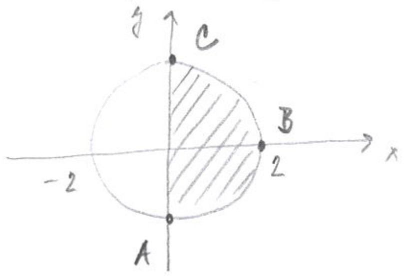
Najděte globální extrémy funkce $f(x,y) = x^2 + y^2 - 2y + 1$ na množině $M = \{[x,y] : x^2 + y^2 \leq 4,\ x \geq 0\}$



• stacionární body:

$$f'_x(x,y) = 2x \qquad 2x = 0$$
$$f'_y(x,y) = 2y - 2 \qquad 2y - 2 = 0$$
$$\Rightarrow x = 0,\ y = 1, \quad S_1 = [0,1] \in M$$

$$\boxed{f(S_1) = 0}$$

• stacionární body na hranici:

a) horní čtvrtkružnice: $y = \sqrt{4-x^2}$, $x \in \langle 0,2 \rangle$

$$f(x,y) = f(x, \sqrt{4-x^2}) = x^2 + 4 - x^2 - 2\sqrt{4-x^2} + 1 = 5 - 2\sqrt{4-x^2}$$

$$f'_x = -2 \cdot \frac{1}{2}(4-x^2)^{-\frac{1}{2}} \cdot (-2x) = \frac{2x}{\sqrt{4-x^2}} \Rightarrow \frac{2x}{\sqrt{4-x^2}} = 0 \Leftrightarrow x = 0 \Rightarrow y = 2$$

$H_1 = [0,2]$, $\boxed{f(H_1) = 1}$

b) dolní čtvrtkružnice: $y = -\sqrt{4-x^2}$, $x \in \langle 0,2 \rangle$

$$f(x,y) = f(x, -\sqrt{4-x^2}) = x^2 + 4 - x^2 + 2\sqrt{4-x^2} + 1 = 5 + 2\sqrt{4-x^2}$$

$$f'_x = 2 \cdot \frac{1}{2} \cdot \frac{-2x}{\sqrt{4-x^2}} = -\frac{2x}{\sqrt{4-x^2}} \Rightarrow -\frac{2x}{\sqrt{4-x^2}} = 0 \Leftrightarrow x = 0 \Rightarrow y = -2$$

$H_2 = [0,-2]$, $\boxed{f(H_2) = 9}$

c) $x = 0$, $y \in \langle -2, 2 \rangle$

$$f(x,y) = f(0,y) = y^2 - 2y + 1$$

$$f'_y = 2y - 2 \Rightarrow 2y - 2 = 0 \Rightarrow y = 1 \Rightarrow H_3 = [0,1],\ \boxed{f(H_3) = 0}$$

• společné body jednotlivých částí hranice:

