HOW TO CHECK STEEL-CONCRETE COMPOSITE BEAM?

...worked examples for CO001

WORKED EXAMPLE 1: BEAM WITH TRAPEZOIDAL SHEET

Secondary beam is composed of IPE section (S235) and concrete (C25/30) slab of 50 mm high on trapezoidal sheet VSŽ 11002 which is oriented perpendicular to the IPE beam. Shear force is transferred using shear studs Köco 19/75.

Geometry

\[
\begin{align*}
L &= 6000 \text{ mm} & \text{secondary beam span} \\
I &= 1800 \text{ mm} & \text{secondary beam spacing} \\
h_c &= 50 \text{ mm} & \text{concrete slab high} \\
h_p &= 50 \text{ mm} & \text{trapezoidal sheet high} \\
h_a &= 200 \text{ mm} & \text{IPE beam high}
\end{align*}
\]
**Loads**

**Dead loads**

IPE weight \[ g_a = A_a \cdot 78.50 = 2848.10^{-6} \cdot 78.50 = 0.22 \text{ kN/m} \]

Trapezoidal sheet weight \[ g_{\text{trap}} = 0.22 \text{ kN/m} \]

Concrete slab weight \[ g_c = h_{\text{eq}} \cdot 25 \cdot l = 0.08225 \cdot 25 \cdot 1.8 = 3.70 \text{ kN/m} \]

where equivalent concrete slab high \( h_{\text{eq}} \) is constant high of concrete slab of cross section area equal to cross section area of concrete slab concreted into trapezoidal sheet according to the figure below.

Overall dead load \[ g_k = g_a + g_{\text{trap}} + g_c = 0.22 + 0.22 + 3.7 = 4.14 \text{ kN/m} \]

**Imposed loads**

Service load on ceiling \[ q_{\text{serv}} = 3.0 \text{ kN/m}^2 \]

\[ q_k = q_{\text{serv}} \cdot l = 3.0 \cdot 1.8 = 5.40 \text{ kN/m} \]

**Loads combinations**

For SLS \[ f_k = 1.0 \cdot g_k + 1.0 \cdot q_k = 4.14 + 5.4 = 9.54 \text{ kN/m} \]

For ULS \[ f_d = 1.35 \cdot g_k + 1.5 \cdot q_k = 1.35 \cdot 4.14 + 1.5 \cdot 5.4 = 13.69 \text{ kN/m} \]

**Internal forces**

\[ M_{\text{Ed}} = \frac{1}{8} \cdot f_d \cdot L^2 = \frac{1}{8} \cdot 13.69 \cdot 6^2 = 61.61 \text{ kNm} \] bending moment

\[ V_{\text{Ed}} = \frac{1}{2} \cdot f_d \cdot L = \frac{1}{2} \cdot 13.69 \cdot 6 = 41.08 \text{ kN} \] shear force
**Alternative a: Elastic solution**

**Shear resistance**

Shear resistance of composite beam is the same as for steel IPE section – shear resistance of concrete slab is neglected. Shear stress in IPE web should not exceed shear strength of steel.

\[
\tau_{Ed} = \frac{V_{Ed} \cdot S_y}{I_y \cdot t_w} = \frac{41.08 \cdot 10^3 \cdot 104.8 \cdot 10^3}{19.43 \cdot 10^6 \cdot 5.6} = 39.57 \text{ MPa}
\]

where

\( t_w \) is web thickness

\( I_y \) is second moment of area

\( S_y \) is first moment of area for horizontal cut in the centroidal axis

**Reliability criterion**

\[
\frac{\tau_{Ed}}{f_y / \sqrt{3}} \cdot \frac{235 / \sqrt{3}}{\gamma_{Mo} \cdot 1.0} \leq 0.29 \leq 1.0
\]

\( \Rightarrow \text{condition is satisfied} \)

**Bending resistance**

Normal stress in IPE flanges should not exceed yield strength of steel and normal stress in concrete slab should not exceed compressive strength of concrete. The calculation is carried out with effective width of concrete slab \( b_{eff} \) due to shear lug.

\[
b_{eff} = b_0 + \sum b_{ei} = 0 + 750 + 750 = 1500 \text{ mm}
\]

where

\( b_{e1} = b_{e2} = l_0 / 8 = 6000 / 8 = 750 \text{ mm} \leq b_1 = b_2 = 900 \text{ mm} \)

where \( l_0 = L \) for simple beam

The composite section is substituted by all-steel section with reduced slab width using \( n \) ratio.

\[
n = \frac{E_a}{E_c} = \frac{210}{15} = 14.0
\]
where

\[ E_s = 210 \text{ GPa} \quad \text{Young's modulus of steel} \]

\[ E_c = \frac{E_{cm}}{2} = \frac{30}{2} = 15 \text{ GPa} \quad \text{commonly used simplification to take into account long term effects} \]

Ideal (equivalent) all-steel section:

