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Sensitivity assessment of steel members under compression

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ABSTRACT

The objective of the paper is to analyse the influence of initial imperfections on the behaviour of a steel member under compression. The influence of the variability of initial imperfections on the variability of the load-carrying capacity studied has been calculated by sensitivity analysis. The advantages of Sobol's sensitivity analysis and the most important properties of Sobol's sensitivity indices are described. The Sobol's first order sensitivity indices are evaluated in dependence on the nondimensional slenderness. The Sobol's sensitivity indices are supplemented with a lucid elaboration based on the Monte Carlo method. Material and geometrical characteristics of a steel member IPE 220 were considered to be random quantities the histograms of which were obtained from experiments. Imperfections that have a dominant influence on the load-carrying capacity are identified.

1. Introduction

The properties of structures are influenced by a number of factors which are of random character (material, geometry, effects of the surrounding environment, load action, etc.). If a structure is to reliably fulfil its function during its service life, it is necessary to make provision for this during its design. In the general classification of initial structural imperfections, three fundamental categories of imperfection are considered [8]. They include:

1. Geometrical imperfections: initial curvature of member axis, excentricity of load action, deviation from the theoretical layout of the cross section (tolerance of dimensions and shape of the cross section), etc.
2. Material imperfections: dispersion of the mechanical properties of the material (non-homogeneousness of material characterised by the dispersion of the yield strength, ultimate strength, Young's modulus, etc.), initial stress state (residual stress as a consequence of rolling, welding, straightening and other technological manufacturing processes).
3. Structural imperfections: imperfections in the realization of joints, connections, welds, anchorage and other structural details which are apparent in comparison with the theoretical assumptions introduced in the solution of idealized system, in deviations of the effects of the actual structural system.

Most initial imperfections arise due to inaccuracy during manufacturing process. The influence of uncertainties of input imperfections on the uncertainty of system response expressed by mathematical models is studied using sensitivity and uncertainty analyses [15]. The general definition of sensitivity analysis is the study of the uncertainty of model output (numerical or otherwise) arising from varying sources of model input uncertainties [16]. The uncertainty analysis is aimed rather at the quantification of the uncertainty of model output [2,13]. Sensitivity and uncertainty analyses are employed as formal methods for the evaluation of data and models, because they both enable the evaluation of uncertainty of output variables and provide information on the importance of input variables and their influence on the monitored output [15,16].

Sensitivity and uncertainty analysis methods are divided into two types: (i) deterministic, and (ii) stochastic. Analysis (i) is a relatively known method currently used in structure design. This analysis accompanies the design procedures for which a calculation model is applied, e.g. in [10,22]. We usually speak about a parametric study (sometimes called "what-if-study"). When designing a structure, a parameter (e.g. cross section, steel grade, weld parameters) gets changed, and the influence on input (e.g. load-carrying capacity, deformations) is studied. However, quantified data on the uncertainty and/or sensitivity are not obtained. In the case (ii), we can compute the average output, its standard deviation, the quantiles of its distribution, confidence bounds, plot the distribution itself and so on [16]. Upon obtaining results of this uncertainty analysis, we can then perform the sensitivity analysis to determine which of the input parameters have a more dominant influence on the uncertainty in the model output [16]. For discussions on the existing techniques see, for example, [4,12]. For application of the fuzzy sets theory in models with prevailing epistemic uncertainty, see [5,11].

This article is aimed at stochastic sensitivity analysis. The stochastic sensitivity analysis will be carried out with the aim

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of assessing the relative sensitivity of the random variability of a monitored event to the random variability of individual input variables. The random variability influence of a certain input variable (in comparison with other variables) on the random variability of a monitored output (e.g. load-carrying capacity, failure probability) will be sought.

The sensitivity analysis, thus, also answers the question as to which variables are dominant and should be considered carefully during: (i) the preparation of input variables; (ii) determination and decision making on the improvement of technological processes; (iii) conception and organization of control activities [4].

