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Sensitivity analysis of steel plane frames with initial imperfections

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ABSTRACT

The article presents the sensitivity and statistical analyses of the load-carrying capacity of a steel portal frame. It elaborates a typical stability problem of a system comprising two single-storey columns loaded in compression. The elements of this system mutually influence each other, and this fact, in conjunction with the random imperfections, influences the load-carrying capacity variance. This mutual interaction is analysed using the Sobol' sensitivity analysis. The Sobol' sensitivity analysis is applied to identify the dominant input random imperfections and their higher order interaction effects on the load-carrying capacity. Majority of imperfections were considered according to the results of experimental research. Realizations of initial imperfections were simulated applying the Latin Hypercube Sampling method. The geometrical nonlinear solution providing numerical result per run was employed. The frame was meshed using beam elements. The columns of the plane frame are considered with two variants of boundary conditions. The dependence between mean and design load-carrying capacities and column non-dimensional slenderness is analysed.

1. Introduction

Along with the progress of structural design theories and the technological advancement of steelworks production, more and more large-scale and high-rise steel bar structures are implemented in modern structures. The issue of stability of these structures becomes more apparent due to the utilization of more slender members.

The frame stability requires that all structural members and connections of the frame have adequate strength to resist the applied loads where static equilibrium is satisfied on the deformed geometry of the structure. In order to determine the load-carrying capacity of an actual structure, it is necessary to take into consideration initial imperfections and to consider the geometrically nonlinear analysis.

In general, all imperfections are of random character. The reliability of steel structures depends on the variance of input imperfections which influences the evaluation of limit states of building structures. The attainment of limit states is generally a random phenomenon, which is examined in the field of reliability using probabilistic theories and mathematical computation models.

One of the most important characteristics occurring in probabilistic methods of reliability assessment of steel structures is the variance of the load-carrying capacity which is primarily given by the quality of production. Basic indicators of production quality include the yield strength, tensile strength, ductility and geometrical characteristics of cross sections; see, e.g., [1,2]. Relatively sufficient statistical information is provided for the material and geometrical characteristics of mass produced hot-rolled members of steel structures in comparison to other building branches. Scarcely measurable imperfections of steel plane frames include the inevitable initial crookedness of bar members (bow imperfections) and out-of-plumb inclinations of the columns (sway imperfections) in the same frame [3,4]. Some measurements have been made in connection with testing programmes [5], but very little data is available.

The frames depicted in Figs. 1 and 2 represent a typical stability problem of a structural system consisting of more members. The frames are typical lean-on systems which are characterized by the structural members tied or linked together in such a way that buckling of the column would require adjacent members to buckle with the same lateral displacement. The imperfection interaction effects can have a significant influence on the overall performance of the frames. The steel frame depicted in Fig. 1 has rotation and translation fixed boundary conditions of both column ends. The steel plane frame in Fig. 2 is similar to that in Fig. 1 with the exception that there is no rotation restrain at the column ends. The rotation fixed and rotation free conditions represent the two limits of real anchorage in practice. Let us denote the frame in Fig. 1 as Frame 1 and the frame in Fig. 2 as Frame 2.

In the presented paper, the effects of input imperfections on the load-carrying capacities of Frames 1 and 2 are evaluated

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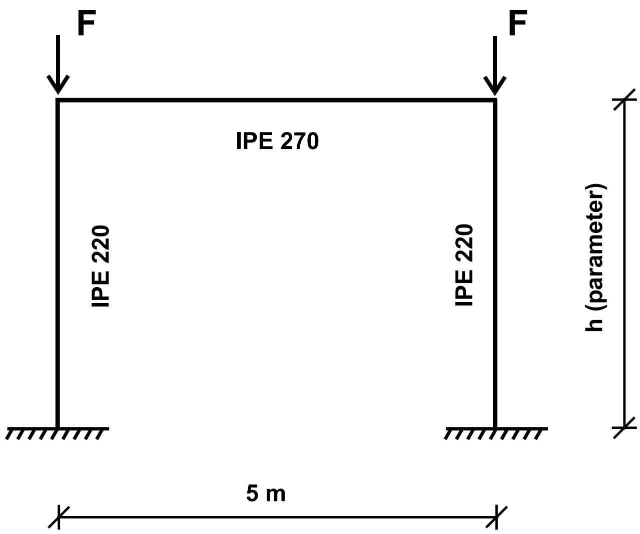

Fig. 1. Frame 1, rotation and translation fixed boundary conditions.

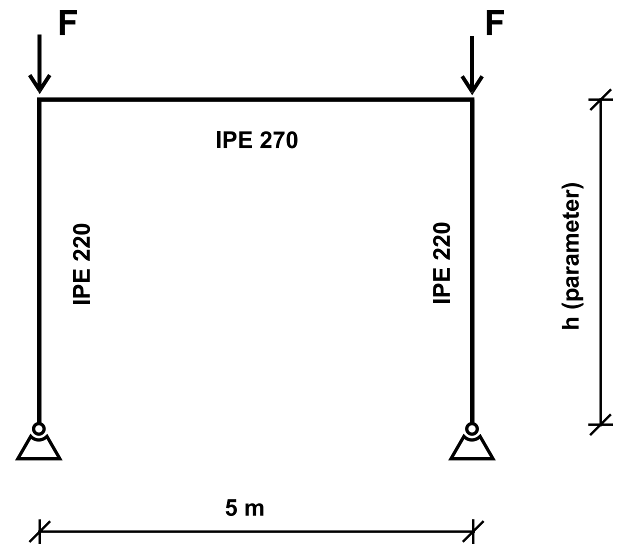

Fig. 2. Frame 2, rotation free and translation fixed boundary conditions.

