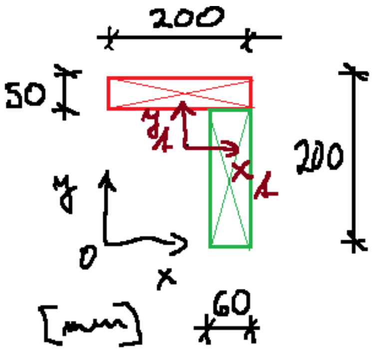
ZÁKLADY STAVEBNÍ MECHANIKY

BDA001

Těžiště, kvadratické a deviační momenty rovinných složených obrazců, aplikace Steinerovy věty.

Zdeněk Kala



$$x_1 = \frac{S_y}{A} = \frac{200\cdot 50\cdot 100 + 150\cdot 60\cdot 170}{200\cdot 50 + 150\cdot 60} = 133{,}158\ mm$$

$$y_1 = \frac{S_x}{A} = \frac{200\cdot 50\cdot 175 + 150\cdot 60\cdot 75}{200\cdot 50 + 150\cdot 60} = 125{,}263\ mm$$

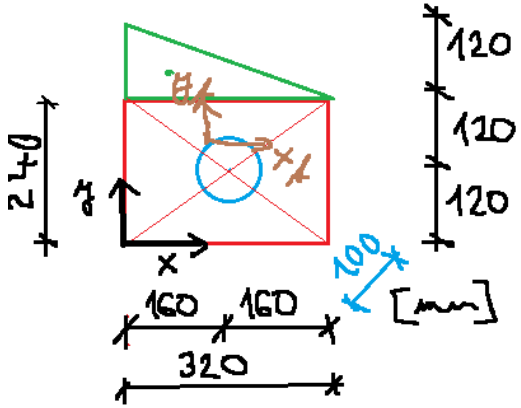
$$D_{xy} = \iint_A x\cdot y\, dA$$

$$D_{xy} = 0$$

$$I_{x_1} = \frac{1}{12}\cdot 200\cdot 50^3 + 200\cdot 50\cdot(200 - 125{,}261 - 25)^2 + \frac{1}{12}\cdot 60\cdot 150^3 + 60\cdot 150\cdot(125{,}263 - 75)^2 = 66{,}433\cdot 10^6\ mm^4 = 66{,}433\cdot 10^{-6}\ m^4$$

$$I_{y_1} = \frac{1}{12}\cdot 50\cdot 200^3 + 200\cdot 50\cdot(100 - 133{,}158)^2 + \frac{1}{12}\,150\cdot 60^3 + 150\cdot 60\cdot(170 - 133{,}158)^2 = 59{,}244\cdot 10^6\ mm^4$$

$$D_{x_1y_1} = 0 + 200\cdot 50\cdot(100 - 133{,}158)\cdot(175 - 125{,}263) + 0 + 150\cdot 60\cdot(170 - 133{,}158)\cdot(75 - 125{,}263) = -33{,}158\cdot 10^6\ mm^4$$



$$x_1 = \frac{S_y}{A} = \frac{320\cdot240\cdot160 + 320\cdot60\cdot106,\bar{6} - \pi\cdot50^2\cdot160}{320\cdot240 + 320\cdot60 - \pi\cdot50^2} = 148,383 \text{ mm}$$

$$y_1 = \frac{S_x}{A} = \frac{120 \qquad 280 \qquad 120}{-\|-} = 154,851 \text{ mm}$$

