Small-sample probabilistic assessment - FREET software

D. Novák, M. Vořechovský & R. Rusina
Institute of Structural Mechanics, Brno University of Technology, Brno, Czech Republic

Keywords: Latin hypercube sampling, reliability software, sensitivity, failure probability, reliability index

ABSTRACT: The aim of the paper is to present methods for efficient structural reliability assessment and to describe a multi-purpose probabilistic software for statistical, sensitivity and reliability analyses of engineering problems. The emphasize is given to the techniques which are developed for an analysis of computationally intensive problems which is typical for a nonlinear FEM analysis. The paper shows the possibility of "randomization" of computationally intensive problems in the sense of the Monte Carlo type simulation. Latin hypercube sampling is used, in order to keep the number of required simulations at an acceptable level. Random variables are randomly generated under their probability distribution functions, statistical correlation among them is imposed by the optimization technique, called the simulated annealing.

1 INTRODUCTION

A large number of efficient stochastic analysis methods have been developed during last years. In spite of many theoretical achievements the acceptability and a routine application in industry is rare. Present reliability software developers should bridge the demands on the usage of effective reliability methods and easy and transparent use by an inexperienced user. Two main categories of stochastic approaches can be distinguished:

- Approaches focused on the calculation of statistical moments of response quantities, like estimation of means, variances etc. Uncertainties input into a response function.
- Approaches aiming at the calculation of estimation of theoretical probability of failure. Uncertainties input into a limit state function.

There are many different methods developed by reliability researchers covering both the approaches. The common feature of all the methods is the fact that they require a repetitive evaluation (simulations) of the response or limit state function. The development of reliability methods is from the historical perspective a struggle for decreasing or to avoiding an excessive number of simulation, eg. Schuëller (1997) etc. From this point of view of the computational demands the major groups of the methods belonging to the first category are as follows:

- Crude Monte Carlo simulation,
- Stratified sampling techniques,
- Perturbation techniques.

The methods of next category for reliability calculations can be distinguished as:

- Crude Monte Carlo simulation,
- Advanced simulation techniques, like importance sampling, adaptive sampling, directional sampling etc. (e.g. Schuëller et al. 1989),
- Approximation techniques,FORM, SORM etc. (e.g. Hasofer & Lind 1974) ,
- Response surface methodologies (e.g. Bucher & Bourgund 1987),
- Curve fitting techniques - evaluating samples of safety margin (e.g. Grigoriu 1983, Li & Lumb 1985).

The lists above are ordered according to the computational demands (the number of simulation required) from the top downwards. There is a decreasing accuracy of the methods in the same direction. In the case of computationally intensive problems there is an overlapping domain of methods, feasible to apply. These techniques are implemented in many different alternatives in reliability software, e.g. COMREL (RCP Munich), VaP (ETH Zurich), SLANG (Weimar University), M-Star (UTAM Prague), Crystall Ball (Decisionerring. Inc.), PROBAN (DNV software) , COSSAN (Innsbruck University).

The intent of the paper is to describe a practical tools to assess response statistics and reliability.

Relevant techniques and the probabilistic software developed for easy handling of practical reliability problems are briefly described, the paper should be considered to have just an informative value.

2 SMALL-SAMPLE STOCHASTIC TECHNIQUE

2.1 *Latin hypercube sampling*

The Latin hypercube sampling (LHS) simulation technique belongs to the category of advanced simulation method (Mc Kay & Conover 1979, Novák & Kijawatworawet 1991). It is a special type of the Monte Carlo numerical simulation which uses the stratification of the theoretical probability distribution function of input random variables. It requires a relatively small number of simulations - repetitive calculations of the structural response resulting from an adopted computational model (tens or hundreds). The utilization of LHS strategy in a reliability analysis can be rather extensive. It is not restricted just for the estimation of the statistical parameters of a structural response. The following topics in which LHS can be applied are outlined as follows (for more details see Novák et. al 1998): Estimation of statistical parameters of a structural response, estimation of the theoretical failure probability, sensitivity analysis, response approximation, preliminary "rough" optimization, reliability-based optimization. There are two stages of LHS: samples for each variable are strategically chosen to represent the variables probability distribution; samples are ordered to match the required statistical correlations among variables.

