ZÁKLADY STAVEBNÍ MECHANIKY

BDA001

Trojkloubový lomený nosník s táhlem
Gerberův nosník, reakce a diagramy vnitřních sil

Zdeněk Kala

Trojkloubový lomený nosník s táhlem

U trojkloubového rámu nebo oblouku vznikají vodorovné složky reakcí. Čím nižší oblouk, tím větší reakce. Zachycení je někdy obtížné, oblouk bývá uložen na zdech nebo štíhlých sloupech

Řešení: použití táhla

Táhlo slouží k odstranění velkých vodorovných složek reakcí.

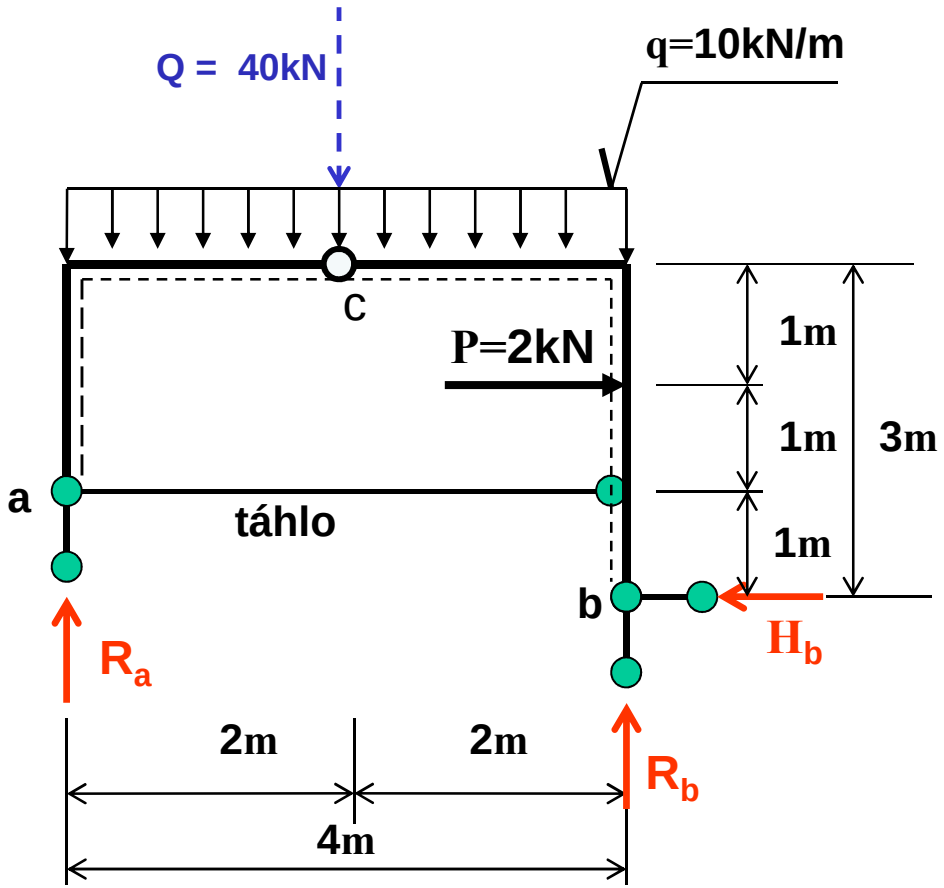
Táhlo je jednonásobná vnitřní vazba zabraňující vzájemnému posunu protilehlých bodů, přenáší pouze tahové normálové síly N.

Trojkloubový lomený nosník s táhlem

1. $\sum F_x = 0$: $-H_b + P = 0$ ⇒ $H_b = 2\ kN\ (\leftarrow)$
2. $\sum M_a = 0$: $-Q.2 - P.1 + 4.R_b - 1.H_b = 0$ ⇒ $(R_b = 21\ kN\ \uparrow)$
3. $\sum M_b = 0$: $-P.2 + Q.2 - 4.R_a = 0$ ⇒ $(R_a = 19\ kN\ \uparrow)$

Kontrola:

$\sum F_z = 0$: $-R_a - R_b + Q = 0$

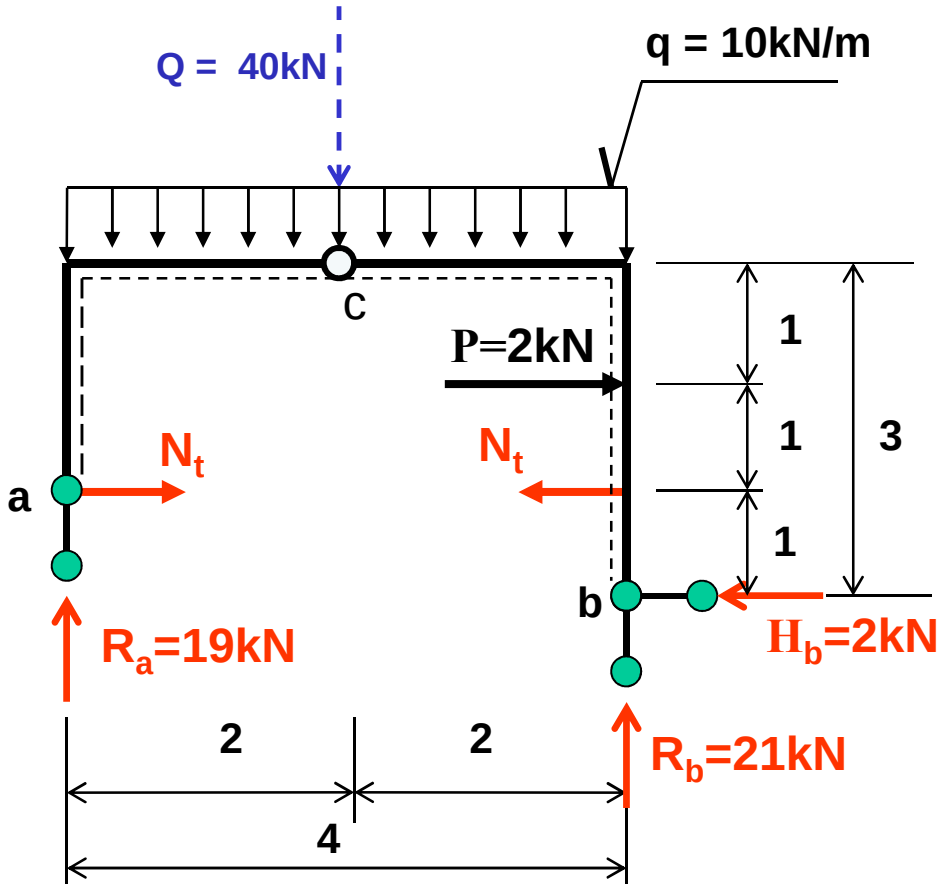


Výpočet síly v táhle

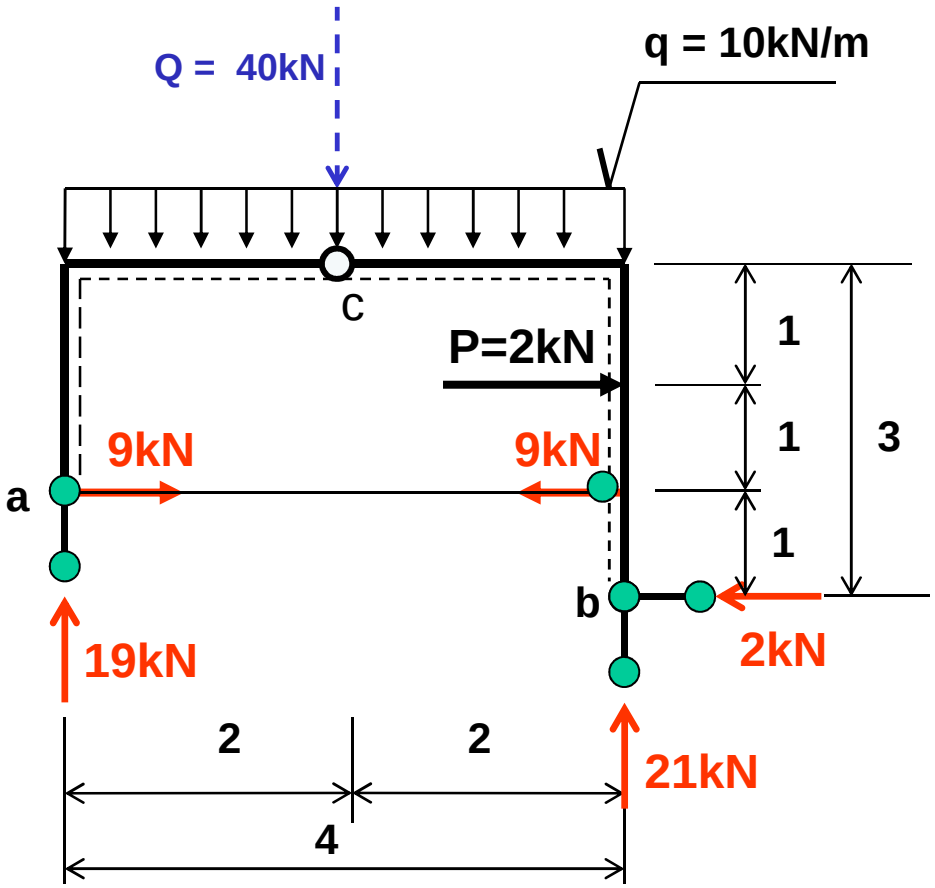
$M_c^L = 0$: $q.2.1 - 2.R_a + 2.N_t = 0$ ⇒ $N_t = 9\ kN$

