Stochastic nonlinear fracture mechanics finite element analysis of concrete structures

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Keywords: Statistical simulation, random fields, concrete, nonlinear fracture mechanics, stochastic finite element analysis

ABSTRACT: The paper summarizes the main outcomes achieved within the framework of the research project "Nonlinear fracture mechanics of concrete using stochastic finite elements and random fields". The project focused on randomization of nonlinear finite element analysis of concrete structures. Theoretical as well as practical application aspects are presented emphasizing the conceptual framework and key points of the solution. Efficient techniques of both nonlinear numerical analysis of concrete structures and stochastic simulation methods of Monte Carlo type have been combined in order to offer an advanced tool for the assessment of the real behavior of concrete structures from statistical and reliability points of view.

1 INTRODUCTION

The transparent and easily understandable concept is the reliability calculation of structures from the stochastically obtained structural resistance and expected load distribution. The stochastic response requires repeated analyses of the structure with random input parameters, which reflects randomness and uncertainties in the input values. The system should use the nonlinear computer simulation for realistic prediction of structural response and its resistance. As the nonlinear structural analysis is computationally very demanding, a suitable technique of statistical sampling should be used, which allows a relatively small number of simulations. Final results are: statistical characteristics of response (stresses, deflections, crack width etc.), information on dominating and non-dominating variables (sensitivity analysis) and estimation of reliability using reliability index and/or theoretical failure probability.

The first aim of the paper is to describe briefly computational methods to simulate:

- uncertainties
- nonlinear behavior of concrete

New and/or updated (significantly improved) theoretical methods that have to be developed, verified and implemented are itemized as follows:

Simulation of uncertainties:

 Small-sample simulation of Monte Carlo type Latin hypercube sampling for both random variables and random fields

- Imposing statistical correlation using the simulated annealing approach
- Small number of random variables to represent random fields based on spectral decomposition of covariance matrix
- Sensitivity analysis based on nonparametric rank-order statistical correlation

Nonlinear fracture mechanics simulation:

- Damage mechanics, fracture mechanics and plasticity theories
- Smeared crack approach, crack band method
- Softening of concrete in both tension and compression
- Combination of nonlinear concrete behavior with discrete and smeared reinforcement in reinforced concrete and pre-stressed structures
- Advanced material models; SBETA (Červenka 2003), fracture-plastic model, microplane model.

The last part of the paper informs on the most interesting applications of the system including both the practical statistical, reliability analysis of complex real structures and the simulation of the laboratory experiments.

The methods were integrated within the complex software system SARA (Pukl et al. 2003ab, Novák at al. 2002, Bergmeister at al. 2004). The system represents a combination of statistical simulation package FREET (Novák et al. 2003, 2005) and nonlinear fracture mechanics software ATENA (Červenka & Pukl 2003, Červenka 2003).

2 STOCHASTIC TECHNIQUES FOR UNCERTAINTIES SIMULATION

2.1 Small-sample simulation of Monte Carlo type

For time-intensive calculations like nonlinear fracture mechanics of concrete, the small-sample simulation techniques based on stratified sampling of Monte Carlo type represent a rational compromise between feasibility and accuracy. Therefore Latin hypercube sampling (LHS) was selected as a key fundamental technique.

The method belongs to the category of stratified simulation methods (e.g. Mc Kay & Conover 1979, Novák et. al 1998). It is a special type of the Monte Carlo simulation which uses the stratification of the theoretical probability distribution function of input random variables. It requires a relatively small number of simulations to estimate statistics of response – repetitive calculations of the structural response (tens or hundreds).

The basic feature of LHS is that the probability distribution functions for all random variables are divided into N_{Sim} equivalent intervals (N_{Sim} is a number of simulations); the values from the intervals are then used in the simulation process (random selection, middle of interval or mean value). This means that the range of the probability distribution function of each random variable is divided into intervals of equal probability. The samples are chosen directly from the distribution function based on an inverse transformation of distribution function. The renresentative parameters of variables are selected randomly, being based on random permutations of integers 1, 2, ..., j, N_{Sim}. Every interval of each variable must be used only once during the simulation. Being based on this precondition, a table of random permutations can be used conveniently, each row of such a table belongs to a specific simulation and the column corresponds to one of the input random variables.