by means of sensitivity analysis. The lean-on imperfect system (left column leans on right column) requires the utilization of sensitivity analysis which enables the evaluation of the influence of individual imperfections on the load-carrying capacity as well as of their higher order interaction effects. An outline of sensitivity analysis methods with examples of their application in a number of scientific fields is listed, e.g., in [6]. With regard to the random character of initial imperfections, the influence of imperfections on the load-carrying capacity of the frame systems will be studied applying the Sobol' sensitivity analysis [7–9]. One of the advantages of Sobol' sensitivity analysis is that it enables the identification of interaction effects among input quantities on the monitored output. The effects of the dominant imperfections, which have the greatest influence on the load-carrying capacities, will be described. Design load-carrying capacities evaluated statistically according to EN1990 [10] and according to the partial factor method of EUROCODE 3 [11] will be compared later on in the article. Obtained results will be discussed in connection with the results of Sobol' sensitivity analysis.

2. Input random imperfections

Imperfections are practically unavoidable, and they represent acceptable construction tolerances. The presence of imperfections in the analysis and design of frame systems with slender members has always been recognized, however, the manner of considering their effect on structural behaviour in computational models differs. Imperfections may generally be considered as deterministic (non-random) variables [12] or as random variables [4,13]. In this article, all imperfections will be considered as random variables.

Geometrical imperfections are, as a general rule, not visible to the naked eye, nor can they be quantified precisely beforehand. The first type of geometrical imperfection is the inclination of each column; see Fig. 3. Let us denote the inclination of the left column as Θ_1 and of the right column as Θ_2. Permitted inclination deviations of columns of a single-storey portal frame are listed in the standard EN1090-2. Let us assume that imperfections Θ_1 and Θ_2 are statistically independent random quantities with mean values equal to zero (perfectly vertical columns) and that both quantities have a Gauss probability density function. Let us further assume that 95% of realizations of Θ_1 and Θ_2 remain within the tolerance limits given by the standard EN1090-2. The classification Class 1 according to standard EN1090-2 was assumed. Detailed probabilistic derivation of the standard deviations $\sigma_{e_1}, \sigma_{e_2}$ is

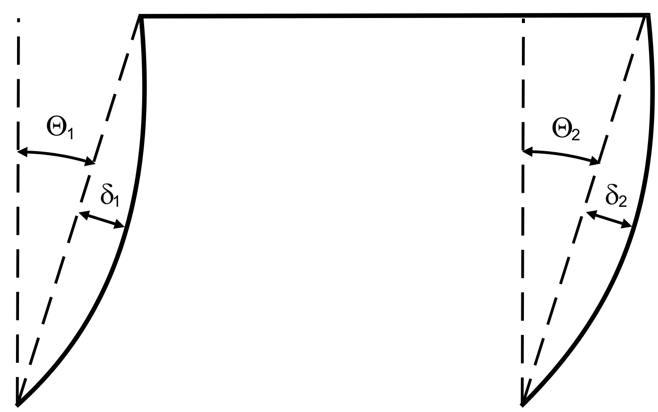

Fig. 3. Initial sway and bow imperfections.

described in [4]. Practical analysis based on the Monte Carlo simulation method may be performed in the following manner. Let us introduce quantities Θ_1 and Θ_2 with $\sigma_{e_1} = \sigma_{e_2} = h/790$. In the case that the random realizations of Θ_1 and Θ_2 have opposite signs, we shall multiply the inclination of each column by the coefficient 79/43; see [4].

The initial bow imperfection (initial crookedness) of member axis was described using a half sine wave; see Fig. 3. Both the positive and negative realizations of the amplitude should occur with the same frequency, which means that the mean value equals zero. The standard deviation of the Gauss probability density function has been selected for the random amplitude such that 95% of the realizations are found within the tolerance limits given by the standard EN 10034. Let us denote the amplitude of initial crookedness of left column as δ_1 and the amplitude of right column as δ_2.

Fig. 4 illustrates eight combinations of positive and negative imperfections from Fig. 3. Fig. 4 provides a basic idea of the shape but not of the magnitude of imperfect geometry. A more accurate notion would be obtained if imperfections Θ_1, Θ_2, δ_1, δ_2 were considered as random variables; see Fig. 5. Fig. 5 illustrates (in a larger scale) eight realizations of initial imperfections generated by the LHS method [14,15]. Random geometrical characteristics of profiles IPE were deduced according to results of experimental research [2]. According to standard EUROCODE 3, hot-rolled cross section is classified as Class 1 cross section which can form a plastic hinge with the rotation capacity required for plastic hinges. Effects