Basic feature of the Latin hypercube sampling strategy: The probability distribution functions for all random variables are divided into N equivalent intervals (N is a number of simulations); the centroids of intervals are then used in the simulation process. This means that the range of the probability distribution function of each random variable is divided into intervals of equal probability. The samples are chosen directly from the distribution function, ie. by using the formula

$$x_{i,k} = \Phi_i^{-1}(\frac{k-0.5}{N}) \qquad (1)$$

where $x_{i,k}$ is the k-th sample of the i-th variable X_i , Φ_i^{-1} is the inverse CPDF for variable X_i .

The representative parameters of variables are selected randomly, being based on random permutations of integers $1, 2, ..., j, N$. Every interval of each variable must be used only once during the simulation. Being based on this precondition, a table of random permutations can be used conveniently, each row of such a table belongs to a specific simulation and the column corresponds to one of the input random variables. It should be noticed that this approach gives samples with a mean close to the desired one while the sample variances might be significantly different.

In order to simulate the variables better to capture the means and variances correctly, the random mean of each section should be chosen (Huntington & Lyrintzis 1998) as

$$x_{i,k} = N \int_{y_{i,k-1}}^{y_{i,k}} x f_i(x)\, dx \qquad (2)$$

where f_i is the probability density function and the limits of integration are given by

$$y_{i,k} = \Phi_i^{-1}(\frac{k}{N}) \qquad (3)$$

The integral above may not be always solvable in the closed form. However, the extra effort to carry the numerical integration is justified by the statistical accuracy gained as it was shown by (Huntington & Lyrintzis 1998).

The second step is reordering LHS samples in such a way that they might match the prescribed statistical correlation as much as possible. It can be done by using different techniques, a robust technique based on the stochastic method of optimization called simulated annealing has been proposed recently by Vořechovský & Novák (2002). This theoretical part is described in details in a separate paper of this conference (Vořechovský & Novák 2003).

2.2 *Nonparametric rank-order sensitivity analysis*

An important task in the structural reliability analysis is to determine the significance of random variables - how they influence the response function of a specific problem. There are many different approaches of sensitivity analysis, a summary of the present methods is given by Novák et. al. (1993). The sensitivity analysis can answer the question "what variables are the most important?". In this way the dominating and non-dominating random variables can be distinguished.

One straightforward and simple approach utilizes the non-parametric rank-order statistical correlation between basic random variables and the structural response variable. The sensitivity analysis is obtained as an additional result in LHS, and no additional computational effort is necessary. The relative effect of each basic variable on the structural response can be measured using the partial correlation coefficient between each basic input variable and the response variable. The method is based on the assumption that the random variable which influences the response variable most considerably (either in a positive or negative sense) will have a higher correlation coefficient than the other variables. In case of a very weak influence the correlation coefficient will be quite close to zero. Because the model for the structural response is generally nonlinear, a non-parametric rank-order correlation is utilized. The key concept of the non-parametric correlation reads: Instead of the actual numerical values we consider the values of its rank among all the other values in the sample, that is $1, 2, ..., N$. Then the resulting list of numbers will be drawn from a perfectly known probability distribution function - the integers are uniformly distributed. In the case of the Latin hypercube sampling the representative values of random variables cannot be identical, therefore there is no necessity to consider mid-ranking. The non-parametric correlation is more robust than the linear correlation, more resistant to the defects in the data and also distribution independent. Therefore it is particularly suitable for the sensitivity analysis based on the Latin Hypercube Sampling. An advantage of this approach is the fact that the sensitivity measure for all random variables can be obtained directly within one simulation analysis. The rank-order statistical correlation can be expressed by the Spearman correlation coefficient or Kendall tau.

2.3 *Reliability estimation*

As it was already mentioned in the introduction, the number of simulations is a crucial point to calculate reliability. Therefore approximation technique of FORM/SORM type are still often utilized (starting by fundamental paper Hasofer & Lind 1974). In the case of an extremely small number of simulations there are the following feasible alternatives only: the calculation of reliability index from the estimation of the statistical characteristics of the safety margin (Cornell reliability index) and the curve fitting procedure. The first technique represents a simplification, and it is well known that it gives an exact result only for the normal distribution of the safety margin.

The curve fitting approach is based on the selection of the most suitable probability distribution of the safety margin. This selection is based on suitable statistical tests. When the mathematical model of probability distribution is selected, the failure probability is calculated as percentile. There are limitations of such approach (Novák & Kijawatworawet 1990).