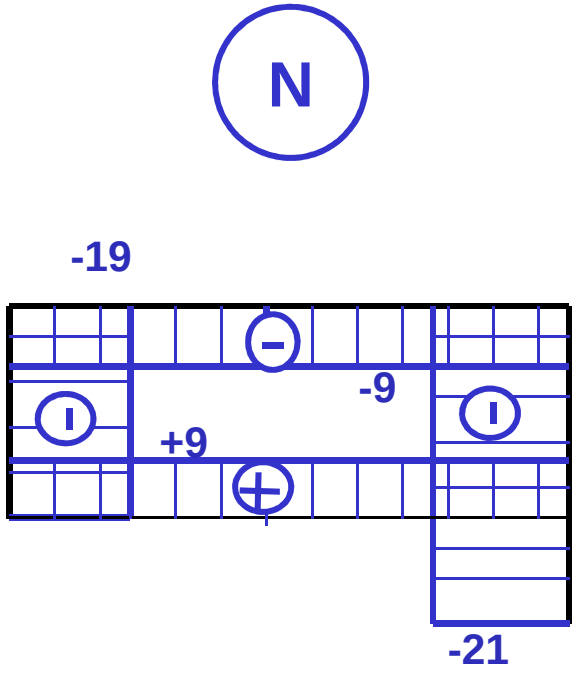
nebo:

