

GEOMETRICKÉ APLIKACE URČITÉHO INTEGRÁLU

	explicitní zadání	parametrické zadání
obsah rovinného obrazce	$P = \int_a^b (f(x) - g(x)) \, dx$ $f(x) \geq g(x) \quad \forall x \in (a, b)$ $P = \int_c^d (f(y) - g(y)) \, dy$ $f(y) \geq g(y) \quad \forall y \in (c, d)$	$P = \int_{\alpha}^{\beta} \psi(t) \varphi'(t) \, dt$ $\psi(t) \geq 0, \varphi'(t) \neq 0 \text{ pro } t \in (\alpha, \beta)$ $P = \int_{\alpha}^{\beta} \varphi(t) \psi'(t) \, dt$ $\varphi(t) \geq 0, \psi'(t) \neq 0 \text{ pro } t \in (\alpha, \beta)$
délka rovinné křivky	$L = \int_a^b \sqrt{1 + (f'(x))^2} \, dx$	$L = \int_{\alpha}^{\beta} \sqrt{(\varphi'(t))^2 + (\psi'(t))^2} \, dt$
objem rotačního tělesa	$V_x = \pi \int_a^b f^2(x) \, dx$ $V_y = \pi \int_c^d g^2(y) \, dy$	$V_x = \pi \int_{\alpha}^{\beta} \psi^2(t) \varphi'(t) \, dt$ $\psi(t) \geq 0, \varphi'(t) \neq 0$ $V_y = \pi \int_{\alpha}^{\beta} \varphi^2(t) \psi'(t) \, dt$ $\varphi(t) \geq 0, \psi'(t) \neq 0$
obsah rotační plochy	$P_x = 2\pi \int_a^b f(x) \sqrt{1 + (f'(x))^2} \, dx$ $P_y = 2\pi \int_c^d g(y) \sqrt{1 + (g'(y))^2} \, dy$	$P_x = 2\pi \int_{\alpha}^{\beta} \psi(t) \sqrt{(\varphi'(t))^2 + (\psi'(t))^2} \, dt$ $\psi(t) \geq 0, t \in (\alpha, \beta)$ $P_y = 2\pi \int_{\alpha}^{\beta} \varphi(t) \sqrt{(\varphi'(t))^2 + (\psi'(t))^2} \, dt$ $\varphi(t) \geq 0, t \in (\alpha, \beta)$

APLIKACE URČITÉHO INTEGRÁLU V MECHANICE (hmotnost, statický moment, moment setrvačnosti soustavy hmotných bodů)

• ROVINNÁ DESKA

	homogenní	nehomogenní
těžisté	$m = \sigma \int_a^b (f(x) - g(x)) \, dx$ $S_x = \frac{1}{2} \sigma \int_a^b (f^2(x) - g^2(x)) \, dx$ $S_y = \sigma \int_a^b x (f(x) - g(x)) \, dx$	$m = \int_a^b \sigma(x) (f(x) - g(x)) \, dx$ $S_x = \frac{1}{2} \int_a^b \sigma(x) (f^2(x) - g^2(x)) \, dx$ $S_y = \int_a^b \sigma(x) x (f(x) - g(x)) \, dx$ $T = [x_T, y_T] = \left[\frac{S_y}{m}, \frac{S_x}{m} \right]$
momenty setrvačnosti	$I_x = \frac{1}{3} \sigma \int_a^b (f^3(x) - g^3(x)) \, dx$ $I_y = \sigma \int_a^b x^2 (f(x) - g(x)) \, dx$	$I_x = \frac{1}{3} \int_a^b \sigma(x) (f^3(x) - g^3(x)) \, dx$ $I_y = \int_a^b \sigma(x) x^2 (f(x) - g(x)) \, dx$

• ROVINNÝ OBLOUK

	homogenní	nehomogenní
ROVINNÝ OBLOUK - explicitní vyjádření		
těžiště	$m = \sigma \int_a^b \sqrt{1 + (f'(x))^2} dx$ $S_x = \sigma \int_a^b f(x) \sqrt{1 + (f'(x))^2} dx$ $S_y = \sigma \int_a^b x \sqrt{1 + (f'(x))^2} dx$ $T = [x_T, y_T] = \left[\frac{S_y}{m}, \frac{S_x}{m} \right]$	$m = \int_a^b \sigma(x) \sqrt{1 + (f'(x))^2} dx$ $S_x = \int_a^b \sigma(x) f(x) \sqrt{1 + (f'(x))^2} dx$ $S_y = \int_a^b \sigma(x) x \sqrt{1 + (f'(x))^2} dx$
momenty setrvačnosti	$I_x = \sigma \int_a^b f^2(x) \sqrt{1 + (f'(x))^2} dx$ $I_y = \sigma \int_a^b x^2 \sqrt{1 + (f'(x))^2} dx$	$I_x = \int_a^b \sigma(x) f^2(x) \sqrt{1 + (f'(x))^2} dx$ $I_y = \int_a^b \sigma(x) x^2 \sqrt{1 + (f'(x))^2} dx$
ROVINNÝ OBLOUK - parametrické vyjádření		
těžiště	$m = \sigma \int_\alpha^\beta \sqrt{(\varphi'(t))^2 + (\psi'(t))^2} dt$ $S_x = \sigma \int_\alpha^\beta \psi(t) \sqrt{(\varphi'(t))^2 + (\psi'(t))^2} dt$ $S_y = \sigma \int_\alpha^\beta \varphi(t) \sqrt{(\varphi'(t))^2 + (\psi'(t))^2} dt$ $T = [x_T, y_T] = \left[\frac{S_y}{m}, \frac{S_x}{m} \right]$	$m = \int_\alpha^\beta \sigma(t) \sqrt{(\varphi'(t))^2 + (\psi'(t))^2} dt$ $S_x = \int_\alpha^\beta \sigma(t) \psi(t) \sqrt{(\varphi'(t))^2 + (\psi'(t))^2} dt$ $S_y = \int_\alpha^\beta \sigma(t) \varphi(t) \sqrt{(\varphi'(t))^2 + (\psi'(t))^2} dt$
momenty setrvačnosti	$I_x = \sigma \int_\alpha^\beta \psi^2(t) \sqrt{(\varphi'(t))^2 + (\psi'(t))^2} dt$ $I_y = \sigma \int_\alpha^\beta \varphi^2(t) \sqrt{(\varphi'(t))^2 + (\psi'(t))^2} dt$	$I_x = \int_\alpha^\beta \sigma(t) \psi^2(t) \sqrt{(\varphi'(t))^2 + (\psi'(t))^2} dt$ $I_y = \int_\alpha^\beta \sigma(t) \varphi^2(t) \sqrt{(\varphi'(t))^2 + (\psi'(t))^2} dt$