Both the approaches are well known in reliability literature. In spite of the fact that the calculation of the failure probability using these elementary techniques does not belong to the category of very accurate reliability techniques, they represent a feasible alternative in many practical cases. In cases when we are constrained by an extremely small number of simulations.

3 SOFTWARE FREET

3.1 *General remarks*

The multipurpose probabilistic software for statistical, sensitivity and reliability analyses of engineering problems FREET (Novák et al. 2002) is based on the efficient reliability techniques described above. This probabilistic software has been recently successfully integrated with an advanced nonlinear fracture mechanics solution of concrete structures - the finite element program ATENA (Červenka & Pukl 2002). The modeling of uncertainties by random fields is under development using efficient simulation techniques (Novák et al. 2000).

FREET is now developed in two versions: the first version FREET M - the analysed response or limit state function is defined completely outside as a subroutine written in C++ or FORTRAN programming languages. This concept allows to work with complicated problems which have already been tested deterministically at the level of programming languages. In the second version FREET A - the ATENA computational model of nonlinear fracture mechanics for concrete is fully integrated.

Basic user's dialog parts in the present version of FREET, Figure 1, are described in the following subsections. All the steps of the probabilistic analysis will be illustrated using particular windows of software.

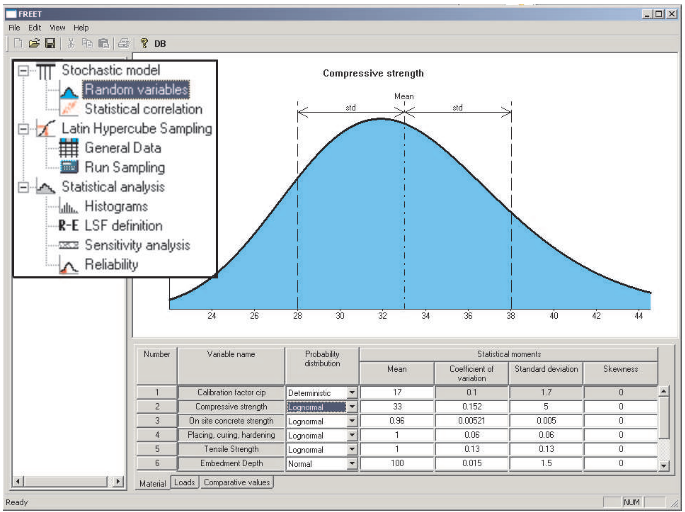

Figure 1: Basic software tools and window Random Variables.

3.2 *Stochastic Model*

The window "Random Variables" allows the user-friendly input of the basic random variables of the analysed problem. The uncertainties are modelled as random variables described by their probability density functions (PDF). The user can choose from the set of selected theoretical models like normal, lognormal, Weibull, rectangular, etc. Random variables are

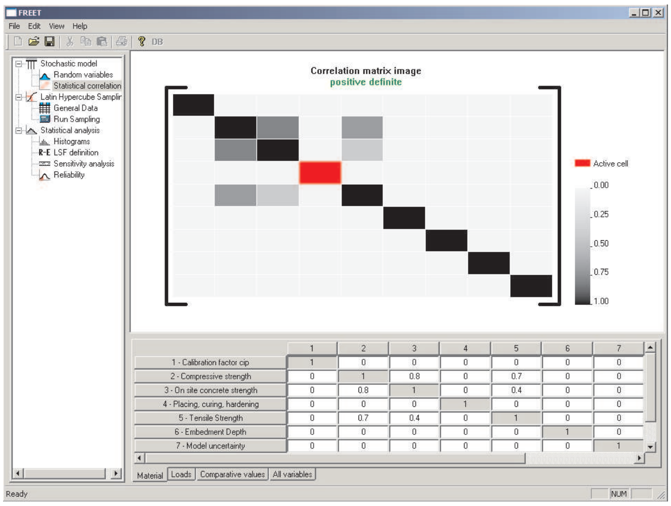

Figure 2: Window Statistical Correlation.