$M_c^P = 0$: $P.1 - q.2.1 + 2.R_b - 3.H_b - 2.N_t = 0$ ⇒ $N_t = 9\ kN$

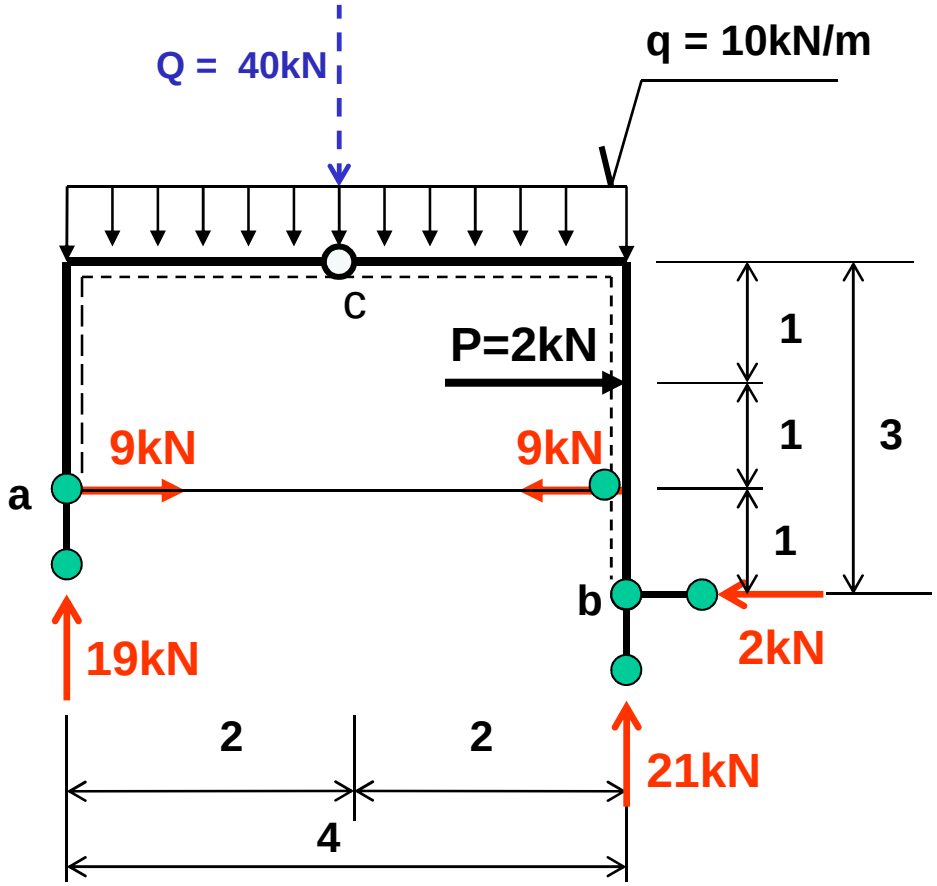


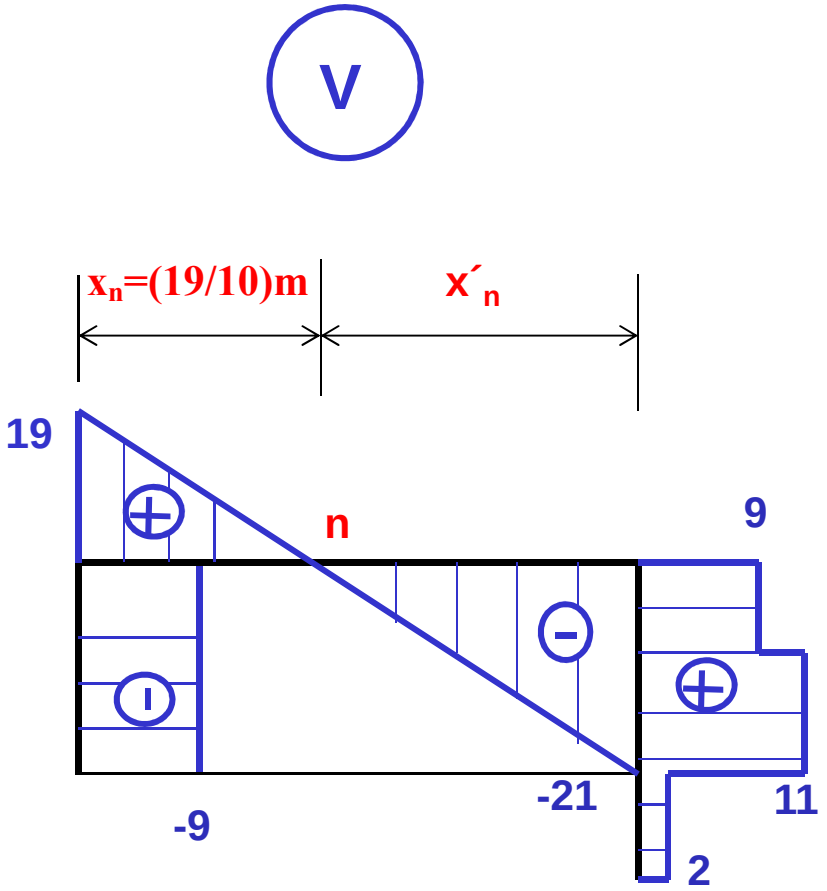
Normálové síly



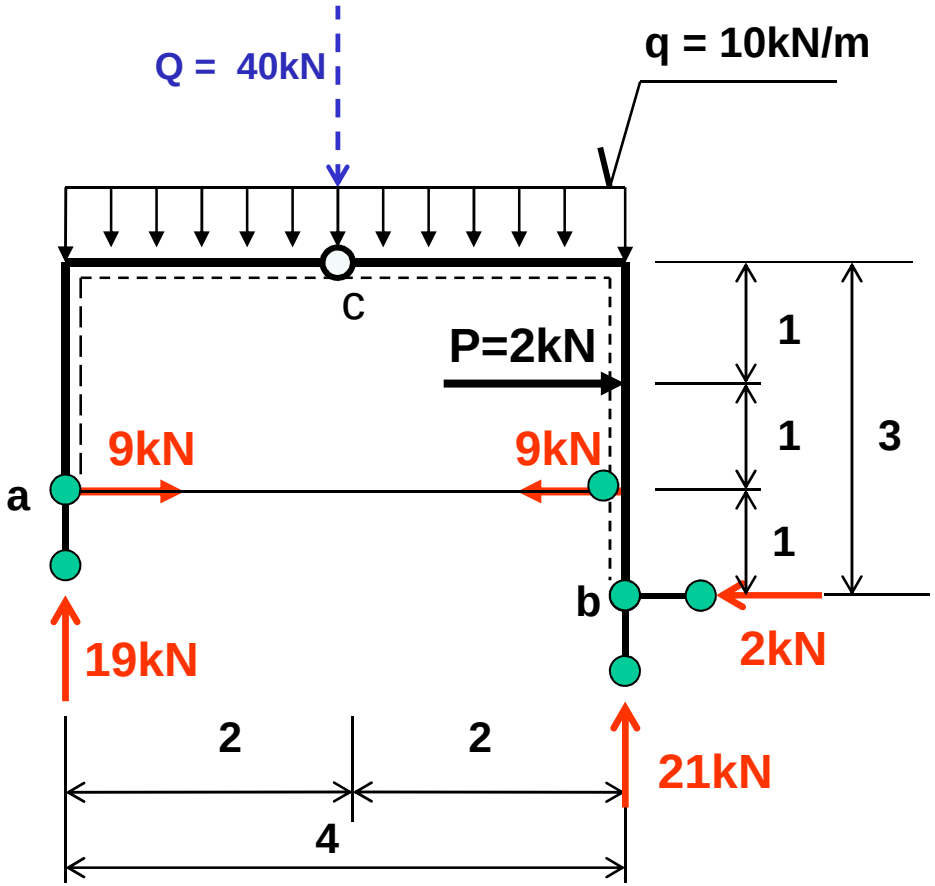


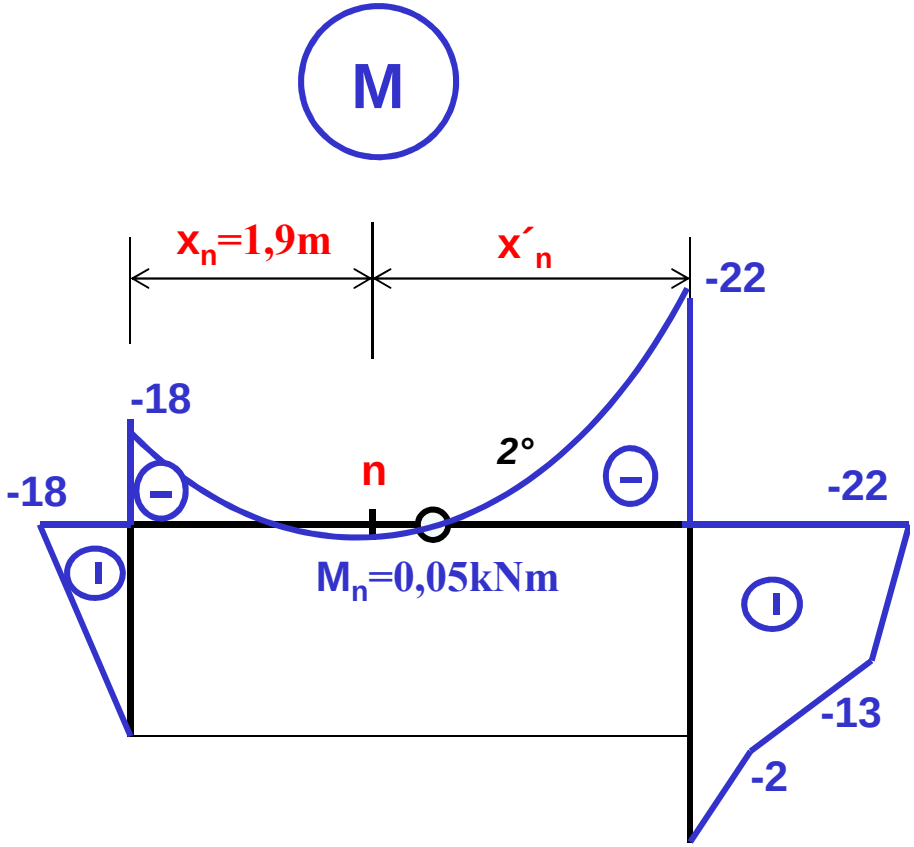
Posouvající síly





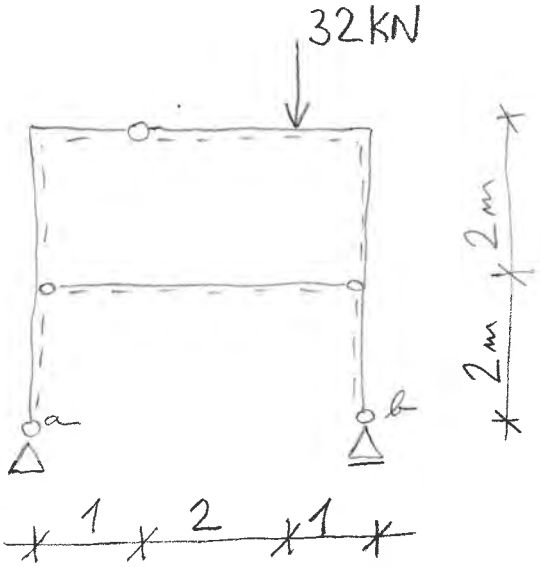
Ohybové momenty

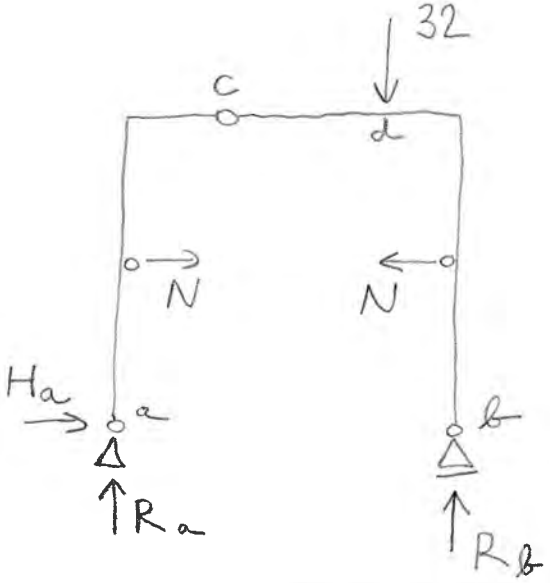




$M_n^L = 19.\ 1{,}9 - 18 - 10\ .\ 1{,}9^{\,2} / 2 = 0{,}05 kNm$

$M_n^P = 21.\ 2{,}1 - 22 - 10\ .\ 2{,}1^{\,2} / 2 = 0{,}05 kNm$





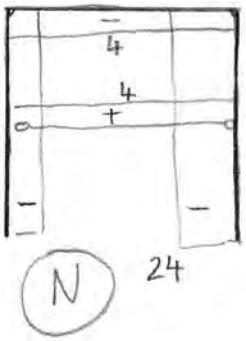
$\Sigma M_a = 0: \quad -32 \cdot 3 + R_b \cdot 4 = 0 \Rightarrow R_b = 24\,kN$

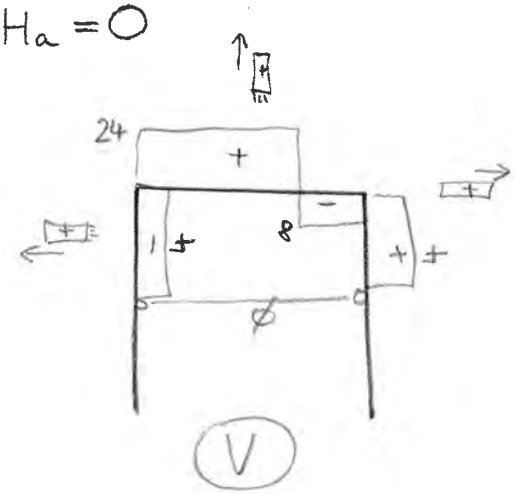
$\Sigma M_b = 0: \quad 32 \cdot 1 - R_a \cdot 4 = 0 \Rightarrow R_a = 8\,kN$

