• TEČNÁ ROVINA A NORMÁLA PLOCHY

- funkce $z = f(x,y)$ určená implicitně rovnicí $F(x,y,z) = 0$ a bodem $T = [x_0, y_0, z_0]$
- obecná rovnice tečné roviny: $\boxed{(x-x_0)\,F'_x(T) + (y-y_0)\,F'_y(T) + (z-z_0)\,F'_z(T) = 0}$
- normála:

$$\boxed{\begin{aligned} x &= x_0 + p \cdot F'_x(T) \\ y &= y_0 + p \cdot F'_y(T) \\ z &= z_0 + p \cdot F'_z(T);\ p \in \mathbb{R} \end{aligned}}$$

• TEČNA A NORMÁLOVÁ ROVINA KE KŘIVCE

- prostorová křivka $\gamma: \vec{r} = \vec{r}(t) = (x(t), y(t), z(t)),\ t \in \langle \alpha, \beta \rangle,\ T = [x_T, y_T, z_T] \in \gamma$
- tečna:

$$\boxed{\begin{aligned} x &= x_T + p \cdot x'(t_T) \\ y &= y_T + p \cdot y'(t_T) \\ z &= z_T + p \cdot z'(t_T);\ p \in \mathbb{R} \end{aligned}}$$

- normálová rovina: $\boxed{\rho:\ (x-x_T)\,x'(t_T) + (y-y_T)\,y'(t_T) + (z-z_T)\,z'(t_T) = 0}$

(43) Naleznete rovnice tečné roviny plochy $z = f(x,y)$ určené implicitně danou rovnicí

$x^2 + 4y^2 + z^2 - 36 = 0$, která je rovnoběžná s rovinou ρ : $x + y - z = 0$.

Řešení:

$\vec{n}_\rho = (1,1,-1) \Rightarrow \vec{n} = (k,k,-k)$, $k \in R - \{0\}$ $\qquad T = [x_0, y_0, z_0]$

$$\left.\begin{array}{lll} F'_x(x,y,z) = 2x & & F'_x(T) = 2x_0 = k \\ F'_y(x,y,z) = 8y & \Rightarrow & F'_y(T) = 8y_0 = k \\ F'_z(x,y,z) = 2z & & F'_z(T) = 2z_0 = -k \end{array}\right\} \Rightarrow T = \left[\frac{k}{2}, \frac{k}{8}, -\frac{k}{2}\right] + \text{musí splňovat}$$

rovnici $x^2 + 4y^2 + z^2 - 36 = 0$

$$\left(\frac{k}{2}\right)^2 + 4\cdot\left(\frac{k}{8}\right)^2 + \left(-\frac{k}{2}\right)^2 - 36 = 0$$

$$\frac{k^2}{4} + \cancel{4}\cdot\frac{k^2}{\cancel{64}\,16} + \frac{k^2}{4} - 36 = 0$$

$$\frac{4k^2 + k^2 + 4k^2}{16} = 36$$

$$\frac{9}{16}k^2 = 36 \Rightarrow k^2 = \frac{\overset{4}{\cancel{36}}\cdot 16}{\cancel{9}} = 64$$

$$k = \pm 8 \Rightarrow T_1 = [-4, -1, 4], \vec{n} = (1,1,-1)$$

$$T_2 = [4, 1, -4], \vec{n} = (1,1,-1)$$

τ_1 : $x + 4 + y + 1 - (z - 4) = 0$

$\underline{x + y - z + 9 = 0}$

τ_2 : $x - 4 + y - 1 - (z + 4) = 0$

$\underline{x + y - z - 9 = 0}$

50. Určete tečnou rovinu grafu fce $e^z - z + xy = 3$ kolmou k přímce $x = 1 + t$

$y = 1 + 2t$

$z = 1.$

Řešení:

$\vec{s} = (1,2,0) = \vec{n}$

$F'_x = y \qquad y = k \qquad y = k$

$F'_y = x \qquad x = 2k \quad \Rightarrow \quad x = 2k$

$F'_z = e^z - 1 \qquad \underline{e^z - 1 = 0} \qquad z = 0$

$T = [2k, k, 0] \in F$

$e^0 - 0 + 2k \cdot k = 3$

$1 + 2k^2 - 3 = 0$

$2k^2 - 2 = 0 \Rightarrow \underline{k = \pm 1} \quad \ldots \quad T_1 = [-2, -1, 0]$