It has been proved that best LHS strategy, which simulates the means and variances very well, is the approach suggested by Keramat & Kielbasa (1997) and Huntington & Lyrintzis (1998). The mean of each interval should be chosen as (Fig. 1):

$$x_{i,k} = \frac{\int_{y_{i,k-1}}^{y_{i,k}} x \cdot f_i(x) \, dx}{\int_{y_{i,k-1}}^{y_{i,k}} f_i(x) \, dx} = N_{Sim} \cdot \int_{y_{i,k-1}}^{y_{i,k}} x \cdot f_i(x) \, dx \tag{1}$$

where f_i is PDF of the variable X_i , and the integration limits are:

$$y_{i,k} = F_i^{-1} \left(\frac{k}{N_{Sim}} \right) \tag{2}$$



Figure 1. Illustration of sampling.

The estimated mean value is achieved accurately and the variance of the sample set is much closer to the target one. For some probability density functions (inclusive e.g. Gaussian, Exponential, Laplace, Rayleigh, Logistic, Pareto, etc.) the integral (1) can be solved analytically. For others, the extra effort of doing the numerical integration is definitely worthwhile.

2.2 Imposing statistical correlation

Once samples are generated, the correlation structure according to the target correlation matrix must be taken into account. There are generally two problems related to the statistical correlation: First, during sampling an undesired correlation can occur between the random variables. For example, instead of the correlation coefficient zero for the uncorrelated random variables, i.e. an undesired correlation, can be generated. It can happen especially in the case of a very small number of simulations (tens), where the number of interval combination is rather limited. The second task is to introduce the prescribed statistical correlation between the random variables defined by the correlation matrix. The columns in LHS simulation plan should be rearranged in such a way that they may fulfill the following two requirements: to diminish the undesired random correlation and to introduce the prescribed correlation. It can be done by using different techniques published in literature on LHS (e.g. Huntington & Lyrintzis 1998, Iman & Conover, 1982) but we found some serious limitations while using them.

A robust technique to impose statistical correlation based on the stochastic method of optimization called simulated annealing has been proposed recently by Vořechovský & Novák (2003). The imposition of the prescribed correlation matrix into the sampling scheme can be understood as an optimization problem: The difference between the prescribed K and the generated S correlation matrices should be as small as possible. A suitable measure of quality of the overall statistical properties can be introduced, e.g. a norm which utilizes the deviations of all correlation coefficients:

$$E_{overall} = \sqrt{\sum_{i=1}^{N_{v}-1} \sum_{j=i+1}^{N_{v}} (S_{i,j} - K_{i,j})^{2}}$$
(3)

The norm *E* has to be minimized from the point of view of the definition of the optimization problem using simulated annealing optimization approach, N_{ν} random variables realizations are related to the ordering in the sampling scheme.

2.3 Simulation of random fields

At higher level of uncertainties modeling, the spatial variability of mechanical and geometrical properties of a system and intensity of load should be represented by means of random fields. Because of the discrete nature of the finite element formulation, the random field must also be discretized into random variables. This process is commonly known as random field discretization. The computational effort in reliability problem generally increases with the number of random variables. Therefore it is desirable to use small number of random variables to represent a random field. To achieve this goal, the transformation of the original random variables into a set of uncorrelated random variables can be performed through a well-known eigenvalue orthogonalization procedure. It is demonstrated that a few of these uncorrelated variables with largest eigenvalues are sufficient for the accurate representation of the random field.

Let us consider the fluctuating components of the homogenous random field, which is assumed to model the material property variation around its expected value. Correlation characteristics can be specified in terms of the covariance matrix *C*_{aa} constructed by discretization using autocorrelation function and geometry of FEM mesh. An eigenvalue orthogonalization procedure will transform variables into uncorrelated space:

$$\mathbf{C}_{\mathbf{X}\mathbf{X}} = \mathbf{\Phi}\mathbf{\Lambda}\mathbf{\Phi}^{\mathrm{T}} \tag{4}$$

The covariance matrix in the uncorrelated space Y is diagonal matrix $\Lambda = C_{yy}$. Then the vector of uncorrelated Gaussian random variables Y can be simulated in the traditional way (Monte Carlo simulation). The transformation back into correlated space yields the vector X using eigenvectors Φ :

$$\mathbf{X} = \mathbf{\Phi} \mathbf{Y} \tag{5}$$

The utilization of LHS method for simulation of Gaussian uncorrelated variables is the new simple idea of improvement of random field simulation using orthogonal transformation of covariance matrix suggested by Novák et al. (2000). The superiority of this stratified technique remains here also for accurate representation of random field, thus leading to the decrease of number of simulations needed. This was proved numerically by Vořechovský & Novák (2005).

