Pro funkci $f(x) = \operatorname{arctg} \frac{x-1}{x-2}$ určete:

a) $D(f)$ a vodorovnou asymptotu

b) tečna a normála v $x_0 = 1$

a)

$D(f) = R - \{2\}$

$\lim_{x\to\infty} \operatorname{arctg} \frac{x-1}{x-2} = \operatorname{arctg} 1 = \frac{\pi}{4}$

$\lim_{x\to\infty} \operatorname{arctg} \frac{x-1}{x-2} = \operatorname{arctg} 1 = \frac{\pi}{4}$

vodorovná asymptota $y = \frac{\pi}{4}$ pro x $\to \pm\infty$

b)

$$f'(x) = \frac{1}{1+\left(\frac{x-1}{x-2}\right)^2} \cdot \frac{1\cdot(x-2)-(x-1)\cdot 1}{(x-2)^2} = \frac{1}{\frac{x^2-4x+4+x^2-2x+1}{(x-2)^2}} \cdot \frac{x-2-x+1}{(x-2)^2} =$$

$$= \frac{-1}{2x^2-6x+5}$$

$f'(1) = \frac{-1}{2-6+5} = \frac{-1}{1} = -1$

t: $y - y_0 = f'(x_0) \cdot (x - x_0)$ t: $y - 0 = -1 \cdot (x - 1)$

$\underline{y = 1 - x}$

n: $y - y_0 = -\frac{1}{f'(x_0)} \cdot (x - x_0)$ n: $y - 0 = 1 \cdot (x - 1)$

$\underline{y = x - 1}$

$f(1) = \operatorname{arctg} 0 = 0$

Pro funkci $f(x) = \arcsin\left(x^2\right)$ určete:

a) $D(f)$, intervaly monotonie a lokální extrémy

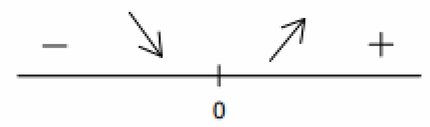
b) rovnici tečny a normály v $x_0 = \frac{1}{\sqrt{2}}$

a)

$D(f) = < -1\,,\ 1 >$

$f'(x) = \frac{1}{\sqrt{1-x^4}} \cdot 2x$

$f'(x) = 0$ pro $x = 0$




rostoucí $< 0\,,\ 1 >$

klesající $< -1\,,\ 0 >$

$f(0) = \arcsin 0 = 0$ lokální minimum $[0\,,\ 0]$

$f(1) = \arcsin 1 = \sin\frac{\pi}{2} = 1$ lokální maximum $\left[1\,,\ \frac{\pi}{2}\right], \left[-1\,,\ \frac{\pi}{2}\right]$

b)

$$f'\left(\frac{1}{\sqrt{2}}\right) = \frac{2\cdot\frac{1}{\sqrt{2}}}{\sqrt{1-\frac{1}{4}}} = \frac{\sqrt{2}}{\sqrt{\frac{3}{4}}} = \frac{\sqrt{2}}{\frac{\sqrt{3}}{2}} = \frac{2\cdot\sqrt{2}}{\sqrt{3}}$$

$y_0 = f(x_0) = \arcsin\frac{1}{2} = \frac{\pi}{6}$

t: $y - y_0 = f'(x_0)\cdot(x - x_0)$ t: $y - \frac{\pi}{6} = \frac{2\cdot\sqrt{2}}{\sqrt{3}}\cdot\left(x - \frac{1}{\sqrt{2}}\right)$

n: $y - y_0 = -\frac{1}{f'(x_0)}\cdot(x - x_0)$ n: $y - \frac{\pi}{6} = -\frac{\sqrt{3}}{2\cdot\sqrt{2}}\cdot\left(x - \frac{1}{\sqrt{2}}\right)$

Pro funkci $f(x) = \ln \frac{x+1}{1-x}$ určete:

a) $D(f)$

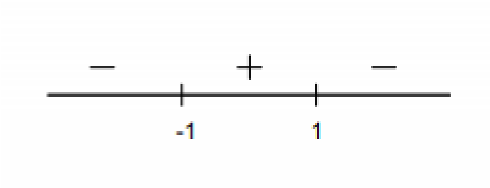
b) lokální extrémy a intervaly monotonie

c) inflexní body a intervaly konvexnosti a konkávnosti

a)

$\frac{x+1}{1-x} > 0$

$D(f) = (-1\ ,\ 1)$




b)

$$f'(x) = \frac{1}{\frac{x+1}{1-x}} \cdot \frac{1\cdot(1-x)-(x+1)\cdot(-1)}{(1-x)^2} = \frac{1-x}{x+1} \cdot \frac{1-x+x+1}{(1-x)^2} = \frac{2}{(x+1)\cdot(1-x)} = \frac{2}{1-x^2}$$

$$= 2 \cdot \left(1 - x^2\right)^{-1}$$

$\Rightarrow$ nemá extrémy, všude rostoucí, $f'(x) > 0$ v (-1 , 1)

c)

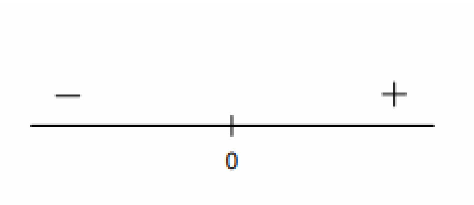
$$f''(x) = -2 \cdot \left(1 - x^2\right)^{-2} \cdot (-2x) = \frac{4x}{\left(1-x^2\right)^2}$$

$f''(x) = 0$ pro $x = 0$

konvexní v $(0\ ,\ 1)$

konkávní v $(-1\ ,\ 0)$

inflexní bod I $[0\ ,\ 0]$



Pro funkci $f(x) = e^{\frac{1}{x}}$ určete:

a) inflexní body, intervaly konvexnosti a konkávnosti

b) vodorovnou asymptotu

a)