Ilya M. Sobol' [19–21], a Russian mathematician, elaborated one of the most coherent sensitivity analyses. In the paper presented, the Sobol's sensitivity analysis will be applied for study of the influence of input random quantities on the load-carrying capacity of a steel grade S235 member. The sensitivity analysis will be carried out using statistical material and geometrical characteristics obtained by physical experiment research [9,17].

2. Variance–based sensitivity indices

Let us consider a computational model with input variables $(X_1, X_2, \ldots, X_M)$ of non-zero variance (or uncertainty), and let us monitor the influence of these variables on the output variable Y utilizing the response function f:

$$Y(X) = f(X_1, X_2, \ldots, X_i, \ldots, X_M). \tag{1}$$

We study the case with statistically fully uncorrelated input random variables. Let us consider the response function $f(1)$, the integral of which can be performed on its function set Ω^M:

$$\Omega^M = (X | 0 \le x_i \le 1;\ i = 1; \ldots, M). \tag{2}$$

Sobol's concept is based on the hierarchical decomposition of the response function (1) into a form with increasing dimension:

$$f = f_0 + \sum_i f_i + \sum_i \sum_{j>i} f_{ij} + \cdots + f_{12\ldots M} \tag{3}$$

where each member is a function of input variables of given indices: $f_i = f_i(X_i)$, $f_{ij} = f_{ij}(X_i, X_j)$, etc. Each member of the decomposition of function f should also have an integral on its functional subset. The decomposition (3) is not a series expansion, because it has 2^M (finite number) of members: f_0 is a constant, number of members f_i is M, number of f_{ij} members is $\binom{M}{2}$ etc. Each input variable has a density function $p_i(X_i) \ge 0$ defined on interval $\langle 0;\ 1\rangle$, $p_i(X_i) = 0$ outside this interval. As a result of the Fubini theorem on the double integral, it holds that if each member of the decomposition apart from constant has a zero mean value:

$$E(f(x_i)) = \int_0^1 p_i(x_i) \cdot f(x_i)\, dx_i = 0 \tag{4}$$

then all members of decomposition are orthogonal in pairs:

$$E\left(f(x_i) \cdot f(x_j)\right) = \int_0^1 \int_0^1 p_i(x_i) \cdot p_j(x_j) \cdot f(x_i) \cdot f(x_j)\, dx_i dx_j = 0, \quad i \ne j \tag{5}$$

Members of the decomposition (3) can thus be rewritten utilizing the conditional realization of the response function:

$$f_0 = E(Y) \tag{6a}$$

$$f_i = E(Y|X_i) - E(Y) \tag{6b}$$

$$f_{ij} = E(Y|X_i X_j) - f_i - f_j - E(Y). \tag{6c}$$

The condition that the density $p_i(X_i)$ is zero outside interval $\langle 0;\ 1\rangle$ is not limiting, because transformation of each input variable into another suitable distribution type (Gaussian, lognormal, histogram, etc.) can be performed utilizing the response function (1). For this purpose, it is practical to consider the density function $p_i(X_i)$ within the interval $\langle 0;\ 1\rangle$ in the simplest possible form with Rectangular distribution.

How can the decomposition of the response function (3) be utilized for sensitivity analysis? Sensitivity analysis can be carried out by substituting deterministic values into (3) and subsequently by comparing individual members of the decomposition $f_0, f_i, f_{ij}, \ldots, f_{12\ldots M}$ with the value of output f. Let us recall that decomposition (3) is not worked out based on the analysis of the response function (1), but by the analysis of change of output Y arising from quantified changes of input variables given by functions $p_i(X_i)$. Change of function $p_i(X_i)$ during decomposition leads to change of member f_0 and of all members with index i, i.e. f_i, f_{ij} etc. In practice, this means that the decomposition of the response function (1) into (3) can also be carried out with an unknown algorithm, for which only input and output are known (black box), whereas the sensitivity analysis can be carried out by quantifying the influence of change in input variables to change of output variables.