Cross section area of equivalent (ideal) all-steel section

\[ A_i = A_s + A_c = 2848 + \frac{75000}{14} = 8205 \text{ mm}^2 \]

where

\[ A_c = b_{eff} \cdot h_c = 1500 \cdot 50 = 75000 \text{ mm}^2 \quad \text{is cross section area of concrete slab} \]

\[ A_s = 2848 \text{ mm}^2 \quad \text{is cross section area of IPE beam} \]

First moment of area of equivalent all-steel section

\[ S_i = S_s + \frac{S_c}{n} = 569.6 \cdot 10^3 + \frac{1875.10^3}{14} = 703.5 \cdot 10^3 \text{ mm}^3 \]

where

\[ S_s = A_s \cdot z_s = 2848 \cdot 200 = 569.6 \cdot 10^3 \text{ mm}^3 \quad \text{is first moment of steel part to the upper fibres of slab} \]

\[ S_c = A_c \cdot z_c = 75000 \cdot 25 = 1875.10^3 \text{ mm}^3 \quad \text{first moment of concrete part to the upper fibres of slab} \]
Position of centroid axis of equivalent all-steel section

\[ z_{\text{top}} = \frac{S_i}{A_i} = \frac{703.5 \times 10^3}{8205} = 85.7 \text{ mm} \]

Second moment of area (moment of inertia) of equivalent all-steel section

\[ I_i = I_{a,i} + I_{c,i} = \frac{292.4 \times 10^6}{14} = 77.50 \times 10^6 \text{ mm}^4 \]

where

\[ I_{a,i} = I_a + A_a \cdot z_a^2 = 19.43 \times 10^6 + 2848 \times 114.3^2 = 56.61 \times 10^6 \text{ mm}^4 \]

is moment of inertia of steel part

\[ I_{c,i} = I_c + A_c \cdot z_c^2 = 15.63 \times 10^6 + 75000 \times 60.7^2 = 292.4 \times 10^6 \text{ mm}^4 \]

is moment of inertia of concrete part

\[ I_a = 19.43 \times 10^6 \text{ mm}^4 \]

moment of inertia of IPE to the own centroid axis (taken from static tables)

\[ I_c = \frac{1}{12} \cdot b_{\text{eff}} \cdot h_c^3 = \frac{1}{12} \cdot 1500 \cdot 50^3 = 15.63 \times 10^6 \text{ mm}^4 \]

moment of inertia of concrete slab to the own centroid axis

\[ \sigma_1 = \frac{M_{\text{Ed}}}{I_i} \cdot z_1 = \frac{61.61 \times 10^6}{77.50 \times 10^6} \cdot 214.3 = 170.4 \text{ MPa} \]

\[ \sigma_2 = \frac{M_{\text{Ed}}}{I_i} \cdot z_2 = \frac{61.61 \times 10^6}{77.50 \times 10^6} \cdot 14.3 = 11.4 \text{ MPa} \]

\[ \sigma_3 = \frac{1}{n} \cdot \frac{M_{\text{Ed}}}{I_i} \cdot z_3 = \frac{1}{14} \cdot \frac{61.61 \times 10^6}{77.50 \times 10^6} \cdot (-35.7) = -2.03 \text{ MPa} \]

\[ \sigma_4 = \frac{1}{n} \cdot \frac{M_{\text{Ed}}}{I_i} \cdot z_4 = \frac{1}{14} \cdot \frac{61.61 \times 10^6}{77.50 \times 10^6} \cdot (-85.7) = -4.87 \text{ MPa} \]

\[ b_{\text{eff}} = 1500 \text{ mm} \]
Criterion for steel part

\[
\frac{\sigma_i}{f_{\gamma_{Mo}}} = \frac{170,4}{235/1,0} = 0,73 \leq 1,0
\]

=> condition is satisfied

Criterion for concrete part

\[
\frac{\sigma_4}{f_{\gamma_c}} = \frac{4,87}{20/1,5} = 0,37 \leq 1,0
\]

=> condition is satisfied

Shear studs

Will be added

Deflection in serviceability limit state

Overall deflection of composite beam depends on the way of erectionning. In the case of erectionning without intermediate supports some load cases act on steel IPE section with no composite action and other load cases act on composite section.

