

$$\int \frac{3x+1}{(x^2+2)^2} dx = \int \frac{\frac{1}{2}(3 \cdot 2x+2)}{(x^2+2)^2} dx = \frac{1}{2} \int \left(\frac{3 \cdot dx}{(x^2+2)^2} + \frac{2}{(x^2+2)^2} \right) dx =$$

$$= \frac{3}{2} \int \frac{dx}{(x^2+2)^2} + \int \frac{1}{(x^2+2)^2} dx = \frac{3}{2} I_1 + I_2 =$$

$$I_1 = \int \frac{dx}{(x^2+2)^2} dx = \left| \begin{array}{l} x^2+2 = t \\ 2x dx = dt \\ dx = \frac{1}{2x} dt \end{array} \right| = \int \frac{1}{t^2} dt = \int t^{-2} dt = -\frac{1}{t} + c = -\frac{1}{x^2+2} + c$$

$$I_2 = \int \frac{1}{(x^2+2)^2} dx = \frac{1}{2 \cdot 1 \cdot 2} \left(\frac{x}{(x^2+2)^1} + (2 \cdot 1 - 1) \int \frac{1}{(x^2+2)} dx \right) =$$

$$= \frac{1}{4} \cdot \left(\frac{x}{x^2+2} + \int \frac{1}{2\left(\frac{x^2}{2}+1\right)} dx \right) = \frac{1}{4} \left(\frac{x}{x^2+2} + \frac{1}{2} \cdot \sqrt{2} \arctg \frac{x}{\sqrt{2}} \right) + c$$

$$= \frac{-3}{2} \cdot \frac{1}{x^2+2} + \frac{1}{4} \cdot \frac{x}{x^2+2} + \frac{\sqrt{2}}{8} \arctg \frac{x}{\sqrt{2}} + c =$$

$$= \frac{1}{4} \cdot \frac{x-6}{x^2+2} + \frac{\sqrt{2}}{8} \arctg \frac{x}{\sqrt{2}} + c$$