$D(f) = R - \{0\}$

$f'(x) = e^{\frac{1}{x}} \cdot \left(\frac{1}{x}\right)' = -1 \cdot x^{-2} \cdot e^{\frac{1}{x}} = -\frac{1}{x^2} \cdot e^{\frac{1}{x}}$

$f''(x) = -1 \cdot (-2) \cdot x^{-3} \cdot e^{\frac{1}{x}} + \left(-1 \cdot x^{-2} \cdot e^{\frac{1}{x}} \cdot (-1) \cdot x^{-2}\right) = \frac{2 \cdot e^{\frac{1}{x}}}{x^3} + \frac{e^{\frac{1}{x}}}{x^4} =$

$= \frac{(2x+1) \cdot e^{\frac{1}{x}}}{x^4}$

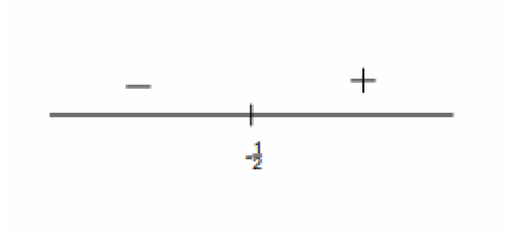
$f''(x) = 0$ pro $2x + 1 = 0$

$x = -\frac{1}{2}$

konvexní $\left(-\frac{1}{2}, 0\right) \cup (0, \infty)$

konkávní $\left(-\infty, -\frac{1}{2}\right)$

inflexní bod I $\left[-\frac{1}{2}, e^{-2}\right]$




b)

$\lim_{x\to\infty} e^{\frac{1}{x}} = e^0 = 1$ $\underline{y = 1}$ pro $x \to \pm\infty$

$\lim_{x\to-\infty} e^{\frac{1}{x}} = e^0 = 1$

Pro funkci $f(x) = \frac{1}{e^x}$ určete:

a) inflexní body, intervaly konvexnosti a konkávnosti

b) vodorovnou asymptotu

a)

$\boldsymbol{D}(f) = \boldsymbol{R}$

$f'(x) = \frac{-e^x}{(e^x)^2} = -\frac{1}{e^x}$

$f''(x) = \frac{e^x}{(e^x)^2} = \frac{1}{e^x}$

$\Rightarrow$inflexní body nemá, všude konvexní

b)

$\lim_{x\to\infty} \frac{1}{e^x} = 0$

$\lim_{x\to-\infty} \frac{1}{e^x} = \infty$

vodorovná asymptota $y = 0$ pro $x \to \infty$

Je daná funkce $f(x) = \frac{e^{3x}}{x^2}$ určete:

a) $D(f)$, lokální extrémy a intervaly monotonie

b) inflexní body a intervaly konvexnosti a konkávnosti

$D(f) = R - \{0\}$

a)

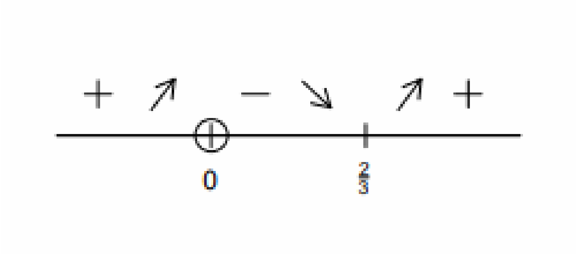
$$f'(x) = \frac{\left(e^{3x}\right)' \cdot x^2 - e^{3x} \cdot 2x}{x^4} = \frac{3 \cdot e^{3x} \cdot x^2 - 2x \cdot e^{3x}}{x^4} = \frac{e^{3x} \cdot x \cdot (3x-2)}{x^4} = \frac{e^{3x} \cdot (3x-2)}{x^3}$$

$f'(x) = 0$ pro $3x - 2 = 0$

$$x = \frac{2}{3}$$

rostoucí v $(-\infty\ ,\ 0)$ a $\left(\frac{2}{3}\ ,\ \infty\right)$

klesajíci v $\left(0\ ,\ \frac{2}{3}\right)$




$f\left(\frac{2}{3}\right) = \frac{e^{3 \cdot \frac{2}{3}}}{\left(\frac{2}{3}\right)^2} = \frac{9}{4} \cdot e^2$ lokální minimum $E\left[\frac{2}{3}\ ,\ \frac{9}{4} \cdot e^2\right]$

b)

$$f''(x) = \frac{\left(e^{3x} \cdot (3x-2)\right)' \cdot x^3 - \left(e^{3x} \cdot (3x-2) \cdot 3x^2\right)}{x^6} = \frac{\left(e^{3x} \cdot 3 \cdot (3x-2) + e^{3x} \cdot 3\right) \cdot x^3 - \left(e^{3x} \cdot \left(9x^3 - 6x^2\right)\right)}{x^6} =$$

$$= \frac{e^{3x} \cdot x^2 \cdot \left(9x^2 - 6x + 3x - 9x + 6\right)}{x^6} = \frac{e^{3x} \cdot \left(9x^2 - 12x + 6\right)}{x^4} = \frac{3 \cdot e^{3x} \cdot \left(3x^2 - 4x + 2\right)}{x^4}$$

$f''(x) = 0$ pro $3x^2 - 4x + 2 = 0$

$$D = b^2 - 4 \cdot a \cdot c = 16 - 4 \cdot 3 \cdot 2 = -8 < 0$$

$\Rightarrow$ nemá inflexní body, všude $f''(x) > 0$, všude konvexní $(-\infty\ ,\ 0) \cup (0\ ,\ \infty)$