Load cases acting on steel IPE section: weight of concrete mixture, trapezoidal sheet weight and IPE weight

\[
f_{k,1} = g_a + g_{\text{trap}} + g_c = 0,22 + 0,22 + 3,7 = 4,14 \text{ kN/m}
\]

\[
w_1 = \frac{5}{384} \cdot \frac{f_{k,1} \cdot L^4}{E_a \cdot I_a} = \frac{5}{384} \cdot \frac{4,14 \cdot 6000^4}{210.10^3 \cdot 93.10^6} = 17,1 \text{ mm}
\]

Load cases acting on composite section: imposed loads

\[
f_{k,2} = q_k = 5,40 \text{ kN/m}
\]

\[
w_2 = \frac{5}{384} \cdot \frac{f_{k,2} \cdot L^4}{E_i \cdot I_i} = \frac{5}{384} \cdot \frac{5,40 \cdot 6000^4}{210.10^3 \cdot 77,50.10^6} = 4,3 \text{ mm}
\]

Overall deflection:

\[
w = w_1 + w_2 = 17,1 + 4,3 = 21,4 \text{ mm}
\]

Limit value of deflection:

\[
w_{\text{lim}} = \frac{L}{250} = \frac{6000}{250} = 24 \text{ mm}
\]

Criterion for deflection

\[
\frac{w}{w_{\text{lim}}} = \frac{21,4}{24} = 0,89 \leq 1,0
\]

=> condition is satisfied
Alternative b: Plastic solution

Shear resistance

Shear resistance of composite beam is the same as for steel IPE section – shear resistance of concrete slab is neglected.

\[
V_{pl,Rd} = \frac{A_v \cdot f_y}{\gamma_{M0}} = \frac{1025 \cdot 235}{1,0} = 235 \text{ kN}
\]

where
\(A_v\) is shear affected area of IPE = web area

Reliability criterion

\[
\frac{V_{Ed}}{V_{pl,Rd}} = \frac{41,08}{139,07} = 0,30 \leq 1,0 \quad => \text{condition is satisfied}
\]

Bending resistance

Effective width of concrete slab due to shear lug

\[
b_{eff} = b_0 + \sum b_{ei} = 0 + 750 + 750 = 1500 \text{ mm}
\]

where

\[b_{e1} = b_{e2} = l_0/8 = 6000/8 = 750 \text{ mm} \leq b_{1} = b_{2} = 900 \text{ mm}\]

where \(l_0 = L\) for simple beam

High of compressed part of concrete slab is calculated from equation where normal force in IPE section is equal to normal force in concrete slab for assumption that neutral axis passes through concrete slab.

\[
F_a = F_c \quad A_a \cdot f_{yd} = A_c \cdot 0,85 \cdot f_{cd}
\]

from there

\[
x = \frac{A_a \cdot f_{yd}}{0,85 \cdot b_{eff} \cdot f_{cd}} = \frac{2848 \cdot 235}{0,85 \cdot 1500 \cdot 13,3} = 39 \text{ mm}
\]

where
The design of composite beams is discussed. The shear stud dimensions and characteristic bearing resistance are calculated. The reliability criterion is also analyzed.
Design bearing resistance for trapezoidal sheet oriented perpendicular to the IPE beam

\[
P_{rd} = \frac{P_{rk} \cdot k_t}{\gamma_v} = \frac{80.24 \cdot 0.85}{1.25} = 54.56 \text{ kN}
\]

where \( k_t \) is factor for concrete slabs concreted into trapezoidal sheet oriented perpendicular to the IPE beam

\[
k_t = \frac{0.7}{\sqrt{n_t}} \cdot \frac{b_0}{h_p} \left( \frac{h_{sc}}{h_p} - 1 \right) = \frac{0.7}{\sqrt{1}} \cdot \frac{129}{50} \cdot \left( \frac{75}{50} - 1 \right) = 0.903 \geq k_{t,\text{max}} = 0.85 \quad \Rightarrow \quad k_t = 0.85
\]

Overall shear force between IPE section and concrete slab

\[
N_{cf} = \min \left\{ \frac{A_s \cdot f_{yd}}{A_c \cdot 0.85 \cdot f_{yd} + A_c \cdot f_{yd}} \right\} = \min \left\{ \frac{2848 \cdot 235}{39 \cdot 1500 \cdot 0.85 \cdot 13.3 + 0} \right\} = \min \left\{ \frac{669.28}{669.28} \right\} = 669.28 \text{ kN}
\]

Minimum number of shear studs for full composite action

\[
n_t = \frac{N_{cf}}{P_{rd}} = \frac{669.28}{54.56} = 12.3
\]

Number of shear studs which is possible (maximal realizable number) to put onto beam is influenced by shape of trapezoidal sheet. The wave length of VSŽ 11002 is 200 mm, thus shear studs spacing is 200 mm too.

\[
n_{\text{max}} = \frac{L/2}{l_{\text{wave}}} = \frac{6000/2}{200} = 15
\]

\( \Rightarrow \) full composite action

Other way

Maximal spacing of shear studs for full composite action

\[
b_{\text{max}} = \frac{L/2}{n_t} = \frac{6000/2}{12.3} = 243.9 \text{ mm} \geq l_{\text{wave}} = 200 \text{ mm}
\]

\( \Rightarrow \) full composite action

Deflection in serviceability limit state

The same as for elastic solution

\[
\frac{w}{w_{\text{lim}}} = \frac{21.4}{24} = 0.89 \leq 1.0
\]

\( \Rightarrow \) condition is satisfied