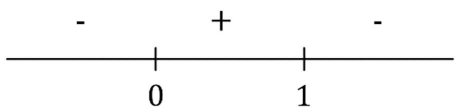
$$f(x) = arctg\ \frac{1-x}{x}$$

1)

$D(f) = R - \{0\}$

znaménko:



2)

$$f'(x) = \left(arctg\ \frac{1-x}{x}\right)' = \frac{1}{1+\left(\frac{1-x}{x}\right)^2} \cdot \left(\frac{1-x}{x}\right)' = \frac{1}{1+\left(\frac{1-x}{x}\right)^2} \cdot \frac{-1 \cdot x - (1-x) \cdot 1}{x^2} = \frac{-1}{2x^2-2x+1}$$

$f' \neq 0 \implies$ nemá lokální extrém, <u>všude klesající</u>

3)

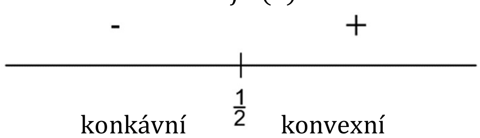
$$f''(x) = \left(\frac{-1}{2x^2-2x+1}\right)' = -((1-2x+2x^2)^{-1})' = -\frac{1}{(1-2x+2x^2)^2} \cdot (1-2x+2x^2)' =$$

$$= -\left(-\frac{1}{(1-2x+2x^2)^2} \cdot (4x-2)\right) = \frac{4x-2}{(2x^2-2x+1)^2}$$

$f''(x) = 0$ pro $4x - 2 = 0 \Leftrightarrow x = \frac{1}{2}$

$f\left(\frac{1}{2}\right) = arctg\ 1 = \frac{\pi}{4}$ inflexní bod $[\frac{1}{2}, \frac{\pi}{4}]$

znaménko $f''(x)$:



4)asymptoty:

- svislé

$$\lim_{x\to 0^+} arctg \frac{1-x}{x} = arctg \lim_{x\to 0^+}\left(\frac{1}{x} - 1\right) = arctg\infty = \frac{\pi}{2}$$

$$\lim_{x\to 0^-} arctg \frac{1-x}{x} = arctg \lim_{x\to 0^-}\left(\frac{1}{x} - 1\right) = arctg - \infty = -\frac{\pi}{2}$$

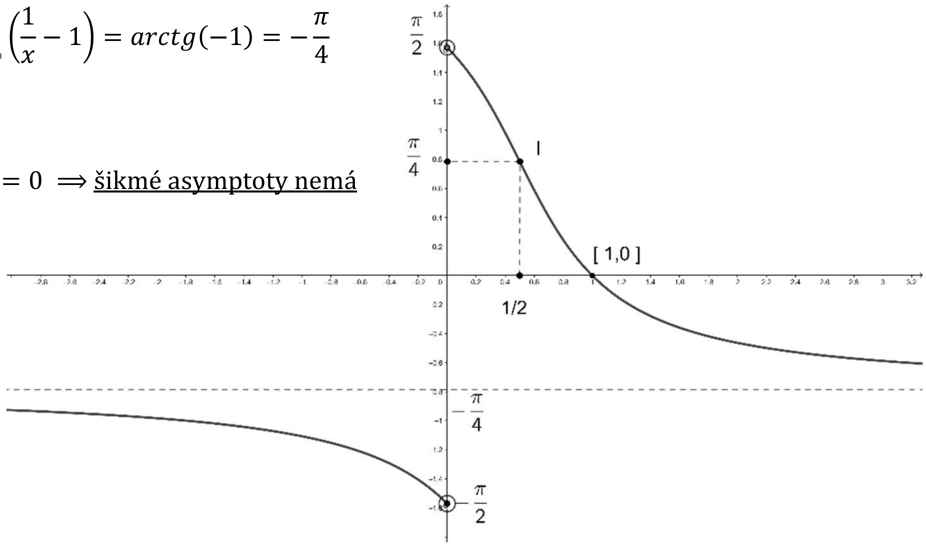
- vodorovné:

$$\lim_{x\to \infty} arctg \frac{1-x}{x} = arctg \lim_{x\to \infty}\left(\frac{1}{x} - 1\right) = arctg(-1) = -\frac{\pi}{4}$$

$$\lim_{x\to -\infty} arctg \frac{1-x}{x} = arctg \lim_{x\to -\infty}\left(\frac{1}{x} - 1\right) = arctg(-1) = -\frac{\pi}{4}$$