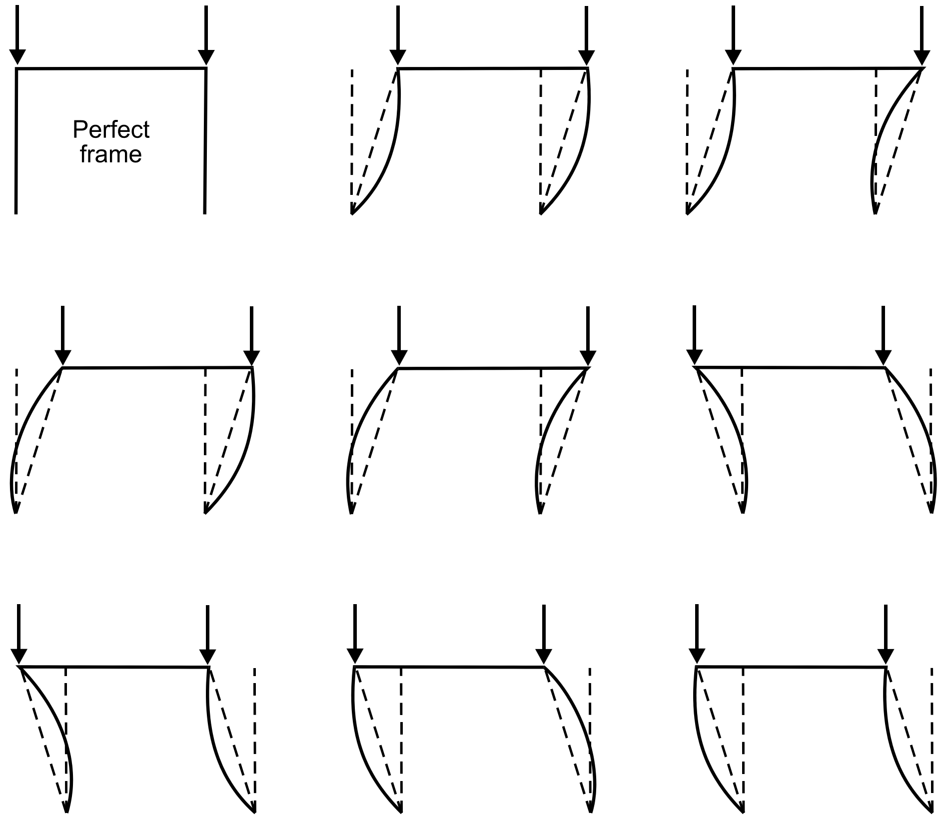

Fig. 4. Eight combinations of plus and minus initial imperfections Θ_1, Θ_2, δ_1, δ_2.

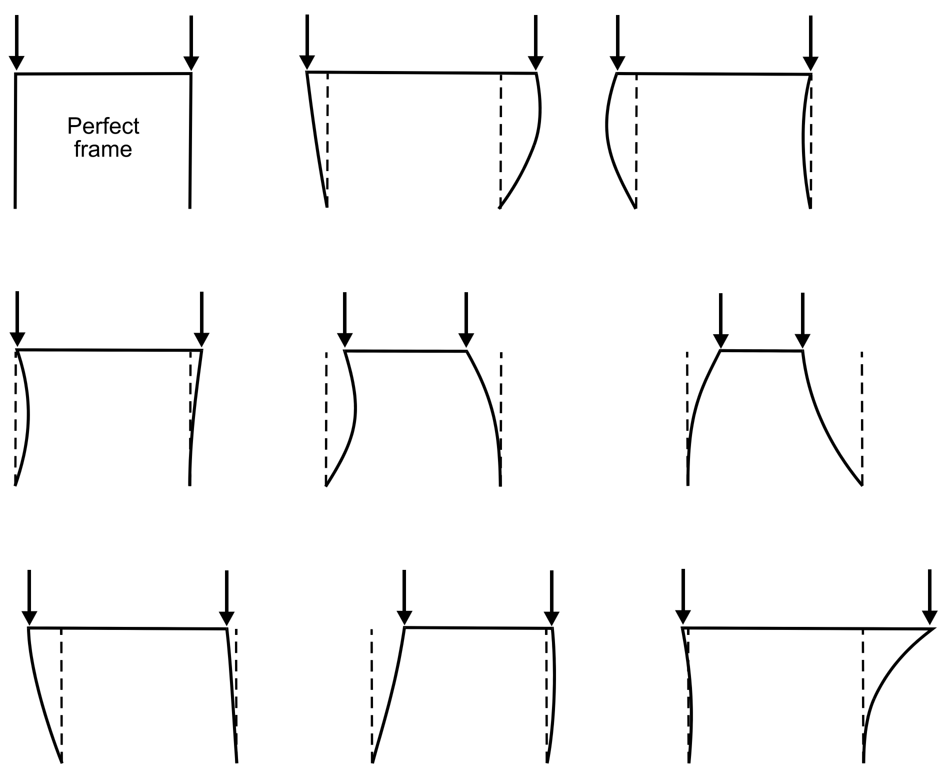

Fig. 5. Eight LHS random realizations of initial imperfections Θ_1, Θ_2, δ_1, δ_2.

of local plated imperfections can therefore be neglected for Class 1 cross sections. Let us assume that both columns and the cross beam are prismatic (the cross section is constant from one end to the other). Let us note that, as a result of the aforementioned random geometrical imperfections, the frames from Figs. 1 and 2 have an asymmetrical geometry.

Material properties in the Czech Republic have been statistically studied for a long time [1,2]. The yield strength is the most

Table 1
Input random quantities.