2.4 Sensitivity analysis

An important task in the structural reliability analysis is to determine the significance of random variables. With respect to the small-sample simulation techniques described above the straightforward and simple approach uses the non-parametric rank-order statistical correlation between the basic random variables and the structural response variable (Iman & Conover 1980, Novák et al. 2004). The sensitivity analysis is obtained as an additional result of LHS, and no additional computational effort is necessary.

The relative effect of each basic variable on the structural response can be measured using the partial correlation coefficient between each basic input variable and the response variable. The method is based on the assumption that the random variable which influences the response variable most considerably (either in a positive or negative sense) will have a higher correlation coefficient than the other variables. Because the model for the structural response is generally nonlinear, a non-parametric rank-order correlation is used by means of the Spearman correlation coefficient or Kendall tau.

2.5 Reliability analysis

In cases when we are constrained by small number of simulations (tens, hundreds) it can be difficult to estimate the failure probability. The following approaches are therefore utilized here; they are approximately ordered from elementary (extremely small number of simulations, inaccurate) to more advanced techniques.

- Cornell's reliability index the calculation of reliability index from the estimation of the statistical characteristics of the safety margin
- The curve fitting approach based on the selection of the most suitable probability distribution of the safety margin.
- FORM approximation (Hasofer-Lind's index)
- · Importance sampling techniques
- Response surface methods

These approaches are well known in reliability literature and also providing all details is beyond the aim of this paper. In spite of the fact that the calculation of the failure probability (or/and reliability index) using some of these techniques does not always belong to the category of very accurate reliability techniques (first three in the list), they represent a feasible alternative in many practical cases.

3 NONLINEAR TECHNIQUES AND MATERIAL MODELS FOR CONCRETE

An algorithm for nonlinear analysis is based on three basic parts: Finite element technique, constitutive model, and nonlinear solution methods, which should compose a balanced approximation. Nevertheless, the constitutive models decide about the material behavior, and therefore they are treated here more extensively. Advanced techniques are implemented in finite element software for realistic computer simulation of damage and failure of concrete and reinforced concrete structures (Červenka 2000, 2002). Since concrete is a complex material with strongly nonlinear response, special constitutive models for the finite element analysis of concrete structures are employed. The constitutive relation in a material point (constitutive model) plays the most crucial role in the finite element analysis and decides how the structural model represents reality. Only the most important features and material models are referenced here.

3.1 Crack band method and smeared crack approach

Tensile behavior of concrete is modeled by nonlinear fracture mechanics combined with the crack band method and smeared crack concept, Fig. 2. Main material parameters are tensile strength, fracture energy and shape of the stress-crack opening curve. A real discrete crack is simulated by a band of localized strains. The crack strain is related to the element size (localization limiter). Consequently, the softening law in terms of strains for the smeared model is calculated for each element individually, while the crack-opening law is preserved. This model is objective due to the energy formulation and its dependency on the finite element mesh size is negligible, which was confirmed by numerous studies (e.g. Červenka & Pukl 1995).



Figure 2. Smeared crack model for tensile behavior of concrete.

3.2 Material model SBETA

The concrete in plane stress condition can be well described by a damage model. The model is based on the equivalent uniaxial law, which covers the complete range of the plane stress behavior in tension and compression. The effect of biaxial stress state on the concrete strength is captured by the biaxial failure function, Fig. 3. For the tensile response (cracking) the crack band method described above is applied. Similar method is applied for the compressive softening. Next important features of the model are: reduction of compressive strength after cracking, tension stiffening effect, reduction of the shear stiffness after cracking, fixed and rotated cracks models.