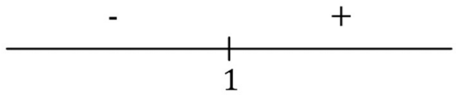
- šikmé:

$$\lim_{x\to\infty} \frac{arctg\frac{1-x}{x}}{x} = \frac{\lim_{x\to\infty} arctg\frac{1-x}{x}}{\lim_{x\to\infty} x} = \frac{-\frac{\pi}{4}}{\infty} = 0$$ $\implies$ <u>šikmé asymptoty nemá</u>



$$f(x) = \frac{1}{\ln x}$$

1) znaménko:



$D(f)$: $(0, \infty)$ $x \neq 1$

2)

$f'(x) = \left(\frac{1}{\ln x}\right)' = (\ln x^{-1})' = -\frac{1}{\ln^2 x} \cdot \frac{1}{x} = -\frac{1}{x \cdot \ln^2 x}$

$f'(x) \neq 0 \Rightarrow$ nemá lokální minimum ani maximum

$f'(x) < 0 \Rightarrow$ všude klesající

3)

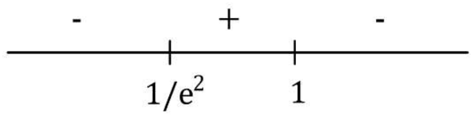
$$f''(x) = \left(-\frac{1}{x \cdot \ln^2(x)}\right)' = -((x \cdot \ln^2(x))^{-1})' = -\frac{1}{(x \cdot \ln^2(x))^2} \cdot (x \cdot \ln^2(x))' =$$

$$= -\left(-\frac{1}{(x \cdot \ln^2(x))^2} \cdot (\ln^2(x) + 2 \cdot \ln(x))\right) = \frac{\ln(x) + 2}{x^2 \cdot \ln^3(x)}$$

$f''(x) = 0 \Leftrightarrow \ln(x) + 2 = 0 \Leftrightarrow \ln(x) = -2 \Leftrightarrow x = e^{-2}$

inflexní bod $[\frac{1}{e^2}, -\frac{1}{2}]$

znaménko $f''(x)$



4)asymptoty:

- se směrnicí: $y = ax + b \Rightarrow$ po dosazení $y = 0$

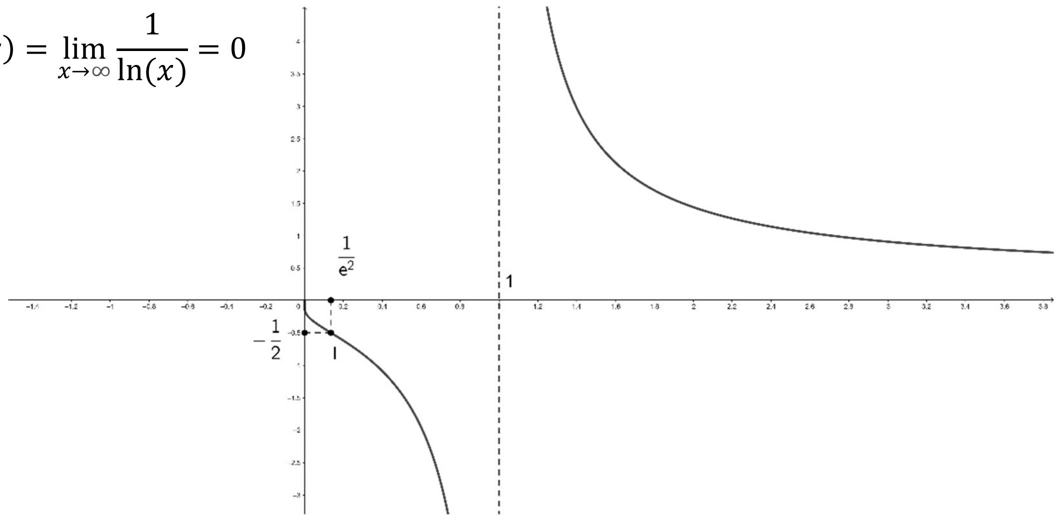
$$a = \lim_{x \to \infty} \frac{f(x)}{x} = \lim_{x \to \infty} \frac{1}{x \cdot \ln(x)} = 0$$