The change of output variable Y is characterised by standard deviation $\sigma_Y$or variance $V(Y) = \sigma_Y^2$. Since (4) and (5) are valid, all members of decomposition (3) are statistically independent random variables and we can write that the variance is equal to the sum of variances of the individual members of decomposition:

$$V(Y) = \sum_i V(f_i(X_i)) + \sum_i \sum_{j>i} V(f_{ij}(X_i, X_j)) + \cdots + V(f_{12\ldots M}(X_1, X_2, \ldots, X_M)) \tag{7}$$

where $V(f_i(X_i)) = V(E(Y|X_i))$ etc. If we express the ratio of individual members of the decomposition (7) to the total variance, Sobol's first order sensitivity indices may be written in the form:

$$S_i = \frac{V(E(Y|X_i))}{V(Y)}. \tag{8a}$$

For users, the significance of S_i is as follows: If the variability X_i is successfully eliminated, the output quantity variance will decrease by $S_i \cdot 100\%$ minimum. In [21], Sobol proposed an alternate definition $S_i = \mathrm{corr}(Y,\ E(Y|X_i))$ based on the evaluation of the correlation between output random variable Y and the conditional random arithmetical mean $E(Y|X_i)$. Analogously as (8a), we can write the second order sensitivity indices:

$$S_{ij} = \frac{V\left(E\left(Y|X_i, X_j\right)\right)}{V(Y)} - S_i - S_j. \tag{8b}$$

Other Sobol's sensitivity indices enabling the quantification of higher order interactions can be expressed similarly. With regard to (3), the decomposition of Sobol's sensitivity indices can be written in the form:

$$\sum_i S_i + \sum_i \sum_{j>i} S_{ij} + \sum_i \sum_{j>i} \sum_{k>j} S_{ijk} + \cdots + S_{123\ldots M} = 1. \tag{9}$$

3. Sensitivity analysis of load-carrying capacity

Let us now apply the Sobol's decomposition to the analysis of the load-carrying capacity of a steel strut of profile IPE220, see Fig. 1.

Let initial deflection of the column be assumed to be half sine wave with the amplitude e_0, as shown in Fig. 1.

$$y_0 = e_0 \sin\left(\frac{\pi \cdot x}{L}\right). \tag{10}$$

The resulting shape of the strut under load action F (see Fig. 1) is given by the differential equation

$$\frac{d^2 y}{dx^2} + \frac{F(y + y_0)}{E \cdot I_z} = 0 \tag{11}$$

$$R = -\frac{\sqrt{A^2 \cdot Q^2 + 2 \cdot A \cdot F_{cr} \cdot W_z \left(|e_0| \cdot F_{cr} - f_y \cdot W_z\right) + F_{cr}^2 \cdot W_z^2} - A \cdot Q - F_{cr} \cdot W_z}{2 \cdot W_z}$$

Box I.

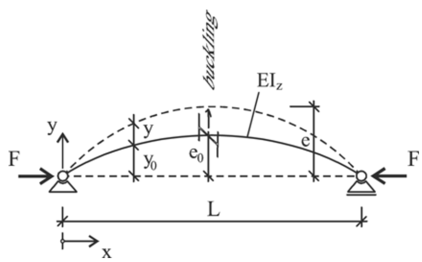

Fig. 1. Buckling of the member with IPE cross-section.

where I_z is the second moment of area to axis Z (axis perpendicular to flange around which the section bends during buckling). After substituting for y_0 from (10), and considering the boundary conditions ($x = 0$; $y + y_0 = 0$ and $x = L$; $y + y_0 = 0$), the solution is obtained:

$$y = \frac{e_0}{\frac{F}{F_{cr}} - 1} \cdot \sin\left(\frac{\pi \cdot x}{L}\right) \quad (12)$$

where F_{cr} is Euler's critical load. The column deflection at mid-length $x = L/2$ [23] is:

$$e = e_0 + \frac{e_0}{\frac{F}{F_{cr}} - 1} = \frac{e_0}{1 - \frac{F}{F_{cr}}}. \quad (13)$$