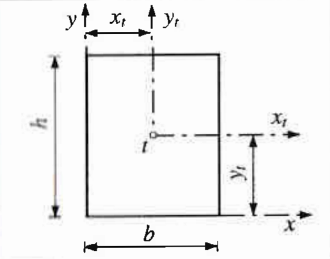
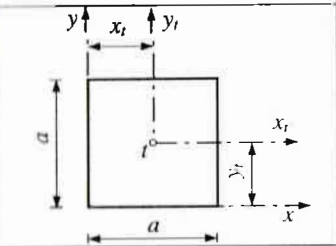
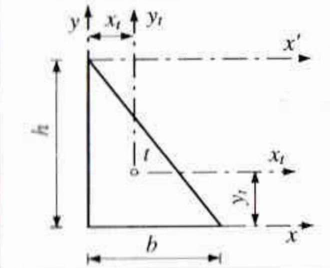
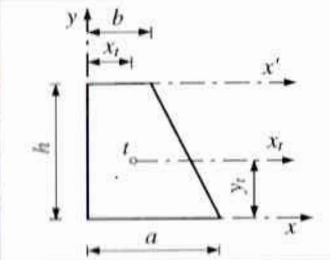
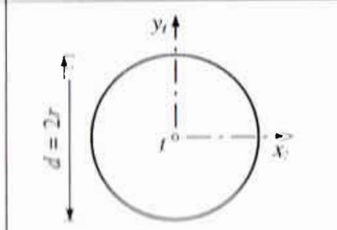
$$I_{x_1} = \frac{1}{12}320\cdot240^3 + 320\cdot240\cdot(120-154,851)^2 + \frac{1}{36}320\cdot120^3 + 320\cdot60\cdot(280-154,851)^2 - \left[\frac{\pi\cdot50^4}{4} + \pi\cdot50^2\cdot(120-154,851)^2\right] = 763,548\cdot10^6 \text{ mm}^4$$

$$I_{y_1} = \frac{1}{12}\cdot240\cdot320^3 + 320\cdot240\cdot(160-148,383)^2 + \frac{1}{36}\cdot120\cdot320^3 + 320\cdot60\cdot(106,\bar{6}-148,383)^2 - \left[\frac{\pi\cdot50^4}{4} + \pi\cdot50^2\cdot(160-148,383)^2\right] = 802,395\cdot10^6 \text{ mm}^4$$

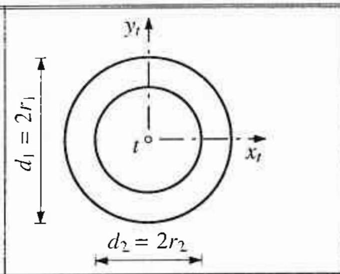
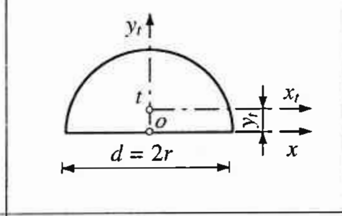
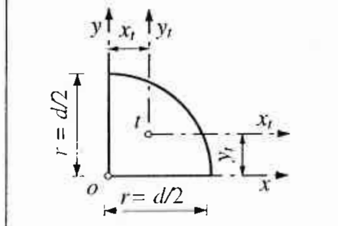
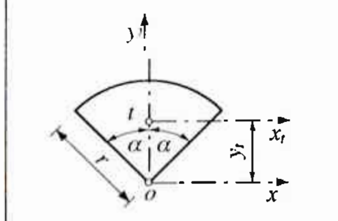
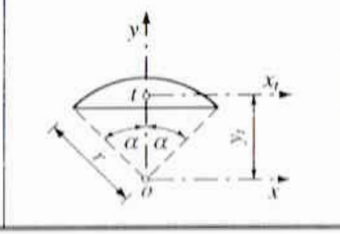
$$-\frac{1}{72}\cdot b^2h^2$$

$$D_{x_1y_1} = 0 + 320\cdot240\cdot(120-154,851)\cdot(160-148,383) - \frac{1}{72}\cdot320^2\cdot120^2 + 320\cdot60\cdot(106,\bar{6}-148,383)\cdot(280-154,851) - \left[0 + \pi\cdot50^2\cdot(160-148,383)\cdot(120-154,851)\right] = -148,632\cdot10^6 \text{ mm}^4$$