$$b = \lim_{x \to \infty} (f(x) - x) = \lim_{x \to \infty} \frac{1}{\ln(x)} = 0$$

- svislá

$$\lim_{x \to 1^+} \frac{1}{\ln x} = +\infty$$

$$\lim_{x \to 1^-} \frac{1}{\ln x} = -\infty$$

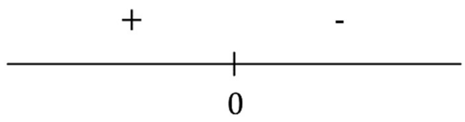


$$f(x) = x + \frac{1}{x}$$

1)

$D(f) = R - \{0\}$

znaménko:



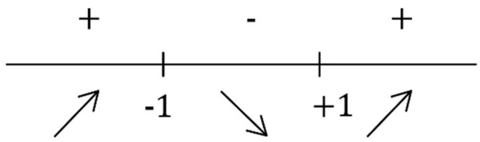
2)

$f'(x) = \left(x + \frac{1}{x}\right)' = (x)' + \left(\frac{1}{x}\right)' = 1 - \frac{1}{x^2}$

$f'(x) = 0$ pro $x = 1$

$x = -1$

znaménko $f'(x)$



3)

$f''(x) = \left(1 - \frac{1}{x^2}\right)' = (1)' - \left(\frac{1}{x^2}\right)' = \frac{2}{x^3}$

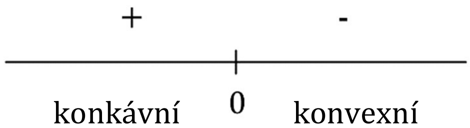
$f''(x) \neq 0$

nemá inflexní bod

$f''(1) > 0$ lokální minimum [1,2]

$f''(-1) < 0$ lokální maximum [-1,-2]

znaménko $f''(x)$



4)asymptoty:

- se směrnicí: $y = ax + b \Rightarrow$ po dosazení $y = x$

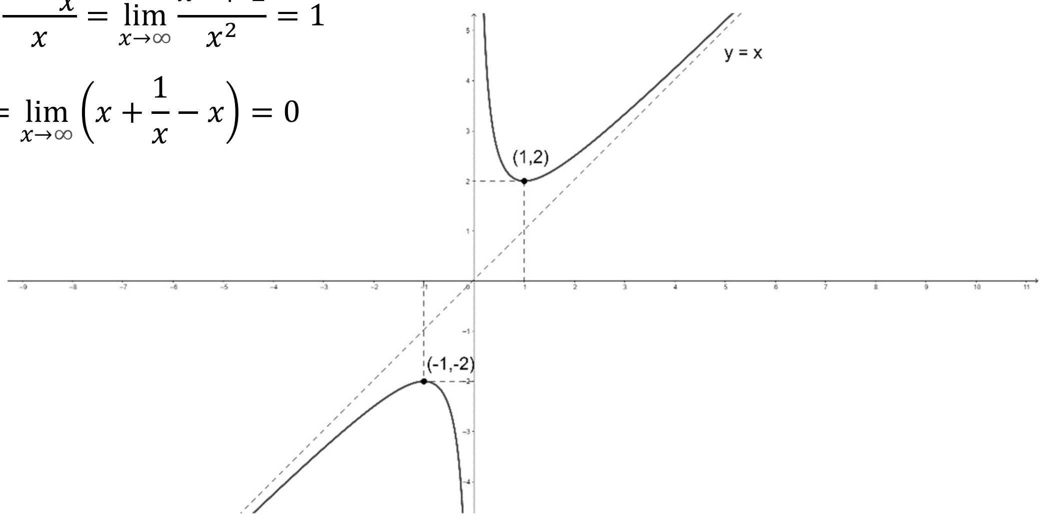
$$a = \lim_{x \to \infty} \frac{f(x)}{x} = \lim_{x \to \infty} \frac{x + \frac{1}{x}}{x} = \lim_{x \to \infty} \frac{x^2 + 1}{x^2} = 1$$