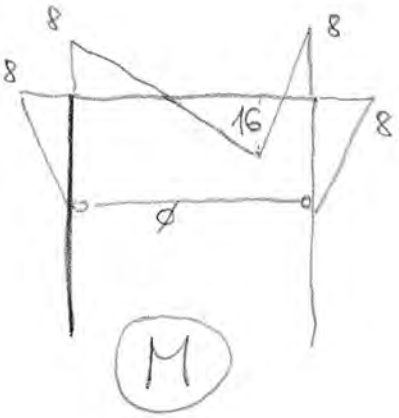
N napočítáme, protože $N - N = 0$

$\Sigma M_c = 0: \quad -32 \cdot 2 - N \cdot 2 + 24 \cdot 3 = 0 \Rightarrow N = +4\,kN$ (tah)

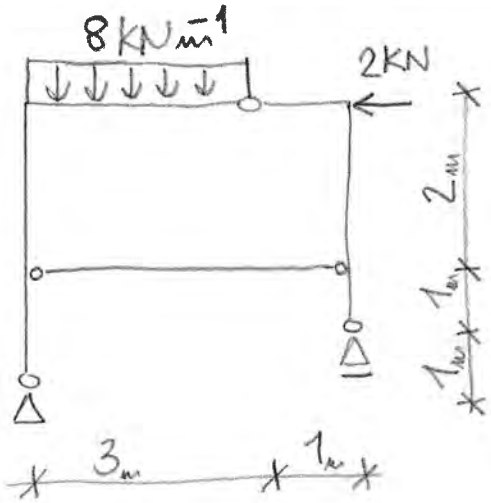
$\Sigma F_x = 0: \quad H_a = 0$

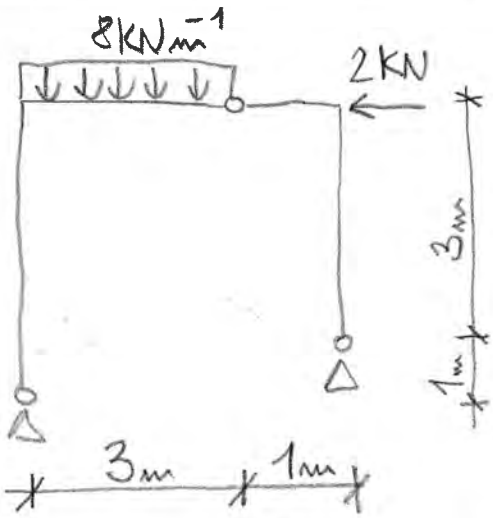




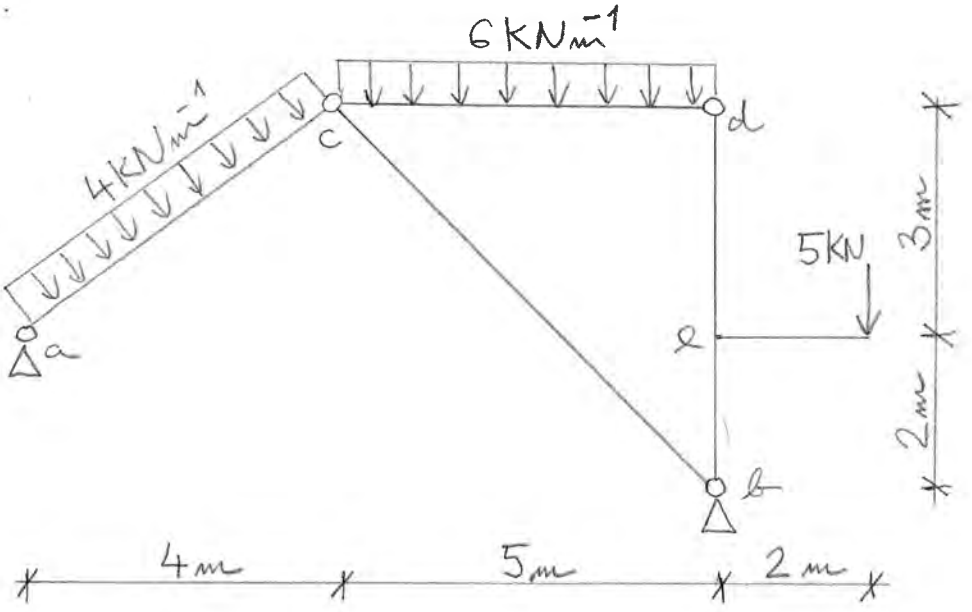


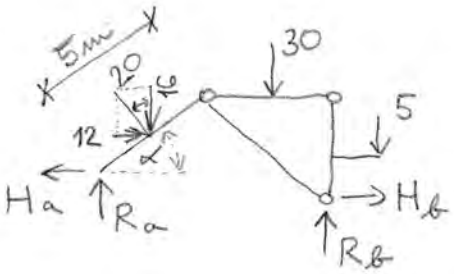
$M_d = 24 \cdot 1 - 4 \cdot 2 = 16\,kNm$





REAKCE: 2 rovnice o 2 neznámých





$\Sigma M_b = 0$

$H_a \cdot 2 - R_a \cdot 9 - 12 \cdot 3{,}5 + 16 \cdot 7 + 30 \cdot 2{,}5 - 5 \cdot 2 = 0$

$H_a \cdot 2 - R_a \cdot 9 = -135$

$H_a = 4{,}5 \cdot R_a - 67{,}5$

$\operatorname{tg}\alpha = \frac{3}{4} \Rightarrow \alpha = 36{,}8699°$

$\Sigma M_c = 0$

$20 \cdot 2{,}5 - R_a \cdot 4 - H_a \cdot 3 = 0$

$50 - R_a \cdot 4 - (4{,}5 \cdot R_a - 67{,}5) \cdot 3 = 0$

$50 - R_a \cdot 4 - 13{,}5 \cdot R_a + 202{,}5 = 0$

$R_a = 14{,}428\ \text{kN}$

$H_a = 4{,}5 \cdot 14{,}428 - 67{,}5 = -2{,}571$

$\Sigma M_a = 0 \quad -12 \cdot 1{,}5 - 16 \cdot 2 - 30 \cdot 6{,}5 - 5 \cdot 11 + R_b \cdot 9 + H_b \cdot 2 = 0$

$R_b \cdot 9 + H_b \cdot 2 = 300$

$H_b = 150 - R_b \cdot 4{,}5$

$\Sigma M_c = 0 \quad -30 \cdot 2{,}5 - 5 \cdot 7 + R_b \cdot 5 + H_b \cdot 5 = 0$

$-110 + R_b \cdot 5 + (150 - R_b \cdot 4{,}5) \cdot 5 = 0$

$-110 + R_b \cdot 5 + 750 - R_b \cdot 22{,}5 = 0$

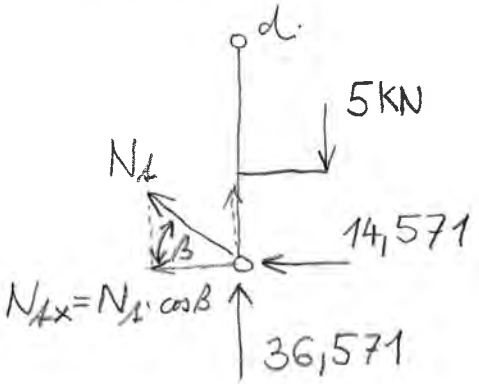
$R_b = 36{,}571\ \text{kN}$

$H_b = 150 - 36{,}571 \cdot 4{,}5 = -14{,}571\ \text{kN}$

$\Sigma F_y = 0 \quad 14{,}428 - 16 - 30 - 5 + 36{,}571 = -0{,}001 \approx 0$ ✓

$\Sigma F_x = 0 \quad 2{,}571 + 12 - 14{,}571 = 0$ ✓

TÁHLO



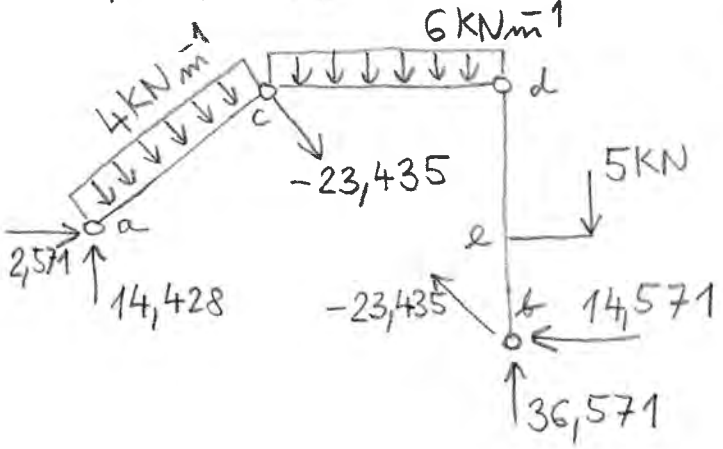
$\Sigma M_d = 0$

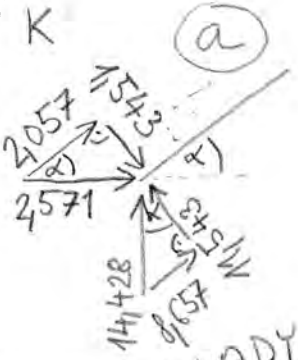
$-(N_1 \cdot \cos 45°)\cdot 5 - 14{,}571 \cdot 5 - 5\cdot 2 = 0$