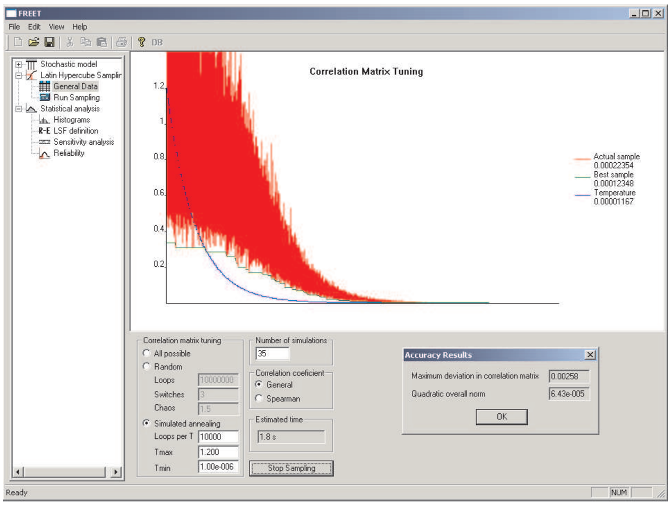

Figure 3: Imposing of statistical correlation by Simulated Annealing.

described by statistical characteristics (statistical moments): Mean value, standard deviation (or coefficient of variation) and higher statistical moments or statistical parameters respectively. Random variables can be divided into groups for easy handling in the case of large number of random variables (see the bottom of the window).

The window "Statistical Correlation" serves for the input of correlation matrix, Figure 2. The user can work at the level of a subsets of correlation matrices (one group of random variables) or at the global level (all random variables resulting in a large correlation matrix; groups are defined by user). The level of correlation during the interactive input is highlighted, the positive definiteness is checked. The result of this step is the defined set of input parameters for computational model - random variables.

3.3 *Sampling*

The sampling proceeds in two steps: First, the Latin hypercube samples are prepared without paying attention to the statistical correlation - spurious

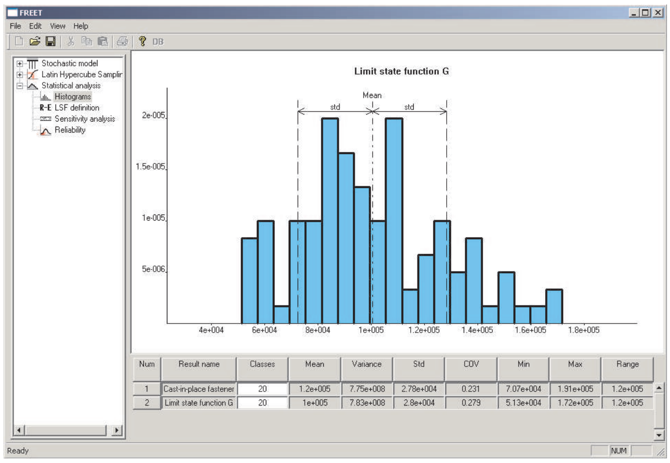

Figure 4: Basic statistical assessment - histogram and statistical characteristics.

sensitivity can be introduced. Second, the samples are reordered by the Simulated Annealing approach in order to match the required correlation matrix as close as possible. The basic parameter - the number of simulations of LHS is on input here. The random input parameters are generated according to their PDF using LHS sampling and the generated realisations of random parameters are used as inputs for the analysed function (a computational model). The deterministic solution is performed repeatedly and the results (e.g. a structural responses) are saved.

3.4 *Statistical Assessment*

At the end of the whole simulation process the resulting set of structural responses is statistically evaluated. The results are: histogram, mean value, variance, cumulative probability density function of structural response, sensitivity and reliability. This basic statistical assessment is visualised through the window Histograms, (Fig. 4).

3.5 *Sensitivity Analysis*

The window Sensitivity Analysis shows the importance of the random variables: positive or negative sensitivity, (Fig. 5). The parallel co-ordinates representation provides an insight into the analysed problem. The random variables are ordered with respect to the sensitivity expressed by the non-parametric rank-order correlation coefficient.

3.6 *Reliability*

The software allows to define the limit state function at the level of program unit in the user-selected programming language. The second option is to define response function only and then to add the so called

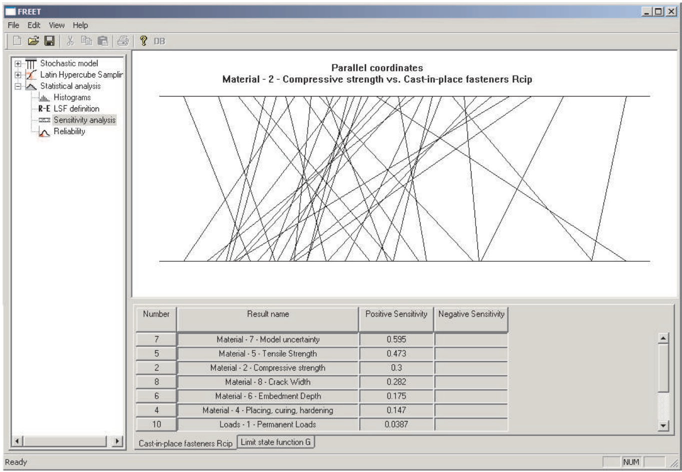

Figure 5: Window Sensitivity Analysis.