$$b = \lim_{x \to \infty} (f(x) - x) = \lim_{x \to \infty} \left(x + \frac{1}{x} - x\right) = 0$$

- svislá

$$\lim_{x \to 0^+} \left(x + \frac{1}{x}\right) = +\infty$$

$$\lim_{x \to 0^-} \left(x + \frac{1}{x}\right) = -\infty$$



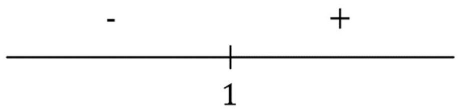
$$f(x) = \frac{lnx}{x}$$

1)

$D(f): x > 0 \ \ (0, \infty)$

$lnx = 0 \Leftrightarrow x = 1$

znaménko:

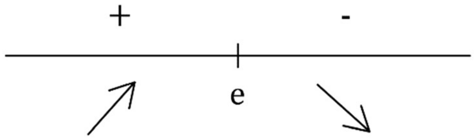


2)

$$f'(x) = \left(\frac{\ln(x)}{x}\right)' = \frac{(\ln(x))' \cdot x - (\ln(x)) \cdot x'}{x^2} = \frac{\frac{1}{x} \cdot x - 1 \cdot \ln(x)}{x^2} = \frac{1 - \ln(x)}{x^2}$$

$f'(x) = 0 \ \Leftrightarrow 1 - lnx = 0 \Leftrightarrow 1 = lnx \Leftrightarrow x = e$

znaménko $f'(x)$



3)

$$f''(x) = \left(\frac{1 - \ln(x)}{x^2}\right)' = \frac{(1 - \ln(x))' \cdot x^2 - (1 - \ln(x)) \cdot (x^2)'}{(x^2)^2} =$$

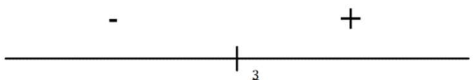
$$= \frac{\left(-\frac{1}{x}\right) \cdot x^2 - 2x \cdot (1 - \ln(x))}{(x^2)^2} = \frac{2\ln(x) - 3}{x^3}$$

$f''(x) = 0 \Leftrightarrow 2lnx - 3 = 0 \Leftrightarrow lnx = \frac{3}{2} \Leftrightarrow x = e^{\frac{3}{2}}$

znaménko $f''(x)$

inflexní bod $[\, e^{\frac{3}{2}}, \frac{3}{2e^{\frac{3}{2}}} \,]$

$f''(e) < 0$ lokální maximum $[\, e, \frac{1}{e} \,]$



4)asymptoty:

- se směrnicí: $y = ax + b \ \Rightarrow$ po dosazení $y = 0$

$$a = \lim_{x \to \infty} \frac{f(x)}{x} = \lim_{x \to \infty} \frac{\ln(x)}{x^2} = \lim_{x \to \infty} \frac{\frac{1}{x}}{2x} = 0$$

$$b = \lim_{x \to \infty} (f(x) - x) = \lim_{x \to \infty} \frac{\ln(x)}{x} = \lim_{x \to \infty} \frac{1}{x} = 0$$

- svislá

$$\lim_{x \to 0^+} \frac{\ln(x)}{x} = -\infty$$