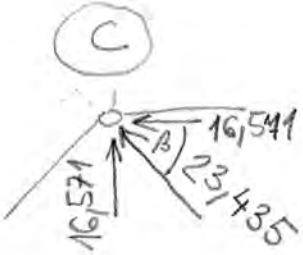
$N_1 = -23{,}435$ KN (tlak v táhle)

$\operatorname{tg}\beta = \frac{5}{5} \Rightarrow \beta = 45°$

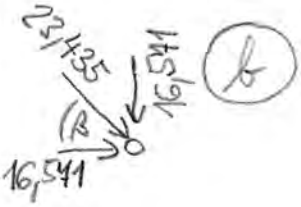
NÁHRADNÍ NOSNÍK

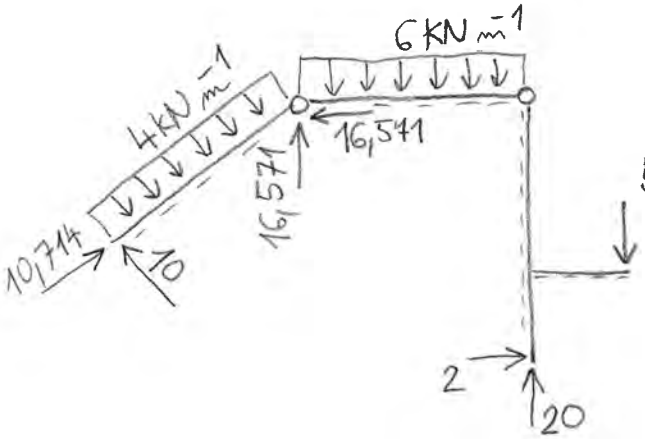




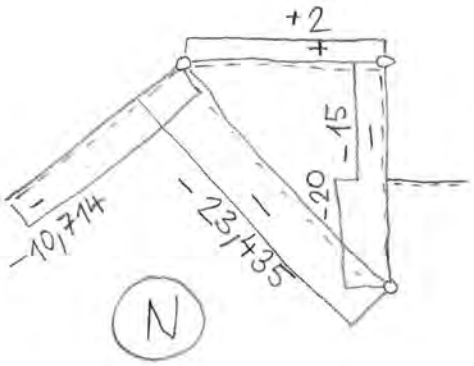


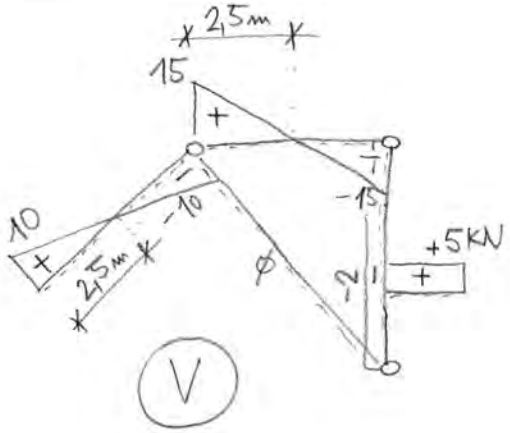
ROZKLADY SIL:

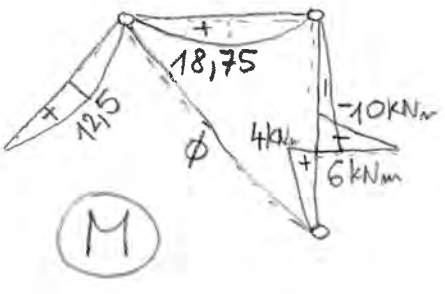




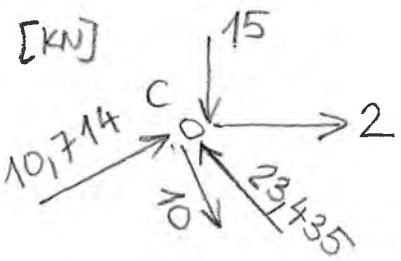
NÁHRADNÍ NOSNÍK PRO VYKRESLOVÁNÍ N_1, V_1, M







STYČNÍK C



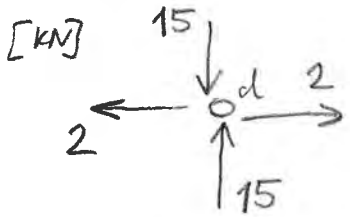
$\Sigma F_x = 0$

$10{,}714 \cdot \cos 36{,}87° + 10 \cdot \sin 36{,}87° - 23{,}435 \cdot \cos 45° + 2 = 0{,}00015 \doteq 0$

$\Sigma F_y = 0$

$10{,}714 \cdot \sin 36{,}87° - 10 \cdot \cos 36{,}87° + 23{,}435 \cdot \sin 45° - 15 = -0{,}00055 \doteq 0$

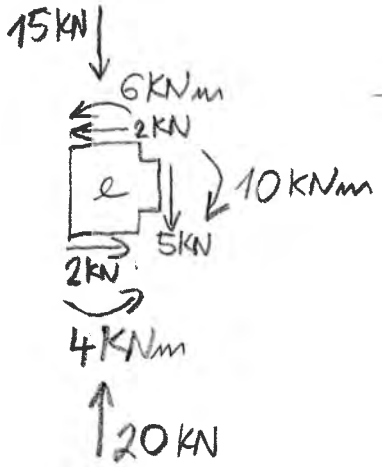
STYČNÍK d



$\Sigma F_x = 0 \quad 2 - 2 = 0$

$\Sigma F_y = 0 \quad 15 - 15 = 0$

STYČNÍK e

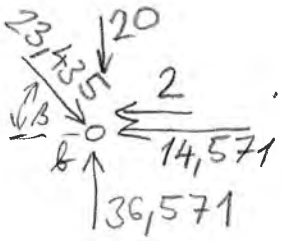


$\Sigma F_x = 0 \quad 2 - 2 = 0$

$\Sigma F_y = 0 \quad 20 - 5 - 15 = 0$

$\Sigma M = 0 \quad 4 - 10 + 6 = 0$

STYČNÍK b



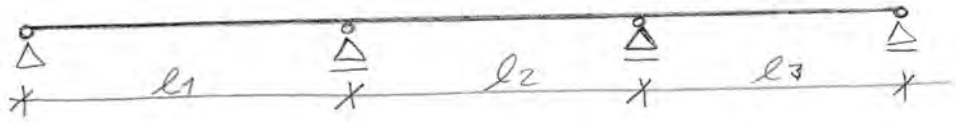
$\Sigma F_x = 0 \quad 23{,}435 \cdot \cos 45° - 14{,}571 - 2 \doteq 0$

$\Sigma F_y = 0 \quad 36{,}571 - 23{,}435 \cdot \sin 45° \doteq 0$

Poznámka: Pro překlad nejsou všude uvedeny jednotky.

Spojitý nosník s vnitřními klouby - Gerberův nosník

Gerberův nosník vzniká z nosníku spojitého, u něhož je statická neurčitost odstraněna vhodným vložením vnitřních kloubů.
Pro řešení V, V, M máme 3 statické podmínky rovnováhy + „moment v kloubu = 0".



$V = 2b + 3d = 2 \cdot 0 + 3 \cdot 1 = 3$ ... počet stat. podm. r.