The maximum stress $\sigma_{\max}$ due to the combination of the axial uniform stress and the bending stress is:

$$\sigma_{\max} = \frac{R}{A} + \frac{R \cdot |e|}{W_z} = f_y. \quad (14)$$

Load-carrying capacity R is maximum load action F of elastic member ($\sigma_{\max}$ is equal to yield strength f_y). R can be evaluated from (14) with used (13):

$$\sigma_{\max} = \frac{R}{A} + \frac{R \cdot |e_0|}{(1 - R/F_{cr}) \cdot W_z} = f_y \Rightarrow R \quad (15)$$

where A is the sectional area, W_z is the sectional module to axis Z. The model uncertainty factor as proposed in [27] has been neglected. By the solution (15), we can obtain the load-carrying capacity R in the form as given in Box I:

$$Q = |e_0| \cdot F_{cr} + f_y \cdot W_z \quad (16a)$$

$$A = 2 \cdot t_2 \cdot b + (h - 2t_2) \cdot t_1 \quad (16b)$$

$$F_{cr} = \pi^2 EI_z / \left(L^2\right) \quad (16c)$$

$$I_z = \left(2 \cdot t_2 \cdot b^3 + (h - 2t_2) \cdot t_1^3\right) / 12 \quad (16d)$$

$$W_z = I_z / (b/2). \quad (16e)$$

E is the Young's modulus, L is the strut length, h is the sectional height, b is the sectional width, t_1 is the web thickness and t_2 is the flange thickness. Input random variables are listed in Table 1. The input variables are fully uncorrelated.

Fully uncorrelated input variables represent one of preconditions for the application of Sobol's sensitivity analysis. However, initial imperfections may generally be considered as random fields, e.g. in [5,14]. How can we proceed when it is necessary to use the correlated input factors? The useful strategy to circumvent the use of correlated samples in sensitivity analysis can be illustrated by an example in [16]. Instead of entering X_1 and X_2 as correlated factors one can enter X_1 and X_3, where X_3 represents a factor describing noise, and model X_2 is a function of X_1 and X_3. It is necessary to note that dependence and correlation are not synonymous. A correlation implies dependence, while the opposite is not true. Dependencies described via correlations are useful for practical numerical computations [16].

Statistical characteristics h, b, t_1, t_2, f_y were considered as histograms according to results of physical experimental research [9]. Statistical characteristics of Young's modulus E were considered according to [1,18]. The standard deviation of the Gaussian distribution of density function of the amplitude of initial curvature e_0 has been considered based on the assumption that 95% of the realizations of this random variable are found within the tolerance limit $\pm 0.15\%\, L$ of standard [26], where the strut length L is a computational parameter.

The dependence of Sobol's sensitivity indices S_i of the first order on dimensionless slenderness $\bar{\lambda} = L/(i_z \cdot 93.9)$ [25] is depicted in Fig. 2, where $i_z = 24.8$ mm is the nominal value of the radius of gyration of profile IPE220. The sensitivity indices S_i were evaluated from the basic definition (8a) utilizing the Monte Carlo method. The conditional random arithmetical mean $E(Y|X_i)$ was evaluated for $N = 100\,000$ simulation runs; the variance $V(E(Y|X_i))$ was calculated for $N = 100\,000$ simulation runs, as well, i.e. the numerically demanding difficulty of the calculation is N^2. In practice, the procedure is such that indices S_i were evaluated for strut length L, which was parametrically increased from zero with the step $\Delta L = 0.094$ m.