No.	Member	Symbol	Mean value	Std. deviation
1.	Left column	h_1[a]	220.20 mm	0.9731 mm
2.		b_1[a]	111.53 mm	1.0855 mm
3.		t_{w1}[a]	6.22 mm	0.2304 mm
4.		t_{f1}[a]	9.13 mm	0.4219 mm
5.		E_1[b]	210 GPa	10.5 GPa
6.		f_{y1}[a]	297.3 MPa	16.8 MPa
7.		δ_1[b]	0	0.76533 h
8.	Cross beam	h_0[a]	270.24 mm	1.194 mm
9.		b_0[a]	136.88 mm	1.3322 mm
10.		t_{w0}[a]	6.96 mm	0.2577 mm
11.		t_{f0}[a]	10.13 mm	0.4678 mm
12.		E_0[b]	210 GPa	10.5 GPa
13.		f_{y0}[a]	297.3 MPa	16.8 MPa
14.	Right column	h_2[a]	220.20 mm	0.9731 mm
15.		b_2[a]	111.53 mm	1.0855 mm
16.		t_{w2}[a]	6.22 mm	0.2304 mm
17.		t_{f2}[a]	9.13 mm	0.4219 mm
18.		E_2[b]	210 GPa	10.5 GPa
19.		f_{y2}[a]	297.3 MPa	16.8 MPa
20.		δ_2[b]	0	0.76533 h
21.	System	Θ_1[b]	0	Chapter 2
22.	System	Θ_2[b]	0	Chapter 2

[a] Histogram.
[b] Gauss pdf.

important quantity during the dimensioning of structures. The yield strength variation depends largely on the chemical composition and rolling conditions. The actual yield strengths are usually much higher than the characteristic values.

Let us further assume that the material is linearly elastic. Statistical characteristics of yield strength of steel grade S235 of the IPE profile used in the study presented hereby were published in [1]. The modulus of elasticity was considered, as based on data obtained from the technical literature [16], to be a random quantity. The effect of residual stresses was not taken into consideration in the numerical study. All the input characteristics, given synoptically in Table 1, are statistically uncorrelated.

3. Computation model

The frame was modelled using beam elements. Each column was meshed into ten beam elements, and the cross beam was meshed using three beam elements. The equilibrium equations were expressed for the deformed frame geometry. The analysis of the deformed geometry was performed by the geometric nonlinear solution using elastic structural analysis with linear stress–strain laws. The geometric nonlinear solution was elaborated and programmed by the author of the present paper [17]. The load-carrying capacity was evaluated using step-by-step Euler Newton–Raphson iterations. The first criterion, i.e., the strength condition, for the load-carrying capacity is given by the load during which plasticization of the flange is initiated. The second criterion, i.e., stability condition, for the load-carrying capacity is given by the load corresponding to a decrease of the determinant of the stiffness matrix to zero which occurs at high yield strength values of slender columns with small initial imperfections Θ_1, Θ_2, δ_1, δ_2. The ultimate one-parametric loading is obtained as the lowest value from the strength and stability criteria of the load-carrying capacity.

Realizations of input variables were computed by applying the LHS method which was used to simulate experiment repetitions. The obtained output is the random load-carrying capacity. The load-carrying capacity was determined with an accuracy of 0.1% in each simulation run.

4. Sensitivity analysis

The information on problems and applications of the sensitivity analysis of steel structures is presented, e.g., in [4,13,17–24]. In general, the sensitivity analysis studies relationships between information flowing in and out of the model [25]. The basic categorization of sensitivity analysis is: the deterministic sensitivity analysis and the stochastic sensitivity analysis. Further possible division includes: the local sensitivity analysis and the global sensitivity analysis [25]. The local sensitivity measures determine the influence of parameters by varying one parameter at a time and keeping the other parameters constant [26]. The global sensitivity analysis, on the other hand, considers a variation of all parameters simultaneously and evaluates their contribution to the uncertainty [26].

The aim of global sensitivity analysis is to apportion the uncertainty in the output variable to the uncertainty in each input variable, described typically by probability density functions [25]. Global sensitivity analysis typically takes an uncertainty range in the input that reflects our imperfect knowledge of the material and geometrical characteristics of steel structures. With regard to the ultimate limit states of steel structures, the resistance is frequently considered to be the important output variable.

4.1. *The Sobol' method*

The Sobol' decomposition has been introduced as a method for global sensitivity analysis [25]. The Sobol' sensitivity analysis enables us to analyse the influence of arbitrary subgroups of input variables on the monitored output. The sensitivity analysis of load-carrying capacity (random output Y) to input imperfections (input random variables X_i in Table 1) was evaluated in the presented study.

4.1.1. *Sensitivity indices*

The Sobol' first-order sensitivity index may be written in the form:

$$S_i = \frac{V(E(Y|X_i))}{V(Y)}. \tag{1}$$

S_i measures the main (e.g. additive) effect of X_i on the model output Y. An important distinction between Sobol' and classical sensitivity is that the Sobol' sensitivity analysis detects interactions of input variables through the second and higher order terms, whilst classical sensitivity methods give only derivatives with respect to single variables. The influences of higher order interactions (e.g., the influence of doubles) on the monitored output are significant in systems comprising more members. Interactions represent important features of models, and are more difficult to detect than first-order effects. The Sobol' second-order sensitivity index is given as

$$S_{ij} = \frac{V(E(Y|X_i, X_j))}{V(Y)} - S_i - S_j. \tag{2}$$

S_{ij}, for $i \neq j$, is called the second-order sensitivity index and measures the interaction effect between a pair of (X_i, X_j). It captures that part of the variation in Y due to X_i and X_j that cannot be explained by the sum of individual effects of X_i and X_j. $V(E(Y|X_i, X_j))$ in Eq. (2) measures the joint effect of the pair (X_i, X_j) on Y. Analogous formulae can be written for higher order Sobol' sensitivity terms, enabling the analyst to quantify higher order interactions.