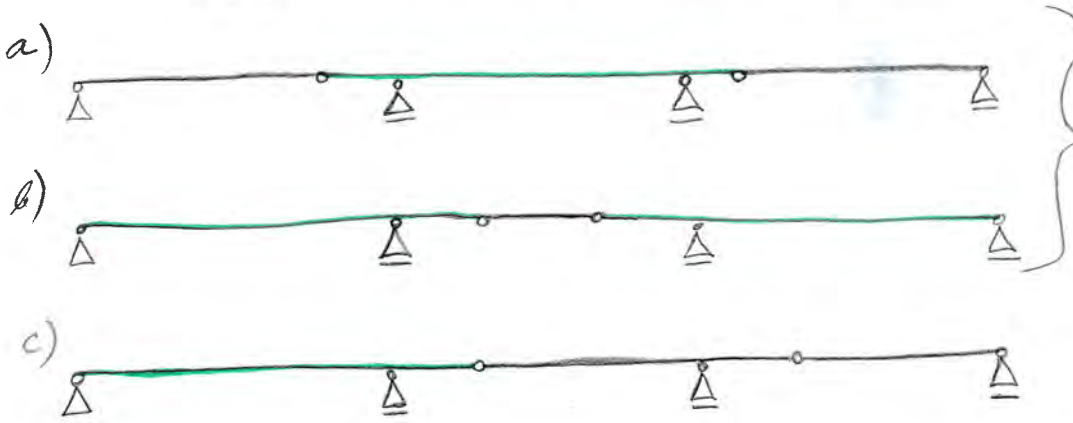
$a = a_1 + 2a_2 + 3a_3 + 2 \cdot \Sigma (n-1) \cdot k_n$ ... počet složek reakcí vazeb

$= 3 + 2 \cdot 1 = 5$

$V = 3, \quad a = 5$

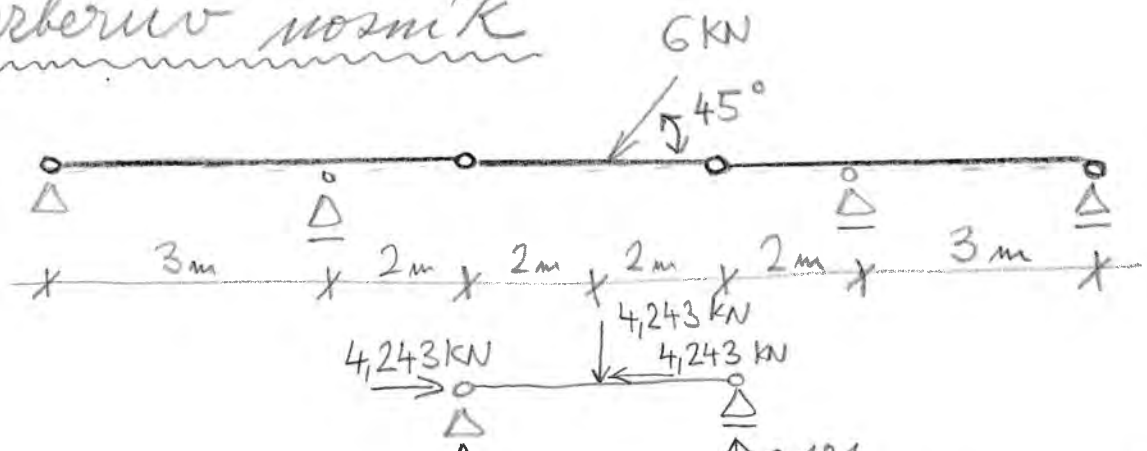
$3 < 5$ ... staticky neurčité. Máme 5 složek reakcí a jen 3 podmínky rovnováhy.

Proto přidáme další podmínku vložením vnitřního kloubu, ve kterém $M = 0$.



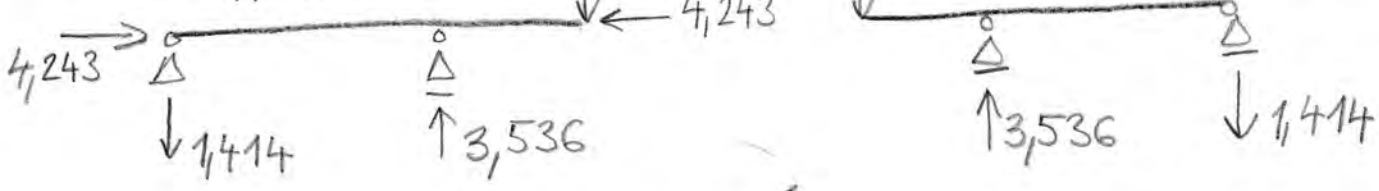
Základní část (nesoucí)
Výpočet složek reakcí začínáme na částech nedlejších (podporových)

Gerberův nosník

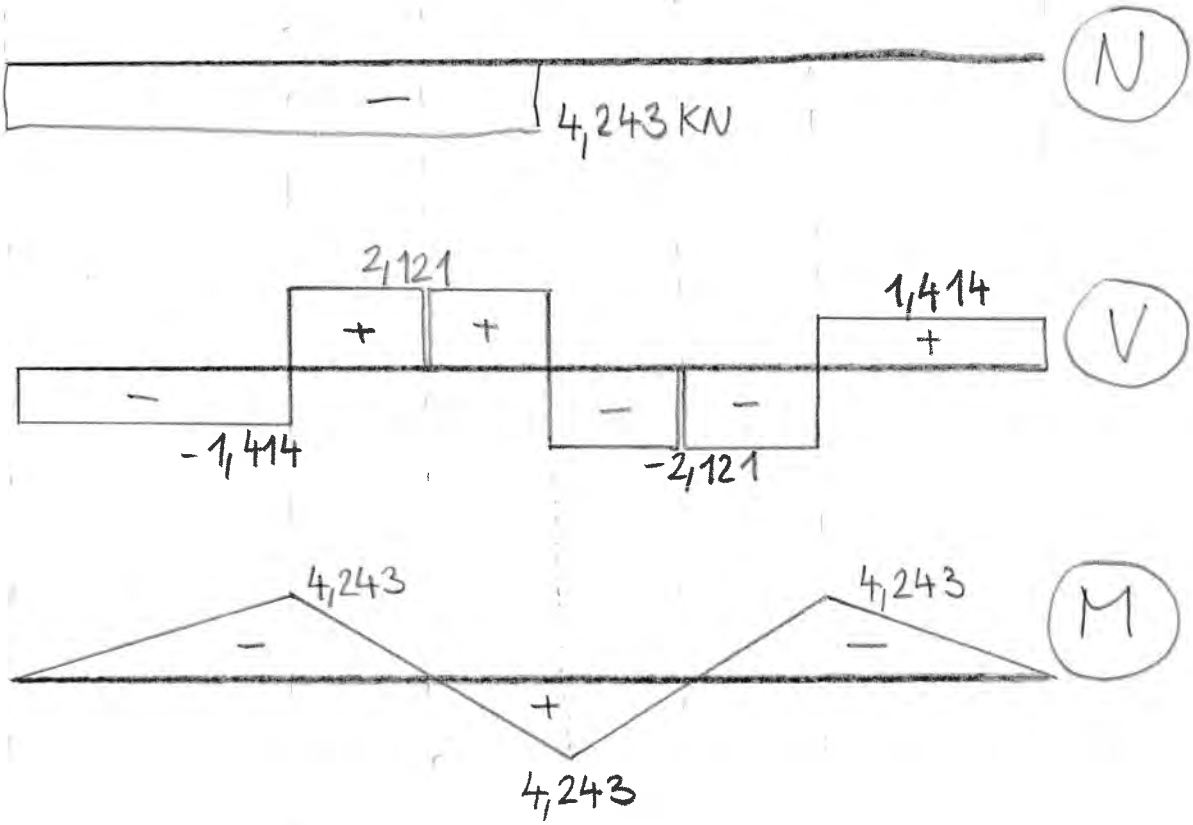


REAKCE: ↑2,121 ↑2,121

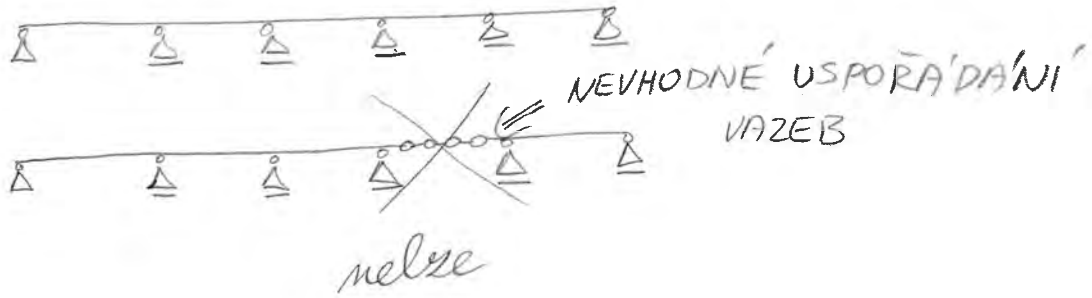
AKCE: 2,121 (stejně velké, ale opačného smyslu)