$T_2 = [2, 1, 0]$

$t_1: \quad 1(x+2) + 2(y+1) = 0$

$x + 2 + 2y + 2 = 0$

$\underline{x + 2y + 4 = 0}$

$t_2: \quad 1(x-2) + 2(y-1) = 0$

$x - 2 + 2y - 2 = 0$

$\underline{x + 2y - 4 = 0}$

48. Nalezněte tečnou rovinu ke grafu fce dané impl. rcí $(x-1)^2+y^2+z^2=2$, kt. j. kolmá k rovinám $2x-2y-z=3$

$$x-y-z=0.$$

Řešení!

$$\vec{n_1}=(2,-2,-1)$$

$$\vec{n_2}=(1,-1,-1)$$

$$\vec{m}\perp(\vec{n_1},\vec{n_2}):\quad \vec{m}=\begin{vmatrix}\vec{e_1} & \vec{e_2} & \vec{e_3}\\ 2 & -2 & -1\\ 1 & -1 & -1\end{vmatrix}=\vec{e_1}(2-1)-\vec{e_2}(-2+1)+\vec{e_3}(-2+2)$$

$$\Rightarrow \vec{m_1}=(1,1,0)$$

$$\vec{m_2}=(-1,-1,0)$$

$$F'_x=2(x-1)$$

$$F'_y=2y$$

$$F'_z=2z$$

I.

$$2(x-1)=k \qquad 2y=k \qquad 2z=0 \quad\Rightarrow\quad x=\frac{k+2}{2} \qquad y=\frac{k}{2} \qquad z=0$$

II.

$$2(x-1)=-k \qquad 2y=-k \qquad 2z=0 \quad\Rightarrow\quad x=\frac{-k+2}{2} \qquad y=-\frac{k}{2} \qquad z=0$$

I. $\left(\frac{k+2}{2}-1\right)^2+\frac{k^2}{4}-2=0$

$$2\frac{k^2}{4}-2=0$$

$$\frac{k^2-4}{4}=0 \Rightarrow k=\pm 2$$

$$T_1=[0,-1,0],\quad T_2=[2,1,0]$$

II. $\left(\frac{-k+2}{2}-1\right)^2+\frac{k^2}{4}-2=0$

$$2\cdot\frac{k^2}{4}-2=0 \Rightarrow k=\pm 2$$

$$T_3=[2,1,0],\quad T_4=[0,-1,0]$$

τ_1: $\underline{x+y+1=0}$

τ_2: $+(x\cancel{z}-2)+(y-1)=0$

$$+x-2+y-1=0$$

$$\underline{x+y-3=0}$$

τ_3: $-(x-2)-(y-1)=0$

$$-x-y+3=0$$

$$\underline{x+y-3=0}$$

τ_4: $-(x-0)-(y\cancel{z}+1)=0$

$$-x-y-1=0$$

$$\underline{x+y+1=0}$$

• Najděte tečnu a obecnou rovnici normálové roviny prostorové křivky

1. $k: x = t^4 + t^2 + 1$

$y = 4t^3 + 5t + 2$

$z = t^4 - t$ v b. $T = [3, -7, 2]$

$3 = t^4 + t^2 + 1 \Rightarrow t^4 + t^2 - 2 = 0 \Rightarrow (t^2+2)(t^2-1) = 0 \Rightarrow t = \pm 1$

$-7 = 4t^3 + 5t + 2$

$2 = t^4 - t$ 1. $\underline{t_0 = -1}$: $2 = 1 + 1$ ✓

2. $t = 1$: $2 \neq 1 - 1$ X

$x' = 4t^3 + 2t$, $x'(t_0) = -4 - 2 = -6$

$y' = 12t^2 + 5$, $y'(t_0) = 12 + 5 = 17$

$z' = 4t^3 - 1$, $z'(t_0) = -4 - 1 = -5$ $\Rightarrow \vec{r}'(t_0) = \vec{r}(-1) = (-6, 17, -5)$

• tečna: $x = 3 - 6t$

$y = -7 + 17t$

$z = 2 - 5t;\ t \in \mathbb{R}$

• normálová rovina: $\rho: -6(x-3) + 17(y+7) - 5(z-2) = 0$

$-6x + 18 + 17y + 119 - 5z + 10 = 0$

$-6x + 17y - 5z + 147 = 0$

$\underline{\underline{6x - 17y + 5z - 147 = 0}}$