overline{\lambda} \approx 1.0$.

The input random imperfections can be divided approximately into two basic groups—those the statistical characteristics of which can be favourably influenced by manufacturing (yield strength, geometrical characteristics), and those not satisfactorily sensitive to manufacturing technology changes (e.g., Young's modulus $E$ variability). The first group of quantities can be subdivided into two subgroups: (i) quantities for which both mean value and standard deviation can be changed by improvement in quality of manufacturing – such a quantity is, e.g., yield strength; (ii) quantities the mean value of which cannot be changed more substantially as the mean value should equal the nominal value, e.g., geometrical characteristics of cross-section dimensions.

According to Figs. 2 to 4 the flange thickness $t_2$ is the important quantity having always a relatively great influence on load-carrying capacity. Lowering the variability of this quantity can be reached by the manufacturing technology change. The variability decrease of yield strength $f_y$ can be recommended for members with lower non-dimensional slenderness above all.

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