The case with statistically independent input random variables X_i (input imperfections) was studied. The decomposition of Sobol' sensitivity indices can be written as:

$$\sum_i S_i + \sum_i \sum_{j>i} S_{ij} + \sum_i \sum_{j>i} \sum_{k>j} S_{ijk} + \cdots + S_{123\ldots M} = 1. \qquad (3)$$

The sum of all sensitivity indices must be equal to 1. The number of terms in (3) increases exponentially with the number of input variables M. The number of terms in (3) is $2^M - 1$, i.e., for $M = 22$ input variables, we would obtain more than four million sensitivity indices; this is excessively large for practical usage. The main limitation in the determination of all members of (3) is the computational demand.

4.1.2. Total effect indices

The high number of sensitivity indices (3) and the numerical demand of their evaluation generally do not allow the listing of the sensitivity indices of all orders but allow only the sensitivity indices (1) at most (2) and the so-called total effect of input variables and their interactions with others on the monitored output [25].

$$S_{Ti} = \frac{E(V(Y|X_{\sim i}))}{V(Y)} = 1 - \frac{V(E(Y|X_{\sim i}))}{V(Y)}. \qquad (4)$$

The total sensitivity index S_{Ti} is defined as the sum of all sensitivity indices involving the load-carrying capacity, in other words, S_{Ti} measures the total effect, i.e., the first and higher order effects (interactions) of input variable X_i. $X_{\sim i}$ are all input variables that do not include the index i. One way to visualize this is by considering that $V(E(Y|X_{\sim i}))$ is the first-order effect of $X_{\sim i}$, so that $V(Y)$ minus $V(E(Y|X_{\sim i}))$ must give the contribution of all terms in the variance decomposition which do include X_i. The difference $S_{Ti} - S_i$ is a measure of how much X_i is involved in interactions with any other input variables. The sum of all S_i is equal to 1 for additive models, and less than 1 for non-additive models. The difference $1 - \sum_i S_i$ is an indicator of the presence of interactions in the model.

5. Sensitivity and statistical analysis results

The frame height h was considered as a computational parameter; see Figs. 1 and 2. In order to enable the mutual comparison of results of Frames 1 and 2, results will not be presented in dependence to frame height h but to the non-dimensional slenderness $\bar{\lambda}$ of columns which is evaluated for a perfect frame according to EUROCODE 3; see also [4]. For example, for $h = 5$ m, we would obtain for Frame 1 from Fig. 1 $\bar{\lambda} = 0.63$, and for Frame 2 from Fig. 2 $\bar{\lambda} = 1.26 = 2 \cdot 0.63$. Let us note that the non-dimensional slenderness $\bar{\lambda}$ is a multi-functional variable linearly dependent on h. By considering the non-dimensional slenderness of columns, more general results are obtained.

5.1. Sensitivity analysis results

The conditional expectation $E(Y|X_i)$ was evaluated by averaging 500 simulation runs of $(Y|X_i)$ within the same slice X_i; the conditional variance $V(E(Y|X_i))$ was calculated for 500 simulation runs, as well. The unconditional variance $V(Y)$ was calculated for 100 000 simulation runs. The second-order sensitivity indices S_{ij} and total effects S_{Ti} were calculated analogously. The LHS method significantly increases the rate of convergence estimates of Sobol' sensitivity indices; see, e.g., [25]. Due to the fact that the frame, load action and boundary conditions are symmetrical, the values of sensitivity indices of both left and right columns are identical, i.e., they are depicted by a single curve only.

5.1.1. Frames 1 and 2

Results of the sensitivity analyses of Frames 1 and 2 are depicted in Figs. 7 and 8. All sensitivity indices (1) and (2) were calculated, however, for greater clarity's sake, very small values of sensitivity indices are not depicted.

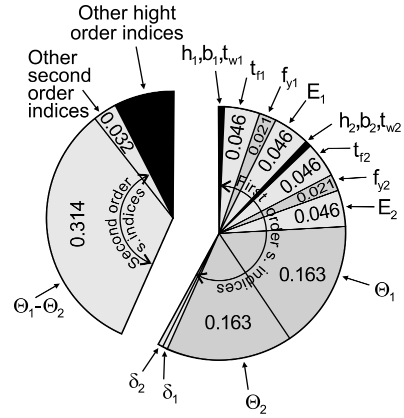

Fig. 6. Sobol' sensitivity indices of Frame 1 for $\bar{\lambda} = 1.02$.

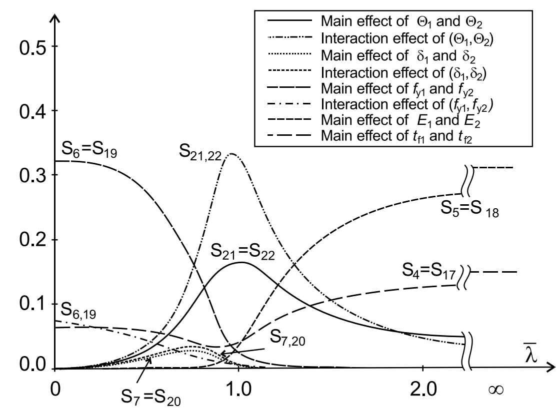

Fig. 7. Sobol' sensitivity indices of Frame 1 vs. non-dimensional slenderness.