All variables in Table 1 apart from the amplitude of initial curvature e_0 with standard deviation σ_{e0}, which was determined according to the tolerance standards [26], were determined experimentally. The dependence of Sobol's sensitivity indices S_i of the first order on the standard deviation σ_{e0} of amplitude e_0 of strut length $L = 2.35$ m is depicted in Fig. 3. The grey background emphasises the interval ($\pm 0.1\%\, L$; $\pm 0.2\%\, L$), where we can expect the standard deviation that would be obtained, with

Table 1
Input random quantities

	Symbol	Density	Mean value	Standard deviation
1.	h	Histogram	220.22 mm	0.9746 mm
2.	b	Histogram	111.48 mm	1.0930 mm
3.	t_1	Histogram	6.2245 mm	0.2467 mm
4.	t_2	Histogram	9.1356 mm	0.4214 mm
5.	f_y	Histogram	297.3 MPa	16.8 MPa
6.	E	Gauss	210 GPa	12.6 GPa
7.	e_0	Gauss	0	0.76533 L

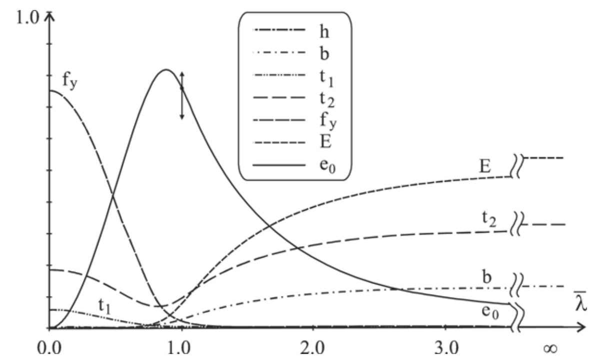

Fig. 2. Sobol's indices S_i versus $\bar{\lambda}$.

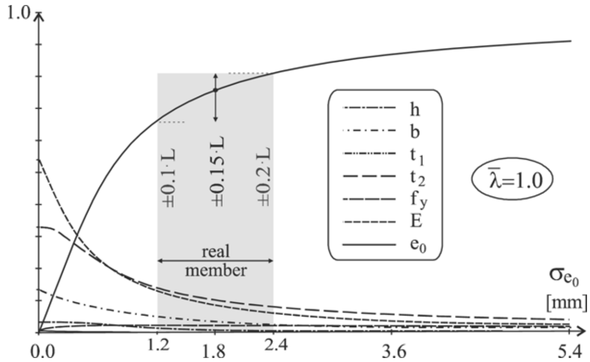

Fig. 3. Sobol's indices S_i versus σ_{e0}.

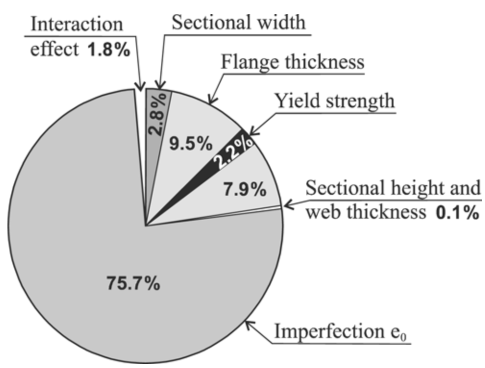

Fig. 4. Sobol's first order sensitivity indices for $\overline{\lambda} = 1.0$.

the highest probability, from measurements on real struts. The inequality $0.98 < \sum_i S_i < 1$ indicates the presence of weak higher interactions for all analysed slenderness.

Sensitivity indices S_i for slenderness $\overline{\lambda} = 1.0$ are depicted in Fig. 4. The sum of all coefficients S_i is $100\% \cdot \sum_i S_i \approx 98.2\%$, i.e. 1.8% accounts for higher order interactions S_{ij}, S_{ijk} etc.

4. Conclusion

It is evident from results depicted in Fig. 2 that for struts of slenderness $\overline{\lambda} \leq 0.47$, the dominant variable is the yield strength f_y. If the strut slenderness is equal to zero, it presents the case of simple compression where the load-carrying capacity $R = f_y \cdot A$ is dependent only on the values of the yield strength and of those of sectional area. For $\overline{\lambda} = 0$, the sensitivity coefficient of yield strength is $S_{fy} = 0.75$, and since the interactions of higher orders are very small the sensitivity index of sectional area may be considered approximately equal to $S_A \approx 1 - 0.75 = 0.25$, where the flange thickness $S_{t2} = 0.185$ and the web thickness $S_{t1} = 0.055$ are of dominant influence, see Fig. 2.