Figure 3. Biaxial failure function and equivalent uniaxial law.

3.3 Fracture-plastic model

This constitutive material model for concrete combines the plasticity with fracture (Červenka & Červenka 1999). The fracture is modeled by an orthotropic smeared crack model based on Rankine tensile criterion. Hardening-softening plasticity model based on Menétrey & Willam (1995) threeparameter failure surface is used to model concrete crushing. The model differs from the other published formulations, exhibits the ability to handle also physical changes like for instance crack closure, and it is not restricted to any particular shape of hardening/softening laws.

3.4 Microplane model

The basic idea of the microplane model is to abandon constitutive modeling in terms of tensors and their invariants and formulate the stress-strain relation in terms of stress and strain vectors on planes of various orientations in the material, now generally called the microplanes. The microplane model M4 (Bažant at al. 2000) is implemented into the finite element package, it represents the most advanced material model available for modeling of quasibrittle failure of concrete in the package.

4 SELECTED TYPES OF APPLICATIONS

4.1 Probabilistic analyses of concrete structures

The presented approach has been successfully used for probabilistic nonlinear analysis of concrete structures (Bergmeister et al. 2004, Pukl et al. 2003ab), A practical example of stochastic failure simulation and reliability assessment of existing bridge structure was a cantilever beam bridge on the Brennero highway in Italy with a length of 167.5 m, Fig. 4. For the reliability assessment of the Colle d'Isarco bridge the resistance (maximum line load capacity) with mean value of 235 kN/m and standard deviation of 18 kN/m was obtained from statistical simulation considering the randomness of materials including their statistical correlation. The Cornell's reliability index as a function of the mean line load is plotted in Fig, 5 for different COV of load. The horizontal line represents the target reliability index 4.7 as specified by Eurocode (2001) for 1 year.



Figure 4. Colle d'Isarco bridge. Brennero highway, Italy.



Figure 5. Reliability index vs. load.

A single span bridge located in Vienna was assessed by Pukl et al. (2002). It is a fully posttensioned box-girder bridge made of 18 segments with the lengths of 2.485 m each. The segments were cast from B500 concrete and are reinforced with St 50 mild steel. The post-tensioning tendons consist of 20 strands St 160/180. The total length of the bridge is 44.60 m, the width 6.40 m and the height 2.10 m. Due to the ongoing construction project the bridge has to be demolished. Before the demolition a range of non-destructive tests as well as finally a full-scale destructive load test were performed, Fig. 6. The stochastic simulation served as the predictive numerical study for planning the test set-up. The statistical properties of the selected random variables including statistical correlation were collected from several sources (9 variables for concrete, 4 for prestressing strands).

The application of random fields is very suitable for solution of soil-structure interaction tasks. The influence of spatial variation of Young modulus and material constants of Drucker-Prager criterion (based on cohesion and angle of internal friction) was studied. The stability of concrete tunnel tube in complicated geological conditions has been analyzed. The thickness of geological layers was between 10 and 25 m, the diameter of the tunnel tube was 11 m, the typical wall thickness 0.5 m. The whole analyzed part of the soil with tunnel had the dimensions of 50 x 60 m. It was solved in plane strain state and discretized in 5000 finite elements. Drucker-Prager plasticity was used for modeling of soil behavior. The spatial variability was simulated using Gaussian random fields with correlation length of 2 m. A model sample is illustrated in Fig. 7.



Figure 6. Full-scale destructive load test and virtual computational model.



Figure 7. Realization of random field of soil property around concrete tunnel.

4.2 Statistical size effect studies

The probabilistic simulation approach was used to capture the statistical size effect obtained from experiments. The probabilistic treatment of nonlinear fracture mechanics in the sense of extreme value statistics has been recently applied for two crack initiation problems which exhibits the Weibull-type statistical size effect: Dog-bone shaped concrete specimens in uniaxial tension (Lehký & Novák 2002), Fig. 8, and the size effect of four point bending plane concrete beams due to the bending span. (Novák et al. 2003, Bažant at al. 2004). The usage of system at level of random fields is illustrated for random crack initiation for four-point bending in Fig. 9 (regions with lower concrete strength are red), a bundle of random load-deflection curve for a particular size is presented in Fig. 10. Due to spatial randomness the mean of the peak load decreases comparing to the deterministic capacity. A new size effect law for crack initiation was verified using this approach too (Bažant et al. 2005).



