

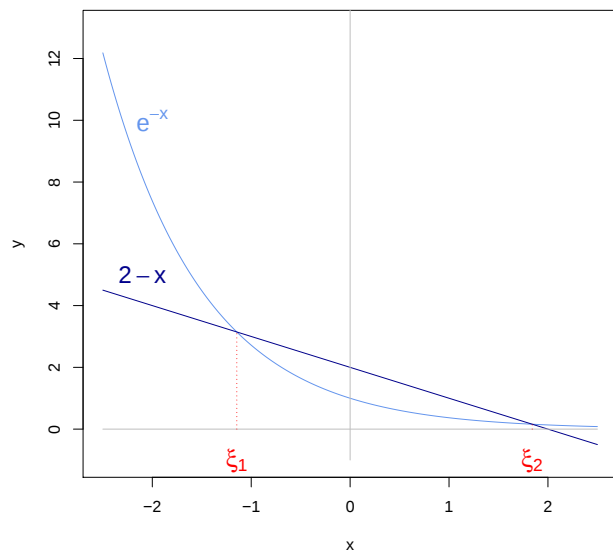
Příklad. Najděte všechny kořeny rovnice $x + e^{-x} - 2 = 0$ s přesností 0.01

- a) metodou bisekce,
- b) metodou regula falsi.

Řešení.

- hrubý odhad intervalu, určení počátečních aproximací:

$$\begin{aligned}x + e^{-x} - 2 &= 0 \\ e^{-x} &= 2 - x\end{aligned}$$



$$f(x) = x + e^{-x} - 2$$

- interval pro odhad záporného kořene:

$$\xi_1 \in \langle -2, -1 \rangle$$

$$\left. \begin{aligned}f(-2) &= 3.3891 \\ f(-1) &= -0.2817\end{aligned} \right\} \quad f(-2) \cdot f(-1) < 0$$

- interval pro odhad kladného kořene:

$$\xi_2 \in \langle 0, 2 \rangle$$

$$\left. \begin{aligned}f(0) &= -1 \\ f(2) &= 0.1353\end{aligned} \right\} \quad f(0) \cdot f(2) < 0$$

- a) metoda bisekce:

$$s_i = \frac{1}{2}(a_i + b_i)$$

$$d_i = \frac{1}{2}(b_i - a_i)$$

STOP kritérium: $d_i < \varepsilon$

- odhad záporného kořene:

odhad počtu kroků:

$$\begin{aligned}\frac{b_0 - a_0}{2^{i+1}} &< \varepsilon \\ \frac{1}{2^{i+1}} &< 0.01 \\ \frac{1}{0.01} &< 2^{i+1} \\ 100 &< 2^{i+1} \\ \log_2 100 &< i + 1 \\ i + 1 &> 6.6439 \\ i &> 5.6439 \\ i &= 6\end{aligned}$$

tabulka hodnot:

i	a_i	b_i	s_i	d_i	$f(a_i)$	$f(b_i)$	$f(s_i)$
0	-2	-1	-1.5	0.5	3.3891	-0.2817	0.9817
1	-1.5	-1	-1.25	0.25	0.9817	-0.2817	0.2403
2	-1.25	-1	-1.125	0.125	0.2403	-0.2817	-0.0448
3	-1.25	-1.125	-1.1875	0.0625	0.2403	-0.0448	0.0914
4	-1.1875	-1.125	-1.1563	0.0313	0.0914	-0.0448	0.0217
5	-1.1563	-1.125	-1.1406	0.0156	0.0217	-0.0448	-0.0119
6	-1.1563	-1.1406	-1.1484	0.0078 < 0.01	0.0217	-0.0119	0.0048

odhad záporného kořene:

$$\hat{x}_1 = -1.1484 \pm 0.0078$$

- odhad kladného kořene:

tabulka hodnot:

odhad počtu kroků:

$$\begin{aligned} \frac{b_0 - a_0}{2^{i+1}} &< \varepsilon \\ \frac{2}{2^{i+1}} &< 0.01 \\ \frac{2}{0.01} &< 2^{i+1} \\ 200 &< 2^{i+1} \\ \log_2 200 &< i + 1 \\ i &> 7.6439 - 1 \\ i &= 7 \end{aligned}$$

i	a_i	b_i	s_i	d_i	$f(a_i)$	$f(b_i)$	$f(s_i)$
0	0	2	1	1	-1	0.1353	-0.6321
1	1	2	1.5	0.5	-0.6321	0.1353	-0.2769
2	1.5	2	1.75	0.25	-0.2769	0.1353	-0.0762
3	1.75	2	1.875	0.125	-0.0762	0.1353	0.0284
4	1.75	1.875	1.8125	0.0625	-0.0762	0.0284	-0.0243
5	1.8125	1.875	1.8438	0.0313	-0.0243	0.0284	0.0020
6	1.8125	1.8438	1.8281	0.0156	-0.0243	0.0020	-0.0112
7	1.8281	1.8438	1.8359	0.0078	-0.0112	0.0020	-0.0046

odhad kladného kořene:

$$\hat{x}_2 = 1.8359 \pm 0.0078$$

b) metoda regula falsi:

$$x_i = a_{i-1} - f(a_{i-1}) \frac{b_{i-1} - a_{i-1}}{f(b_{i-1}) - f(a_{i-1})}, i=1,2,\dots$$

STOP kritérium: $|f(x_i)| < \varepsilon$

- odhad záporného kořene:

i	a_i	b_i	x_i	$f(a_i)$	$f(b_i)$	$f(x_i)$
0	-2	-1	—	3.3891	-0.2817	—
1	-2	-1	-1.0767	3.3891	-0.2817	-0.1416
2	-2	-1.0767	-1.1138	3.3891	-0.1416	-0.0679
3	-2	-1.1138	-1.1312	3.3891	-0.0679	-0.0318
4	-2	-1.1312	-1.1393	3.3891	-0.0318	-0.0148
5	-2	-1.1393	-1.1430	3.3891	-0.0148	$ -0.0068 < 0.01$

odhad záporného kořene:

$$\hat{x}_2 = -1.1430 \pm 0.0068$$

- odhad kladného kořene:

i	a_i	b_i	x_i	$f(a_i)$	$f(b_i)$	$f(x_i)$
0	0	2	—	-1	0.1353	—
1	0	2	1.7616	-1	0.1353	-0.0666
2	1.7616	2	1.8403	-0.0666	0.1353	$ -0.001 < \varepsilon$

odhad kladného kořene:

$$\hat{x}_2 = 1.8403 \pm 0.001$$