Initial strut axial curvature (represented by the amplitude e_0) is the dominant variable for slenderness $\overline{\lambda} = \langle 0.47; 1.67 \rangle$. The maximum sensitivity index $\max S_{e_0} = 0.82$ arises for $\overline{\lambda} = 0.88$. Young's modulus E is the dominant variable for slenderness $\overline{\lambda} > 1.67$. Another important variable among the geometric characteristics is the flange thickness t_2. The load-carrying capacity is equal to the Euler critical force in the limit case $\overline{\lambda} = \infty$ (or $e_0 = 0$), i.e. it is dependent only on variables E, t_2, b, which represent the input variables for the evaluation of stiffness preventing buckling EI_z. Values of sensitivity indices of variables E, t_2, b increase most rapidly for $\overline{\lambda} \approx 1.0$, whereas the sensitivity index of the amplitude e_0 decreases most rapidly. The sensitivity index of yield strength decreases practically to zero for $\overline{\lambda} \approx 1.2$. For $\overline{\lambda} \approx 1.3$, the sensitivity index of e_0 is equal to 0.5, and thus the sum of sensitivity indices of variables E, t_2, b is approximately equal to 0.5. In the case of higher slenderness, variables E, t_2, b have a dominant influence on the load-carrying capacity.

It is important to note that the influence of residual stresses was neglected during evaluation. From the differentiation of curves of normative buckling resistance a, b, c, d of standard EC3 [25], the curves differ most for $\overline{\lambda} = 0.83$, if the influence of residual stresses is taken into consideration. A more elaborate description of the influence of this structural imperfection would require utilization of thin-walled finite elements to model the strut, and the evaluation of the load-carrying capacity applying the geometric non-linear solution; however, it is practically impossible in connection with the numerically demanding evaluation of Sobol's sensitivity indices.

This problem can be solved by applying numerical methods such as the "response surface" where the approximating function should be sufficiently detailed and number of approximation points sufficiently high to enable the description of higher order interactions. The influence of residual stresses can be taken into account in (15) by increasing the absolute value of geometric imperfection of amplitude e_0. This assumption, however, may not be sufficiently apposite for an elaborate sensitivity analysis utilizing Sobol's sensitivity analysis as pointed out by the sensitivity analysis of the load-carrying capacity worked out according to (9), where the Spearman correlation coefficients of residual stress and of amplitude e_0 corresponded only approximately.

Input random imperfections may be divided into two basic groups [4]. The first group includes those variables the statistical characteristics of which can be positively influenced in production (yield strength, geometric characteristics, residual stress) and those that are not sufficiently sensitive to changes in production technology (e.g. variability of Young's modulus E). The first group of variables may be further divided into two subgroups: (i) variables for which mean value and standard deviation can be changed by improvement of production quality [4]. Examples include Young's modulus; (ii) variables, the mean value of which cannot be significantly changed, because it should approximately correspond to the nominal value (geometric characteristics of profile dimensions).

Significant variables in this regard include yield strength, initial axial strut curvature and flange thickness t_2. Decrease in the variability of these variables can be achieved by change in production technology. Decrease of the variability of yield strength f_y is recommended especially for struts with lower dimensionless slenderness.

The sensitivity analysis enables us to identify significant processes and phenomena which influence the reliability of load-carrying structures during service life, and therefore it can be applied to the development of knowledge of real behaviour and limit states [3,24]. At present, the Sobol's sensitivity analysis is one of the most carefully formulated and most coherent concepts which can be applied to the analysis of the majority of stability problems [6,7,14]. It is necessary to try and develop experimental methods aimed at the objectivization of knowledge of the real structure behaviour, and at the verification of theoretical models in relation to definitions of limit states.

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