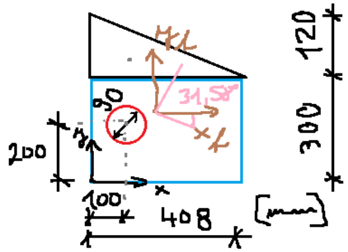
ZÁKLADY STAVEBNÍ MECHANIKY

BDA001

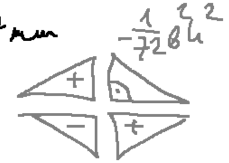
Hlavní kvadratické momenty –početní a grafické řešení. Poloměry setrvačnosti, elipsa setrvačnosti.

Zdeněk Kala



$$x_1 = \frac{S_y}{A} = \frac{408\cdot 300\cdot 204 + 120\cdot 204\cdot 136 - \pi\cdot 45^2\cdot 100}{408\cdot 300 + 120\cdot 204 - \pi\cdot 45^2} = 196{,}862\,mm$$

$$y_1 = \frac{S_x}{A} = \frac{408\cdot 300\cdot 150 + 120\cdot 204\cdot 340 - \pi\cdot 45^2\cdot 200}{-\|-} = 180{,}837\,mm$$

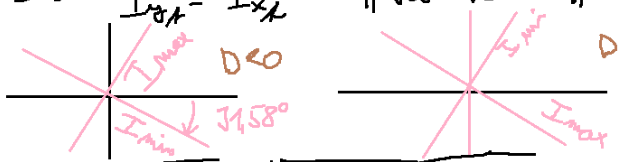


$$I_{x_1} = \frac{1}{12}\cdot 408\cdot 300^3 + 408\cdot 300\cdot(150-180{,}837)^2 + \frac{1}{36}\cdot 408\cdot 120^3 + 120\cdot 204\cdot(340-180{,}837)^2 - \left[\frac{\pi\cdot 45^4}{4} + \pi\cdot 45^2\cdot(200-180{,}837)^2\right] = 1{,}669\cdot 10^9\,mm^4$$

$$I_{y_1} = \frac{1}{12}\,300\cdot 408^3 + 408\cdot 300\cdot(204-196{,}862)^2 + \frac{1}{36}\cdot 120\cdot 408^3 + 120\cdot 204\cdot(136-196{,}862)^2 - \left[\frac{\pi\cdot 45^4}{4} + \pi\cdot 45^2\cdot(100-196{,}862)^2\right] = 1{,}958\cdot 10^9\,mm^4$$

$$D_{x_1y_1} = 0 + 408\cdot 300\cdot(204-196{,}862)\cdot(150-180{,}837) + \frac{-1}{72}\,408^2\cdot 120^2 + 120\cdot 204\cdot(136-196{,}862)\cdot(340-180{,}837) - \left[0 + \pi\cdot 45^2\cdot(200-180{,}837)\cdot(100-196{,}862)\right] = -285{,}564\cdot 10^6\,mm^4$$

$$\mathrm{tg}\,2\alpha_0 = \frac{2\cdot D_{x_1y_1}}{I_{y_1} - I_{x_1}} = \frac{2\cdot(-285{,}564)\cdot 10^6}{1{,}958\cdot 10^9 - 1{,}669\cdot 10^9} = -1{,}976 \Rightarrow 2\cdot\alpha_0 = -63{,}16^\circ \Rightarrow \alpha_0 = -31{,}58$$

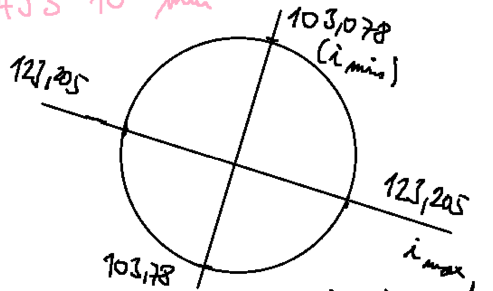


$$I_{max,min} = \frac{I_{x_1}+I_{y_1}}{2} \pm \sqrt{\left(\frac{I_{x_1}-I_{y_1}}{2}\right)^2 + D_{x_1y_1}^2} = \frac{(1{,}669+1{,}958)\cdot 10^9}{2} \pm \sqrt{\left(\frac{1{,}669\cdot 10^9 - 1{,}958\cdot 10^9}{2}\right)^2 + (-285{,}564\cdot 10^6)^2} = \begin{cases} 2{,}133\cdot 10^9\,mm^4 \\ 1{,}493\cdot 10^9\,mm^4 \end{cases}$$

$$i_{y_1} = \sqrt{\frac{I_{y_1}}{A}} = \sqrt{\frac{1{,}958\cdot 10^9\,mm^4}{140{,}518\cdot 10^3\,mm^2}} = 118{,}043\,mm$$

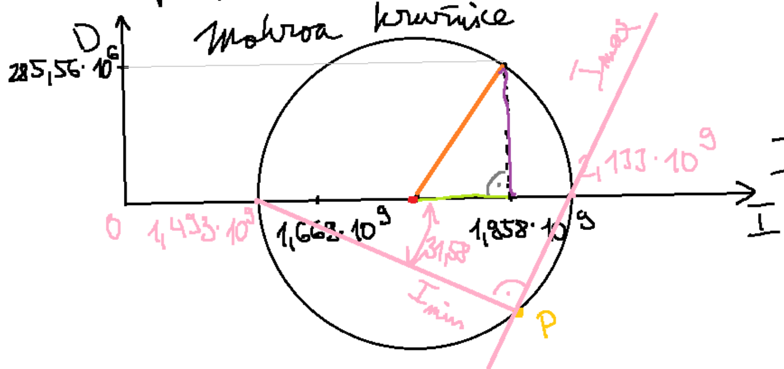
$$i_{max} = \sqrt{\frac{I_{max}}{A}} = \sqrt{\frac{2{,}133\cdot 10^9}{140{,}518\cdot 10^3}} = 123{,}205\,mm$$

$$i_{min} = \sqrt{\frac{I_{min}}{A}} = \sqrt{\frac{1{,}493\cdot 10^9}{-\|-}} = 103{,}078\,mm$$

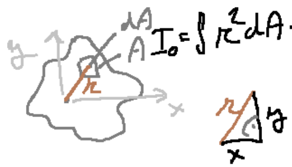


Elipsa setrvačnosti

Mohrova kružnice



$$I_p = I_0 = I_{x_1} + I_{y_1} = I_{max} + I_{min} = 3{,}62\cdot 10^9$$



$$I_0 = \int_A r^2\,dA = \int_A (x^2+y^2)\,dA = \int_A x^2\,dA + \int_A y^2\,dA = I_y + I_x$$