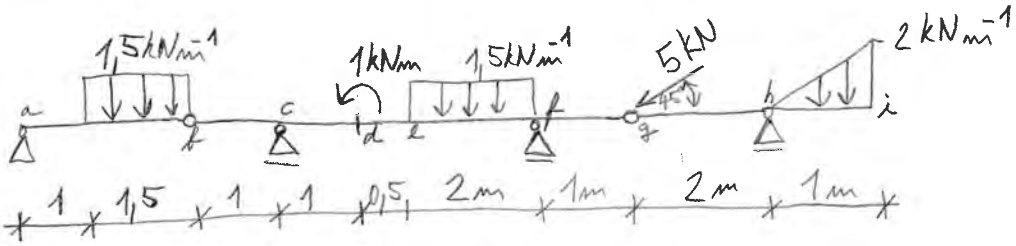
Vnitřní síly vykreslíme na břehu nosníků jako by to byl jeden nosník

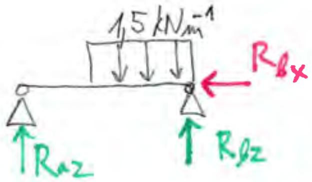


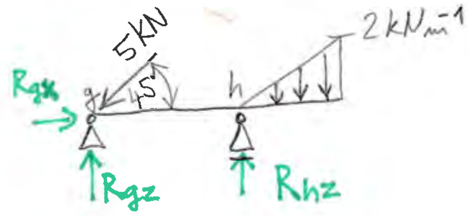
Klouby není možno umístovat zcela libovolně!



Příklad: Stanovte N, V, M na Gerberově nosníku.







$\Sigma M_a = 0$

$-1{,}5 \cdot 1{,}5 \cdot \left(1 + \frac{1{,}5}{2}\right) + R_{bz} \cdot 2{,}5 = 0$

$R_{bz} = 1{,}575\ kN$

$\Sigma F_z = 0$

$R_{az} + 1{,}575 - 1{,}5 \cdot 1{,}5 = 0$

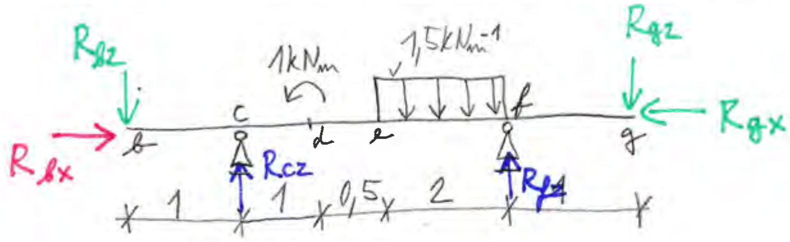
$R_{az} = 0{,}675$

$\Sigma M_g = 0 \quad R_{hz} \cdot 2 - \frac{2 \cdot 1}{2} \cdot 2{,}66\overline{6} = 0$

$R_{hz} = 1{,}333\ kN$

$\Sigma M_h = 0 \quad -R_{gz} \cdot 2 + 5 \cdot \cos 45^\circ \cdot 2 - \frac{2 \cdot 1}{2} \cdot 0{,}66\overline{6} = 0 \Rightarrow R_{gz} = 3{,}2\ kN$

$\Sigma F_x = 0 \quad R_{gx} = 5 \cdot \cos 45^\circ = 3{,}5355\ kN$



$\Sigma M_c = 0$

$1{,}575 \cdot 1 + 1 - 1{,}5 \cdot 2 \cdot 2{,}5 + R_{fz} \cdot 3{,}5 - 3{,}2 \cdot 4{,}5 = 0$

$R_{fz} = 5{,}521\,kN$

$\Sigma M_f = 0$

$1{,}575 \cdot 4{,}5 - R_{cz} \cdot 3{,}5 + 1 + 1{,}5 \cdot 2 \cdot 1 - 3{,}2 \cdot 1 = 0$

$R_{cz} = 2{,}25\,kN$

$\Sigma F_z = 0$

$1{,}111 + 1{,}5 \cdot 2 + 1{,}575 - 3{,}121 - 2{,}787 = 0$ ✓

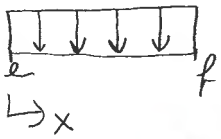
$M_c = R_{az} \cdot 3{,}5 - 1{,}5 \cdot 1{,}5 \cdot (1 + \frac{1{,}5}{2}) = -1{,}575$

$^{L}M_d = R_{az} \cdot 4{,}5 - 1{,}5 \cdot 1{,}5 \cdot (2 + \frac{1{,}5}{2}) + R_{az} \cdot 1 = -0{,}9$

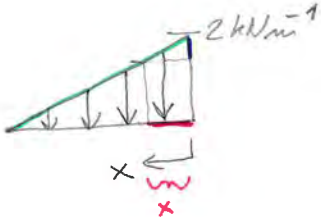
$^{P}M_d = {}^{L}M_d - 1 = -1{,}9$

$M_{ef} = R_{az} \cdot (x - 5) - 1{,}5 \cdot 1{,}5 \cdot (x - 3{,}25) + R_z \cdot (x - 1{,}5) - 1$

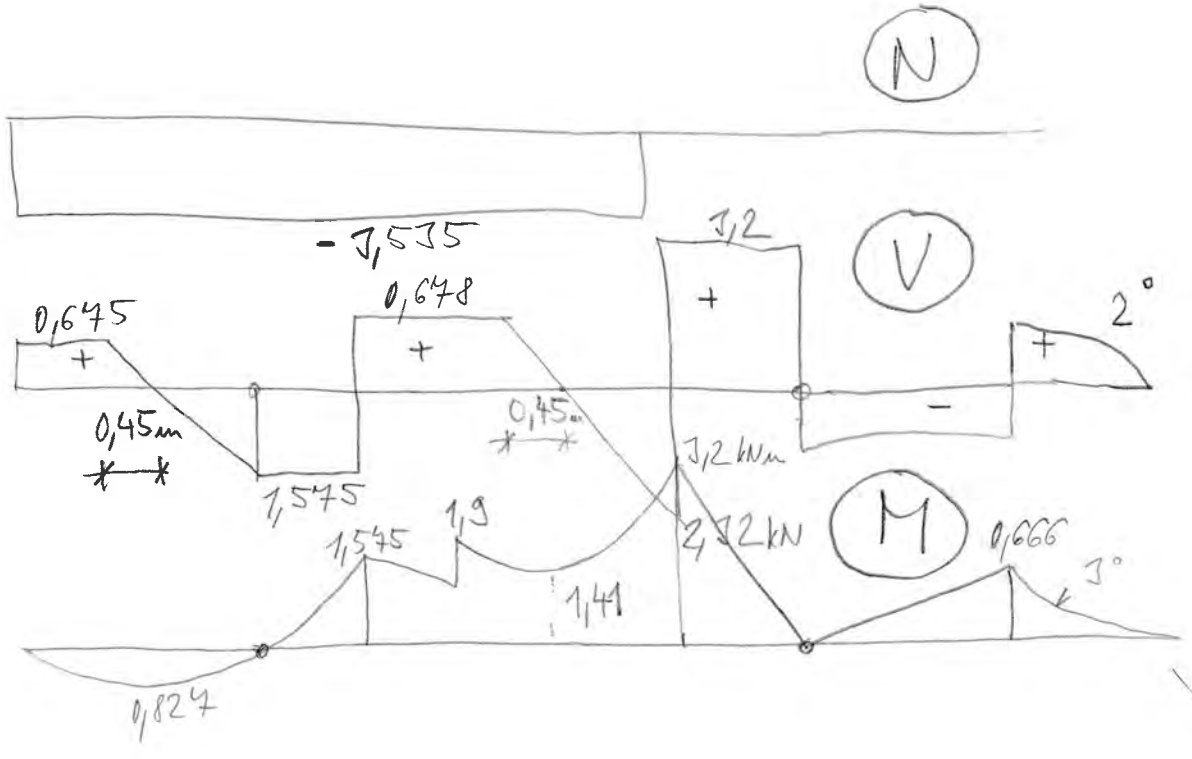
$- 1{,}5 \cdot x \cdot \frac{x}{2}$

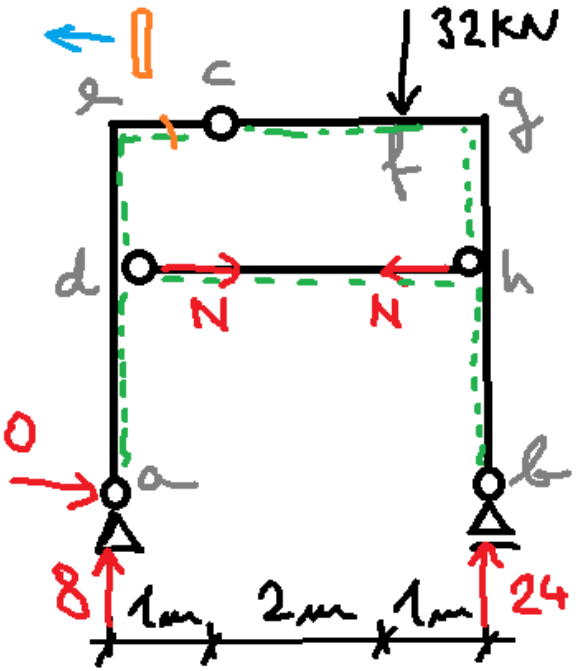


M_{ih}



$M_{ih} = \underbrace{(2 - 2 \cdot x) \cdot x \cdot \frac{x}{2}}_{Q_1} + \underbrace{2 \cdot x \cdot \frac{1}{2}}_{Q_2} \cdot \frac{2}{3} \cdot x +$