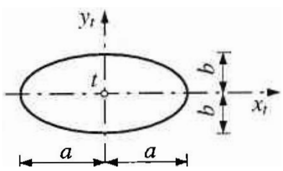
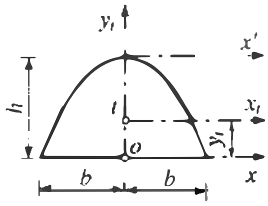
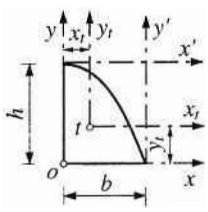
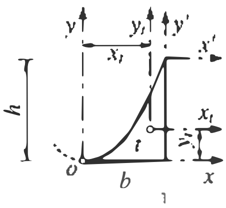
Tab. 5.1 Geometrické charakteristiky rovinných obrazců

Tvar obrazce		Obsah A, poloha těžiště t, momenty setrvačnosti I, polární I_t a deviační D
Obdélník		$A = bh;\; x_t = \frac{b}{2};\; y_t = \frac{h}{2}$ $I_{x_t} = \frac{1}{12}bh^3;\; I_{y_t} = \frac{1}{12}hb^3;\; I_x = \frac{1}{3}bh^3;\; I_y = \frac{1}{3}hb^3$ $D_{xy} = \frac{b^2h^2}{4};\; I_t = \frac{bh}{12}(b^2+h^2)$
Čtverec		$A = a^2;\; x_t = y_t = \frac{a}{2}$ $I_{x_t} = I_{y_t} = \frac{a^4}{12};\; I_x = I_y = \frac{a^4}{3}$ $D_{xy} = \frac{a^4}{4};\; I_t = \frac{a^4}{6}$
Pravoúhlý trojúhelník		$A = \frac{1}{2}bh;\; x_t = \frac{b}{3};\; y_t = \frac{h}{3}$ $I_{x_t} = \frac{1}{36}bh^3;\; I_{y_t} = \frac{1}{36}hb^3;\; D_{x_ty_t} = -\frac{b^2h^2}{72}$ $I_x = \frac{1}{12}bh^3;\; I_y = \frac{1}{12}hb^3;\; D_{xy} = \frac{b^2h^2}{24}$ $I_{x'} = \frac{1}{4}bh^3;\; I_t = \frac{bh}{36}(b^2+h^2)$
Lichoběžník		$A = \frac{1}{2}(a+b)h;\; x_t = \frac{a^2+ab+b^2}{3(a+b)};\; y_t = \frac{(a+2b)h}{3(a+b)}$ $I_{x_t} = \frac{(a^2+4ab+b^2)h^3}{36(a+b)};\; I_{x'} = \frac{(3a+b)h^3}{12}$ $I_x = \frac{(a+3b)h^3}{12};\; D_{xy} = \frac{(a^2+2ab+3b^2)h^2}{24}$
Kruh		$A = \pi r^2 = \frac{\pi d^2}{4};\; I_{x_t} = I_{y_t} = \frac{\pi r^4}{4} = \frac{\pi d^4}{64}$ $I_t = \frac{\pi r^4}{2} = \frac{\pi d^4}{32}$

Tab. 5.1 Geometrické charakteristiky rovinných obrazců (pokračování)