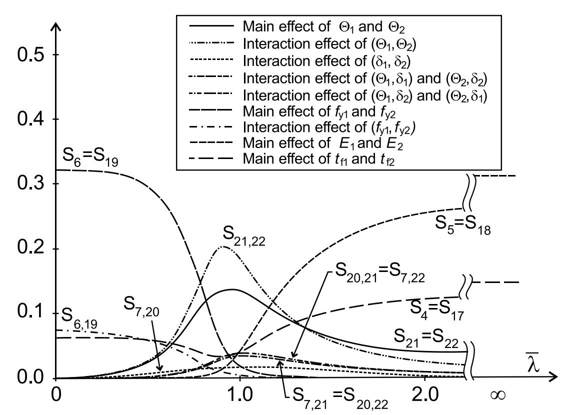

Fig. 8. Sobol' sensitivity indices of Frame 2 vs. non-dimensional slenderness.

It generally holds that the yield strength influence on the load-carrying capacity decreases with increasing $\bar{\lambda}$; see Figs. 7 and 8. For $\bar{\lambda} = 0$, the first-order sensitivity indices S_6 (f_{y1}) and S_{19} (f_{y2}) are dominant, and the second-order effect $S_{6,19}$ between f_{y1} and f_{y2} has the maximum effect on the sum of all second-order sensitivity indices. On the contrary, for high slenderness, the load-carrying capacity approaches the buckling load, and therefore it is more sensitive to changes of Young's modulus values S_5 (E_1) and S_{18} (E_2) and also to changes of flange thickness values S_4 (t_{f1}) and S_{17} (t_{f2}). The graphs of sensitivity indices S_4 (t_{f1}) and S_{17} (t_{f2}) are the same for both frames, the graphs of S_5 and S_{18} are also approximately the same. We can further conclude that for $\bar{\lambda} = 0$ and $\bar{\lambda} = \infty$, results

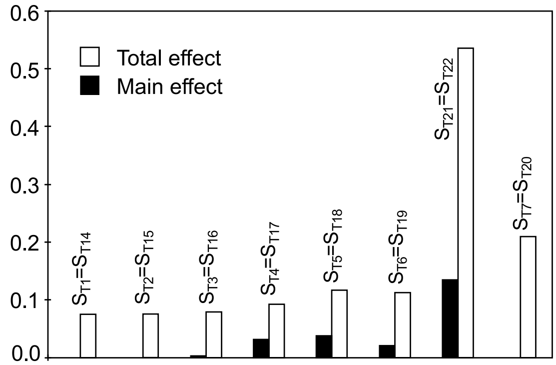

Fig. 9. Main and total indices of Frame 2 for $\overline{\lambda} = 0.93$.

of the sensitivity analyses of both frames are the same. On the other hand, large differences of sensitivity indices were obtained for $\overline{\lambda} \approx 1.0$ for imperfections $\Theta_1, \Theta_2, \delta_1, \delta_2$.

5.1.2. Frame 1

Maximum values of the first-order sensitivity indices $S_{21} = S_{22} = 0.163$ of imperfections Θ_1 and Θ_2 were calculated for $\overline{\lambda} = 1.02$; see Fig. 6. Maximum values of the second-order sensitivity index $S_{21,22} = 0.33$ describing interactions between Θ_1 and Θ_2 were calculated for $\overline{\lambda} = 0.96$. The second-order sensitivity index $S_{21,22}$ is twice greater than the first-order sensitivity indices $S_{21,22} \approx 2 \cdot S_{21} = 2 \cdot S_{22}$; see Fig. 7.

Sensitivity indices S_7, S_{20} and $S_{7,20}$ of imperfections δ_1 and δ_2, in comparison to other imperfections, depicted in Fig. 7, are very small. The maximum values of sensitivity indices S_7, S_{20} and $S_{7,20}$ were obtained for $\overline{\lambda} = 0.75$.

5.1.3. Frame 2

In comparison with results depicted in Fig. 7, the value of second-order sensitivity index $S_{21,22}$ in Fig. 8 is significantly lower. It is apparent from Fig. 8 that interaction effects between Θ_1 and Θ_2 are approximately 50% higher than the main effect of Θ_1 and Θ_2; see Fig. 8. In contrast to Frame 1, in Frame 2, interactions related to variables δ_1, δ_2 are relatively significant. The minimum value of indicator $1 - \sum_i S_i = 0.46$ was calculated for $\overline{\lambda} = 0.93$; this indicates significant higher order interactions.

The analysis of the total effect of input imperfections according to formula (4) for $\overline{\lambda} = 0.93$ is depicted in Fig. 9. It is apparent that the higher order interactions of imperfections Θ_1, Θ_2 are most significant. Variables δ_1, δ_2 are worth noting. Their main effect is practically zero, nevertheless they interact with the other variables, similarly as variables h_1, h_2, b_1, b_2; see Fig. 9.

5.2. Statistical analysis results

Capability for stochastic uncertainty analysis is one of the major features of global sensitivity analysis methods. The aim of stochastic uncertainty analysis is to determine the random load-carrying capacity with regard to the variability of all input random imperfections.