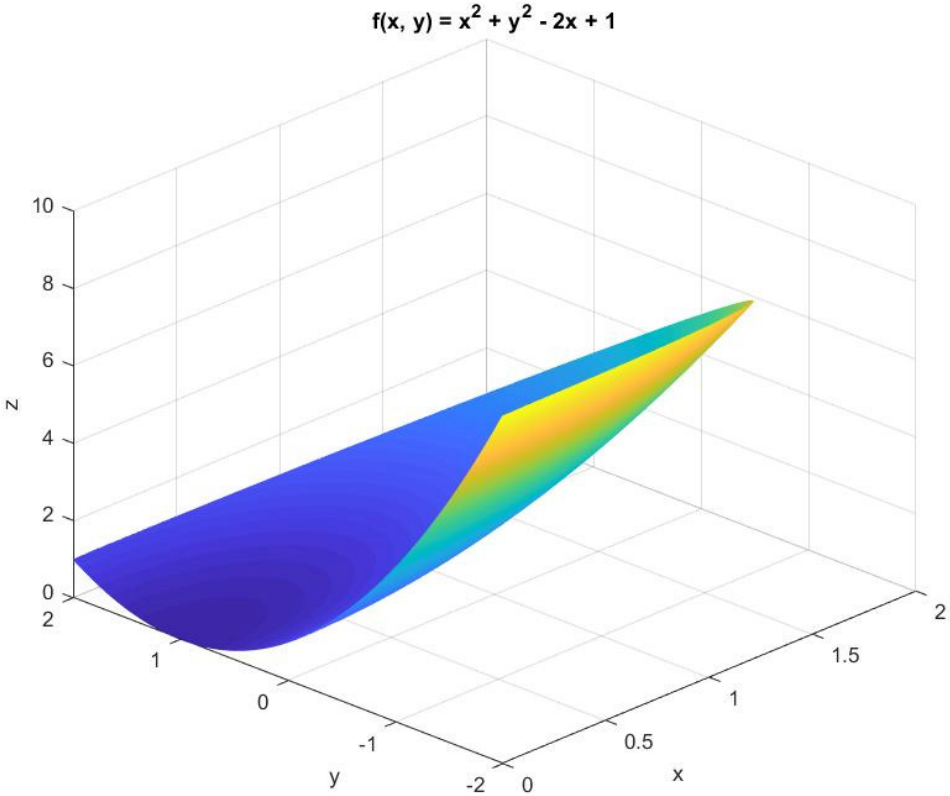
$A = [0,-2]$, $\boxed{f(A) = 9}$

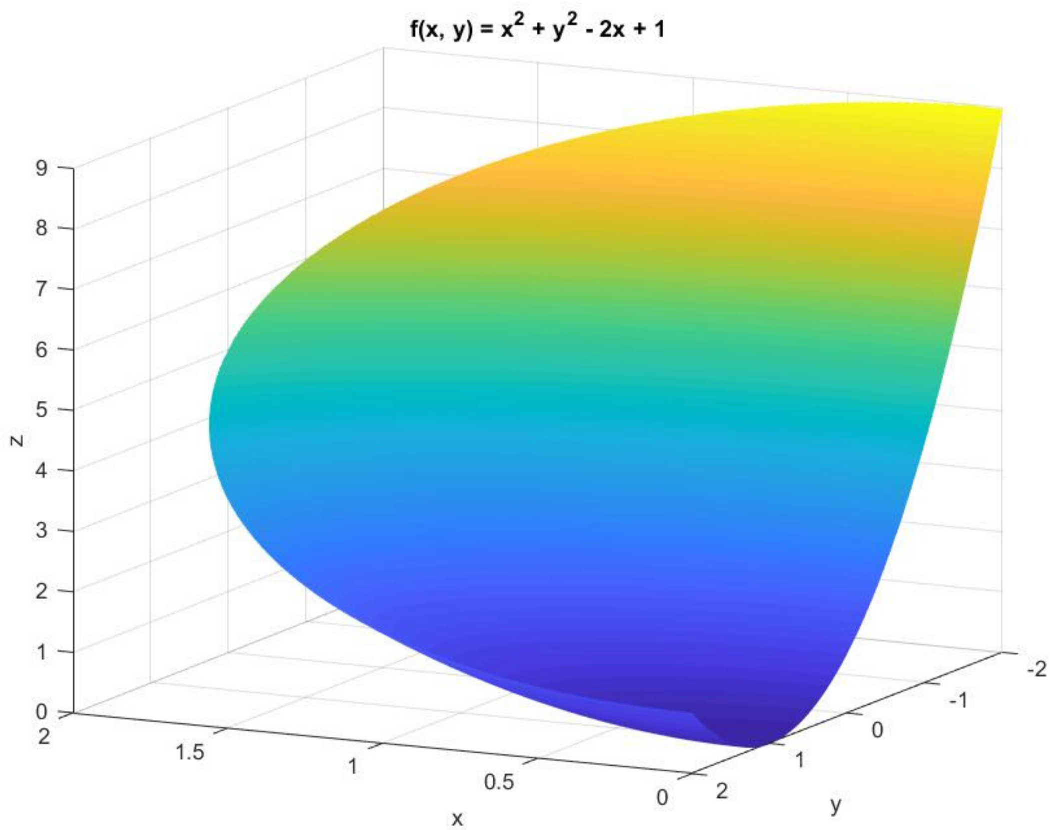
$B = [2,0]$, $\boxed{f(B) = 5}$

$C = [0,2]$, $\boxed{f(C) = 1}$

• absolutní minimum: $S_1 = H_3 = [0,1]$

absolutní maximum: $A = [0,-2]$


f(x, y) = x^2 + y^2 - 2x + 1
10
8
6
4
2
0
z
2
1
0
-1
-2
y
0
0.5
1
1.5
2
x


f(x, y) = x^2 + y^2 - 2x + 1
9
8
7
6
5
4
3
2
1
0
z
2
1.5
1
0.5
0
x
-2
-1
0
1
2
y