Tvar obrazce		Obsah A, poloha těžiště t, momenty setrvačnosti I, polární I_t a deviační D
Mezikruží		$A = \pi(r_1^2 - r_2^2) = \frac{\pi}{4}(d_1^2 - d_2^2)$ $I_{x_t} = I_{y_t} = \frac{\pi}{4}(r_1^4 - r_2^4) = \frac{\pi}{64}(d_1^4 - d_2^4)$ $I_t = \frac{\pi}{2}(r_1^4 - r_2^4) = \frac{\pi}{32}(d_1^4 - d_2^4)$
Půlkruh		$A = \frac{\pi r^2}{2} = \frac{\pi d^2}{8};\; y_t = \frac{4r}{3\pi} = \frac{2d}{3\pi}$ $I_{x_t} = \left(\frac{\pi}{8} - \frac{8}{9\pi}\right)r^4 = \left(\frac{\pi}{8} - \frac{8}{9\pi}\right)\frac{d^4}{16}$ $I_x = \frac{\pi r^4}{8} = \frac{\pi d^4}{128} = I_{y_t};\; I_o = \frac{\pi r^4}{4} = \frac{\pi d^4}{64}$
Čtvrtkruh		$A = \frac{\pi r^2}{4} = \frac{\pi d^2}{16};\; x_t = y_t = \frac{4r}{3\pi} = \frac{2d}{3\pi}$ $I_{x_t} = I_{y_t} = \left(\frac{\pi}{16} - \frac{4}{9\pi}\right)r^4 = \left(\frac{\pi}{16} - \frac{4}{9\pi}\right)\frac{d^4}{16}$ $D_{x_ty_t} = \left(\frac{1}{8} - \frac{4}{9\pi}\right)r^4 = \left(\frac{1}{8} - \frac{4}{9\pi}\right)\frac{d^4}{16}$
Kruhová výseč		$A = \alpha r^2;\; y_t = \frac{2}{3}r\frac{\sin\alpha}{\alpha}$ $I_{x_t} = r^4\left(\frac{2\alpha + \sin 2\alpha}{8} - \frac{4\sin^2\alpha}{9\alpha}\right);\; I_o = \frac{\alpha r^4}{2}$ $I_x = \frac{r^4}{8}(2\alpha + \sin 2\alpha);\; I_y = \frac{r^4}{8}(2\alpha - \sin 2\alpha)$
Kruhová úseč		$A = \left(\alpha - \frac{1}{2}\sin 2\alpha\right)r^2;\; y_t = \frac{4\sin^3\alpha}{3(2\alpha - \sin 2\alpha)}$ $I_{x_t} = r^4\left(\frac{4\alpha - \sin 4\alpha}{16} - \frac{8}{9}\cdot\frac{\sin^6\alpha}{2\alpha - \sin 2\alpha}\right)$ $I_x = \frac{r^4}{16}(4\alpha - \sin 4\alpha);\; I_y = \frac{r^4}{48}(12\alpha - 8\sin 2\alpha + \sin 4\alpha)$

Tab. 5.1 Geometrické charakteristiky rovinných obrazců (pokračování)

	Tvar obrazce	Obsah A, poloha těžiště t, momenty setrvačnosti I, polární I_t a deviační D
		$A = \pi ab$ $I_{x_t} = \frac{\pi}{4} ab^3;\ I_{y_t} = \frac{\pi}{4} ba^3$ $I_t = \frac{\pi}{4} ab\left(a^2 + b^2\right)$
Parabolická úseč		$A = \frac{4}{3} bh;\quad y_t = \frac{2}{5} h$ $I_{x_t} = \frac{16}{175} bh^3;\ I_{y_t} = \frac{4}{15} hb^3$ $I_x = \frac{32}{105} bh^3;\ I_{x'} = \frac{4}{7} bh^3$
pa abolické úseče		$A = \frac{2}{3} bh;\quad x_t = \frac{3}{8} b;\quad y_t = \frac{2}{5} h$ $I_{x_t} = \frac{8}{175} bh^3;\ I_x = \frac{16}{105} bh^3\ \ I_{x'} = \frac{2}{7} bh^3$ $I_{y_t} = \frac{19}{480} hb^3;\ I_y = \frac{2}{15} hb^3;\ I_{y'} = \frac{3}{10} hb^3$
Parabolický trojúhelník		$A = \frac{1}{3} bh;\quad x_t = \frac{3}{4} b;\quad y_t = \frac{3}{10} h$ $I_{x_t} = \frac{37}{2100} bh^3;\ I_x = \frac{1}{21} bh^3;\ I_{x'} = \frac{19}{105} bh^3$ $I_{y_t} = \frac{1}{80} hb^3;\quad I_y = \frac{1}{5} hb^3;\ I_{y'} = \frac{1}{30} hb^3$