According to the rules in the Eurocodes, the design of structures and structural members has to be verified for different limit states [27]. According to EN1990, we can assume statistical independence between load (action) and load-carrying capacity (resistance). Statistical independence of both variables makes possible the separation of their analyses, and enables individual study of them. Pertinent to the ultimate limit state, the design load-carrying capacity (design resistance) can be verified by means of statistical analysis according to standard EN 1990 for target reliability index $\beta_d = 3.8$. The design load-carrying capacity for $\beta_d = 3.8$ is, in practice, obtained as 0.1 percentile [1,2,5]. Various

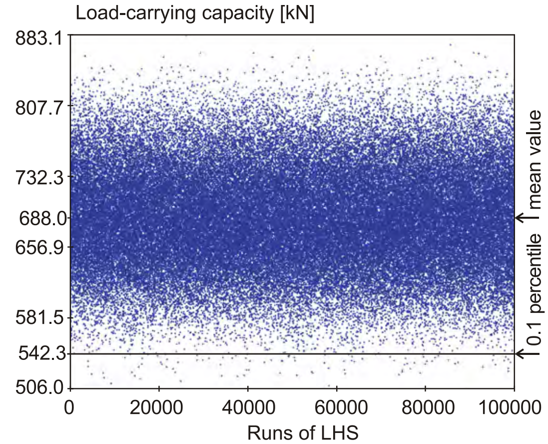

Fig. 10. Runs of LHS of Frame 1 for $\overline{\lambda} = 1.0$.

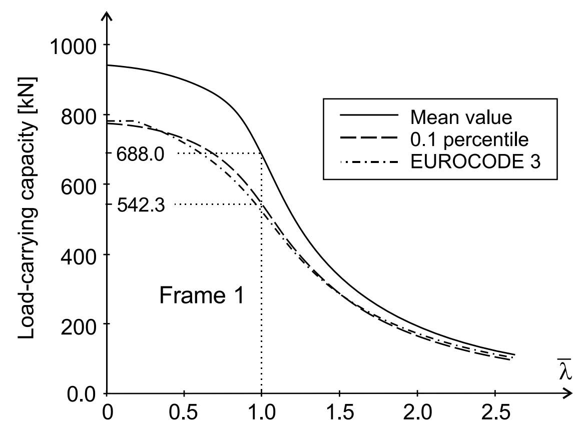

Fig. 11. Mean and design values of Frame 1.

distribution functions for deriving the design values are expressed in Table C3 in EN 1990. For the statistical analysis, we can reliably use only those distribution functions which are not rejected by distribution tests (e.g., Chi-square test).

A more general approach which does not require the evaluation of distribution tests is described in [2,5]. This approach enables the evaluation of the 0.1 percentile directly from the basic probability definition. The numerical studies were evaluated applying the LHS method. Frame 1 with $\overline{\lambda} = 1.0$ was selected for the illustration of the evaluation of the 0.1 percentile. 100 000 runs of the random load-carrying capacity are depicted in Fig. 10. 100 random realizations of the load-carrying capacity have values lower than 542.3 kN; see Fig. 10. In practice, the value 542.3 kN is obtained as the 100th lowest value in the organized ascending file.

In practice, structures are commonly designed according to EUROCODE 3 using the buckling resistance based on buckling length. Design values evaluated according to EN1990 (0.1 percentile) and EUROCODE 3 (buckling resistance) are compared; see Figs. 11 and 12. The step for parameter $\overline{\lambda}$ was chosen as 0.1. 100 000 runs of LHS were used in each step. It is apparent that the design load-carrying capacity evaluated according to EUROCODE 3 and EN1990 closely corresponds with the greatest difference of 7% obtained for $\overline{\lambda} \approx 0.8$; see Fig. 12. This difference may be, to a certain extent, due to the effect of the residual stress which was neglected in the statistical and sensitivity analyses. Residual stress has the dominant influence on load-carrying capacity of the strut when $\overline{\lambda} \approx 0.7$ [28].

The mean value decreases with increasing $\overline{\lambda}$, the decreasing trend is observed also for the 0.1 percentile; see Figs. 11 and 12. The difference between the mean and 0.1 percentile values is influenced, to a certain degree, by the plot of standard deviation

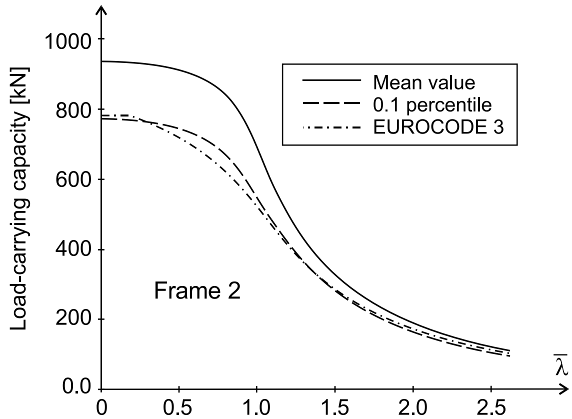

Fig. 12. Mean and design values of Frame 2.

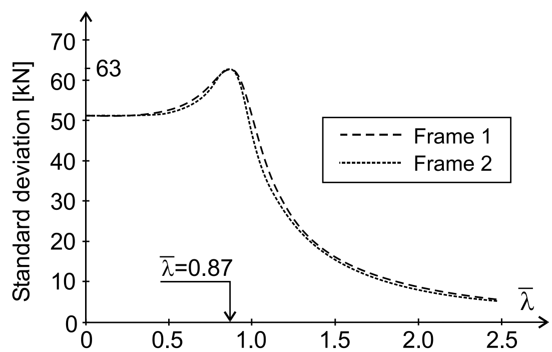

Fig. 13. Comparison of standard deviations of load-carrying capacity.

of the load-carrying capacity. The plots of standard deviations of both frames depicted in Fig. 13 are approximately the same. The maximum standard deviation of 63 kN occurs for $\overline{\lambda} \approx 0.87$.