Figure 8. Random crack initiation and size effect curves – experiment vs. simulation.



Figure 9. Four-point bending and patterns of random fields.



Figure 10. Random load – deflection curves (red curve – deterministic calculation).

4.3 Verification of (code) design formulas

The proposed approach can be efficiently used for the code verification and calibration purposes. The questions always arise concerning the correctness and reliability level of design code formulas vs. newly proposed formulas at academic level. Moreover, recent development introduced the significant inconsistency: Eurocode demands a nonlinear analysis using first mean values and second design values of material parameters. No real guaranties and information on safety can be obtained using the partial safety concept as accepted in the present design codes. The approach generally fails if the internal forces entering safety margin (failure criteria) are not proportional to the load level, as in case of complex nonlinear treatments.

One of the hot topics is certainly shear failure of reinforced concrete beams, where the size effect phenomenon plays an important role and is the target of research. The attempt was made to contribute to the discussion and to treat four different approaches (ACI 318 2002, Eurocode 2 2003, Bažant & Yu 2003, Collins & Kuchma 1999) with respect to a particular experimental data from Toronto University (Collins & Kuchma 1999) and virtual statistical simulation taking into account the randomness. Statistical distributions of nominal strengths for different sizes of beams were obtained using statistical simulation, Fig. 11 shows the scatter for different sizes and how the individual formulas follow the experiment. These first results indicate clearly that ACI approach is absolutely unsafe (naturally, as no size effect is considered) and Eurocode is on rather conservative side with respect to the proposed formulas by Bažant and Collins. The assessment of the result in the probabilistic manner can be done comparing experimental statistical distribution of nominal strength with values obtained from design formulas.



Figure 11. Nominal strength vs. size for different design alternatives: formulas, experiment and simulation.

4.4 Identification of material model parameters

The nonlinear numerical analysis requires the use of an appropriate and realistic material model. Generally, the more sophisticated model the more model parameters are needed. Basic parameters as compressive strength, modulus of elasticity, etc. are usually known. Typically, some other parameters can be estimated using the recommended formulas from literature, but in most cases these formulas can be used only as a first approximation of the parameters. The objective is very often to find such a set of material parameters, which gives the best agreement between the simulated and experimental (if available) loaddeflection curves.

The recently proposed identification strategy is based on a coupling of the stochastic nonlinear fracture mechanics analysis and the artificial neural network (Lehký & Novák 2004, Novák & Lehký 2004, Červenka et al. 2005, Strauss et al. 2004). Fundamental scheme of the approach is shown in Fig. 12, neural network is trained by values of loaddeflection curve and values of identified parameters (considered to be random variables) in repetitive stochastic way using stratified simulation.

An application of such identification approach is the shear wall shown in Fig. 13 (Červenka et al. 2005). Loading by the vertical force was applied first to represent a dead load. Then a horizontal force was applied and increased to failure. The behavior during the experiment reported extensive diagonal cracking prior to failure followed by an explosive crushing of concrete under the maximum load. The experimental and simulated failure and a bundle of load-deflection curves used to train neural network in order to provide best estimates of 10 material parameters are shown in Fig. 13.



Figure 12. Scheme for stochastic training of neural network.



Figure 13. Random load-deflection curve realizations – 20 simulations of LHS, experimental and virtual failure.

5 CONCLUSIONS

Efficient techniques of both nonlinear numerical analysis of concrete structures and stochastic simulation methods were combined in order to offer an advanced tool for the assessment of the real behavior of concrete structures from the statistical and reliability points of view. A wide range of applicability both practical and theoretical, as was shown by selected types of examples, gives an opportunity for further intensive development – bridging first theory and praxis, and second, reliability and nonlinear computation.

ACKNOWLEDGEMENTS

The present research was funded under the grant No.103/02/1030 from 2002 till 2004 and research of section 4.4 under consequent grant No.103/04/2092 from the Grant Agency of Czech Republic. The support is gratefully acknowledged by authors.

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