Určete první a druhé parciální derivace v bodě $A = [1,1]$ funkce dané implicitně rovnicí $xy + y^3 - 2x^2 = 0$.

Řešení!

$$x\,y(x) + y^3(x) - 2x^2 = 0$$

$$\Rightarrow \quad y(x) + x\,y'(x) + 3y^2(x)\,y'(x) - 4x = 0$$

$$A = [1,1]: \quad \underbrace{y(1)}_{1} + 1\underbrace{y'(1)}_{?} + 3\underbrace{y^2(1)}_{1}\underbrace{y'(1)}_{?} - 4\cdot 1 = 0$$

$$4y'(1) = 3 \quad \Rightarrow \quad \underline{\underline{y'(1) = \frac{3}{4}}}$$

$$\Rightarrow \quad y'(x) + y'(x) + x\,y''(x) + 3\cdot 2y(x)\,y'(x)\cdot y'(x) + 3y^2(x)\,y''(x) - 4 = 0$$

$$A = [1,1],\; y'(1) = \frac{3}{4}:$$

$$\underbrace{y'(1)}_{\frac{3}{4}} + \underbrace{y'(1)}_{\frac{3}{4}} + 1\cdot\underbrace{y''(1)}_{?} + 6\underbrace{y(1)}_{1}\underbrace{y'(1)}_{\frac{3}{4}}\underbrace{y'(1)}_{\frac{3}{4}} + 3\underbrace{y^2(1)}_{1}\underbrace{y''(1)}_{?} - 4 = 0$$

$$4y''(1) = -\frac{3}{4} - \frac{3}{4} - 6\cdot\frac{3}{4}\cdot\frac{3}{4} + 4$$

$$4y''(1) = \frac{-24 - 54 + 64}{16}$$

$$y''(1) = -\frac{14}{16}\cdot\frac{1}{4}$$

$$\underline{\underline{y''(1) = -\frac{7}{32}}}$$

Určete první a druhé parciální derivace v bodě $A = [1,1,0]$ funkce dané implicitně rovnicí $x^2 + y^2 + z^2 - 2z = 0$.

Řešení: $z''_{xx}(A) = ?$, $z''_{xy}(A) = ?$, $z''_{yy}(A) = ?$

$z \to z(x,y)$: $\quad x^2 + y^2 + z^2(x,y) - 2z(x,y) = 0$

- $\frac{\partial F}{\partial x}$: $\quad 2x + 2z(x,y)\cdot z'_x(x,y) - 2z'_x(x,y) = 0$

$A = [1,1,0]$: $\quad 2\cdot 1 + 2\underbrace{z(1,1)}_{0}\cdot \underbrace{z'_x(1,1)}_{?} - 2\underbrace{z'_x(1,1)}_{?} = 0$

$-2z'_x(1,1) = -2 \Rightarrow \underline{\underline{z'_x(1,1) = 1}}$

- $\frac{\partial F}{\partial y}$: $\quad 2y + 2z(x,y)\cdot z'_y(x,y) - 2z'_y(x,y) = 0$

$A = [1,1,0]$: $\quad 2\cdot 1 + 2\underbrace{z(1,1)}_{0}\cdot \underbrace{z'_y(1,1)}_{?} - 2\underbrace{z'_y(1,1)}_{?} = 0$

$\underline{\underline{z'_y(1,1) = 1}}$

- $\frac{\partial^2 F}{\partial x^2}$: $\quad 2 + 2z'_x(x,y)\cdot z'_x(x,y) + 2z(x,y)\cdot z''_{xx}(x,y) - 2z''_{xx}(x,y) = 0$

$A = [1,1,0]$, $z'_x(1,1) = 1$:

$2 + 2\underbrace{z'_x(1,1)}_{1}\cdot \underbrace{z'_x(1,1)}_{1} + 2\underbrace{z(1,1)}_{0}\cdot \underbrace{z''_{xx}(1,1)}_{?} - 2\underbrace{z''_{xx}(1,1)}_{?} = 0$

$-2z''_{xx}(1,1) = -4 \Rightarrow \underline{\underline{z''_{xx}(1,1) = 2}}$

- $\frac{\partial^2 F}{\partial x \partial y}$: $\quad 2z'_y(x,y)\cdot z'_x(x,y) + 2z(x,y)z''_{xy}(x,y) - 2z''_{xy}(x,y) = 0$

$2\underbrace{z'_y(1,1)}_{1}\cdot \underbrace{z'_x(1,1)}_{1} + 2\underbrace{z(1,1)}_{0}\cdot \underbrace{z''_{xy}(1,1)}_{?} - 2\underbrace{z''_{xy}(1,1)}_{?} = 0$

$-2z''_{xy}(1,1) = -2 \Rightarrow \underline{\underline{z''_{xy}(1,1) = 1}}$

- $\frac{\partial^2 F}{\partial y^2}$: $\quad 2 + 2z'_y(x,y)\cdot z'_y(x,y) + 2z(x,y)z''_{yy}(x,y) - 2z''_{yy}(x,y) = 0$

$2 + 2\underbrace{z'_y(1,1)}_{1}\cdot \underbrace{z'_y(1,1)}_{1} + 2\underbrace{z(1,1)}_{0}\cdot \underbrace{z''_{yy}(1,1)}_{?} - 2\underbrace{z''_{yy}(1,1)}_{?} = 0$

$-2z''_{yy}(1,1) = -4 \Rightarrow \underline{\underline{z''_{yy}(1,1) = 2}}$