6. Conclusion

The sensitivity analyses of Frames 1 and 2 provide sensitivity information concerning the load-carrying capacity as influenced by initial imperfections. The paper was aimed at the comparison of the influence of individual initial imperfections on the load-carrying capacity of Frames 1 and 2.

Results of the sensitivity analyses of the load-carrying capacities of both frames show that the influence of initial bow imperfections δ_1, δ_2, compared to the influence of the initial sway imperfections Θ_1, Θ_2, is very small; see Figs. 7 and 8. This conclusion is valid for both the main effects as well as the second-order interaction effects. As the non-dimensional slenderness approaches one, the second-order interaction effects between Θ_1 and Θ_2 become significant and the main effect is of secondary importance. Imperfections Θ_1, Θ_2 may generate, as a result of their mutual interactions, extreme values of the load-carrying capacity. This is important for the analysis of reliability and economy of structural design. Variability of imperfections Θ_1, Θ_2 significantly contributes to the output variability, and thus, additional research may be recommended in order to strengthen their knowledge base. However, under heavy service conditions, this is difficult or practically impossible.

Higher order interaction effects were obtained for Frame 2 for $\overline{\lambda} = 0.93$. Mainly imperfections Θ_1, Θ_2 are involved in interactions with other variables; see Fig. 9. The total effect index S_{Ti} is a summarized sensitivity measure which includes the interaction effects of any order. Imperfections that interact with other imperfections with main effect close to zero are worth noticing. Change in such imperfection does not cause any significant change of the load-carrying capacity, if not accompanied by additional changes of one or more significantly interacting imperfections. For example, the main effects of imperfections δ_1, δ_2 of Frame 2 are practically equal to zero but the total effect indices S_{T7} and S_{T20} are the second most significant ones; see Fig. 9.

Let us note that the total effect index is derived from a notion of Sobol' which involved the problem of "freezing" the unimportant factors to their midpoint [8]. Sensitivity indices $S_7 = S_{20} \approx 0$ represent a necessary but insufficient condition for fixing imperfections δ_1, δ_2 of Frame 2. Results depicted in Fig. 9 show that all column imperfections have total effect indices greater than zero, and thus, they cannot be fixed at any value within its range of uncertainty without greater or lesser effect on the value of the variance of the load-carrying capacity.

For $\overline{\lambda} = 0$, the first-order sensitivity indices of yield strength $S_6 = S_{19} = 0.32$ are cardinal; see Figs. 7 and 8. The interaction effect of the second order $S_{6,19} = 0.08$ also exists between the yield strengths of the left and the right columns. The sensitivity indices of flange thickness $S_4 = S_{17} = 0.06$ are the third most important ones among all. Results of the sensitivity and statistical analyses for $\overline{\lambda} = 0$ are the same for both frames and are practically valid for columns under tension.

Young's modulus $S_5 = S_{18} = 0.31$ and flange thickness $S_4 = S_{17} = 0.15$ are the dominant variables for $\overline{\lambda} \to \infty$. Higher order interactions are relatively small. For high slenderness, the load-carrying capacity in limit approaches the Euler's critical force (buckling load), and is thus sensitive to variables preventing buckling. From the point of view of production technology of hot-rolled steel members, we can strive for decrease in the variability of flange thickness, however, the variability of Young's modulus cannot be significantly influenced in practice.

Results of sensitivity analysis of both frames differ, for $\overline{\lambda} \approx 1.0$, most significantly; see Figs. 7 and 8. Sensitivity indices pertinent to flange thickness S_4, S_{17} are small for all analysed slenderness. This may even be the second most significant for high slenderness.

Let us compare the hereby presented results with the results of sensitivity analyses of the strut published in [13]. In the case of the strut, the load-carrying capacity is not significantly influenced by the higher order interactions between initial imperfections.

The evident discrepancies between the mean and design load-carrying capacities are depicted in Figs. 11 and 12. Discrepancies between the design load-carrying capacities evaluated according to EUROCODE 3 and EN1990 (0.1 percentile) are relatively small. The 0.1 percentile yields greater values within the interval $\lambda \in (0.5, 1.0)$; this may be due to the fact that residual stresses were neglected. The 0.1 percentile plots of Frames 1 and 2 differ slightly, however, we cannot conclude that discrepancies for other frame types may not be greater. The increase in the values obtained from the 0.1 percentile can be achieved by decreasing the standard deviation of input imperfections. The influences of individual variables (and their interactions) were quantified applying the tools of sensitivity analysis.

The sensitivity analysis was used to determine where additional information on imperfections (obtained perhaps from measurement) would be most beneficial in terms of uncertainty reduction in probabilistic model results of the frames. By decreasing the standard deviation of the dominant input imperfections, we can significantly increase the reliability of newly designed structures. In practice, sensitivity analysis provides a basis utilizable in production and in the operating of control activities which can thereby be concentrated on the most important input variables with the greatest effect on the load-carrying capacity.

The Sobol' sensitivity is generally suitable for the analysis of majority of stability problems of steel structures with imperfections.

Sampling based methods may be applied for the analysis of the effect of local plated and global bar geometrical imperfections on the ultimate limit state of thin-walled structures [29]. Some researchers [30] introduced multiple local modes into the numerical model to consider possible additional interactions among the global mode and multiple local modes [31]. The Sobol' sensitivity analysis can quantify the interaction effect amongst imperfections formally identical to the buckling modes which cause instability. The solution should be based on measurements of real imperfections; see, e.g., [32].

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