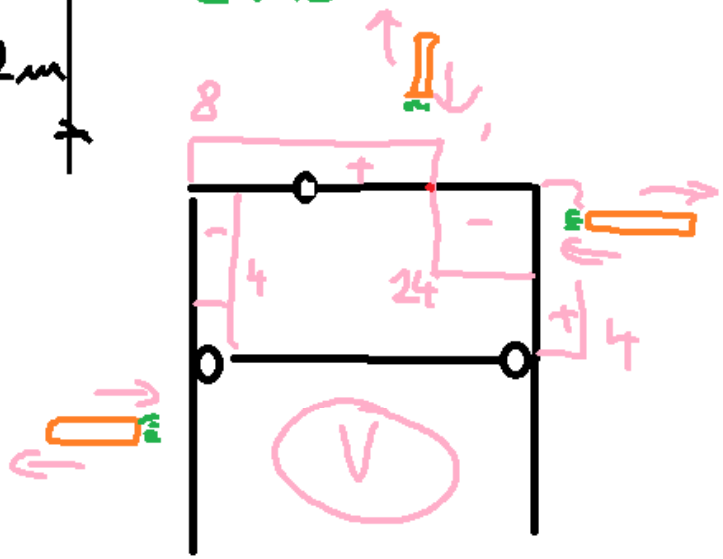


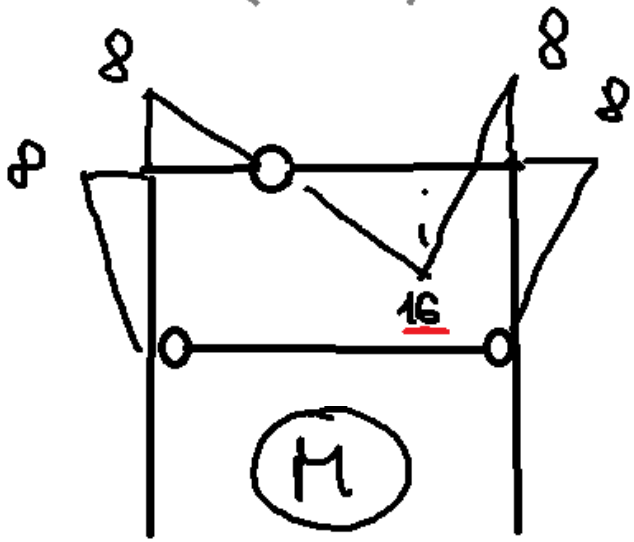
$\Sigma M_a = 0 \quad -12 \cdot 3 + R_b \cdot 4 = 0 \qquad R_b = 24\,kN$

$\Sigma M_b = 0 \quad 12 \cdot 1 - R_a \cdot 4 = 0 \qquad R_a = 8\,kN$

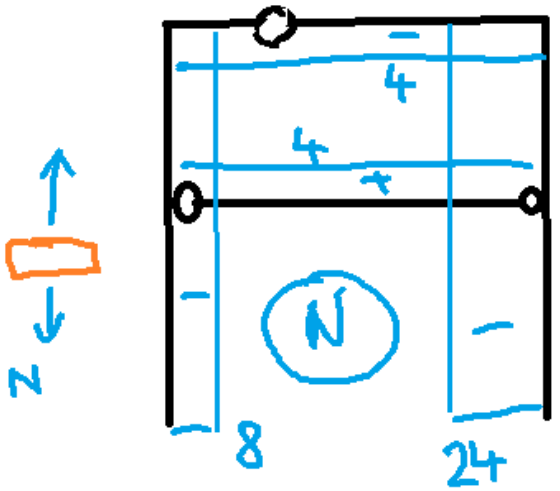
$\Sigma F_x = 0 \quad H_a = 0$

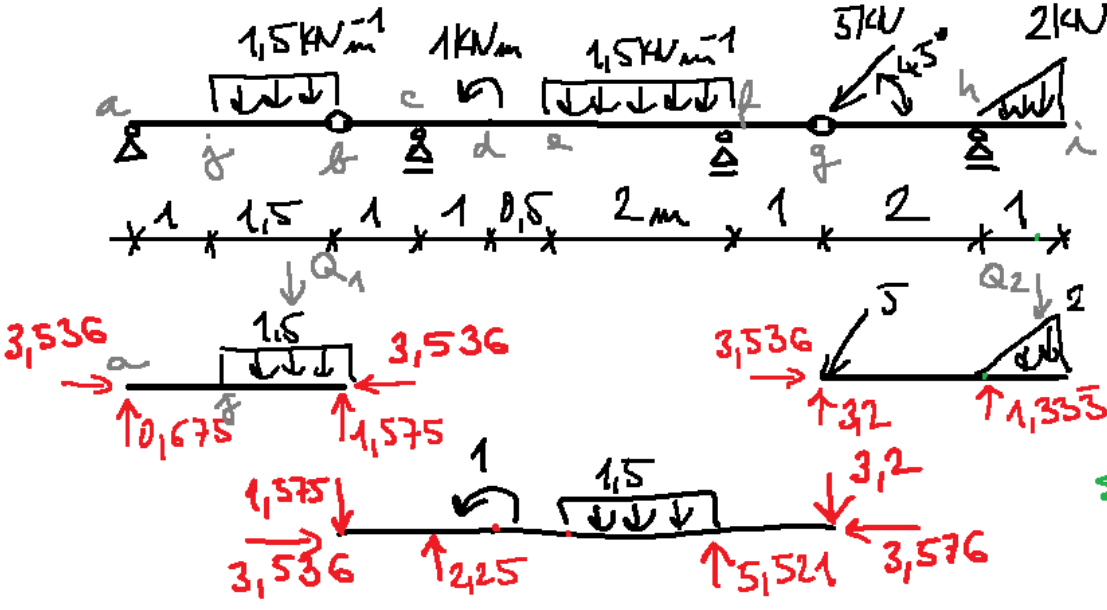
$\Sigma M_c = 0 \quad -8 \cdot 1 + N \cdot 2 = 0 \qquad N = 4\,kN$ (TAH)





$M_{max} = 24 \cdot 1 - 4 \cdot 2 = 16\,kNm$





$\Sigma M_a = 0 \quad -\underbrace{1,5 \cdot 1,5}_{Q_1} \cdot \left(1 + \frac{1,5}{2}\right) + R_B \cdot 2,5 = 0$

$\Sigma F_y = 0 \quad R_a - 1,5 \cdot 1,5 + 1,575 = 0$

$\Sigma M_g = 0 \quad -\underbrace{\frac{2 \cdot 1}{2}}_{Q_2} \cdot \left(2 + \frac{2}{3}\right) + R_A \cdot 2 = 0$

$\Sigma M_A = 0 \quad -R_B \cdot 2 + 5 \cdot \sin 45° \cdot 2 - \frac{2 \cdot 1}{2} \cdot \frac{2}{3} = 0$

$\Sigma F_x = 0 \quad -5 \cdot \cos 45° + H_A = 0$

$\Sigma M_c = 0 = 1,575 \cdot 1 + 1 - 1,5 \cdot 2,25 + R_B \cdot 3,5 - 3,2$

$\Sigma M_f = 0 \quad 1,575 \cdot 4,5 - R_c \cdot 3,5 + 1 + 1,5 \cdot 2 \cdot 1 - 1,2 \cdot 1 = 0$

$M_{max} = -1,575 \cdot 2,952 + 2,25 \cdot 1,952 - 1 - 1,5 \cdot 0,452 \cdot \frac{0,452}{2} = 1,41 \text{ kN m}$

N