Figure 6: Window Reliability Analysis - Response and comparative value histograms.

comparative values later in FREET gui (step Stochastic model). It is possible to define any numbers of such comparative values and then to define any numbers of the limit state functions. Resistance vs. Load is visualized (Fig. 6) and the reliability index and the failure probability estimated (Fig. 7).

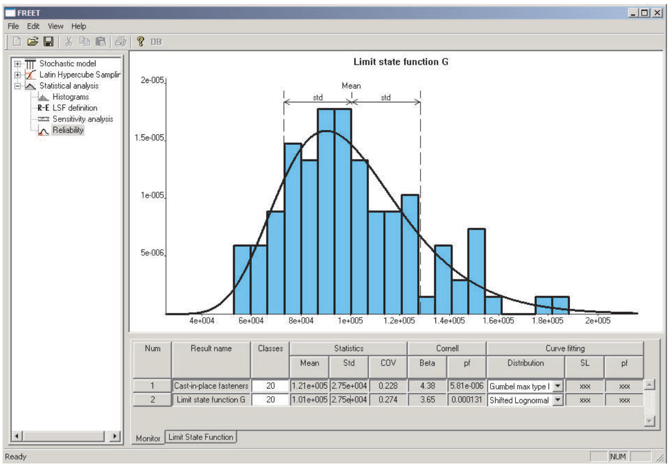

Figure 7: Window Reliability Analysis.

4 SELECTED APPLICATIONS

FREET software has been applied recently for the probabilistic analysis of problems from both the industrial and academic fields. Due to the limited space here, just the list of the selected applications is provided with short descriptive comments.

4.1 *Large concrete subway tunnel in Prague*

An extension of the subway system in Prague, Czech Republic, crosses the River Vltava. A couple of large concrete tunnels, which are curved in plan as well as in elevation, were cast in the dry dock excavated in the bank of the river. After casting the first tube, it was launched into the trench excavated in the river bed. Each tunnel was cast in segments 12 m long. The total number of 14 segments formed a large tube 168 m long. The outside width of the cross-section was 6.48 m and the height was 6.48 m too. The thickness of the walls and the top and the bottom slabs was about 0.7 m. During the launching the tube was suspended in one third of the length from its front on the pontoon and at the back end the tube was supported by the hydraulic telescopic sliding shoes. During the launching the tube was subjected to two actions; i) the self weight and ii) the bouyancy of the water. Both these actions exhibit a statistical scatter due to the difference in dimensions and in the density of concrete. The actual load is a difference of the two actions and therefore it is extremely sensitive to the variability of them - therefore the balancing forces were applied. There was a danger that imperfections might result in an out-of-control situation. A statistical feasibility study was, therefore, needed. The imperfections of the individual segments were modelled as random variables - the total number of variables used in the study was 221. The statistical simulation of this technological process aimed at the decision-making process before launching and thus assessing the reliability of the entire project.

4.2 *Reliability assessment of box-girder bridge in Vienna*

The single span bridge located in Vienna was assessed by Pukl et al. in 2002. It is a fully post-tensioned box-girder bridge made of 18 segments with lengths of 2.485 m each. The segments were cast from B500 concrete and are reinforced with St 50 mild steel. The post-tensioning tendons consist of 20 strands St 160/180. The total length of the bridge is 44.60 m, the width 6.40 m and the height 2.10 m. Due to the ongoing construction project the bridge has to be demolished. Before the demolition a range of non-destructive tests as well as finally a full-scale destructive load test will be performed. The stochastic simulation served as the predictive numerical study for planning and optimizing the test set-up. The statistical properties of the selected random variables including statistical correlation were collected from several sources (9 variables for concrete, 4 for prestressing strands). A statistical simulation of the load-deflection diagram and, consequently, a reliability calculation have been performed.

4.3 *Statistical size effect studies*

The FREET software integrated with the ATENA software were used to capture the statistical size effect obtained from experiments. Probabilistic treatment of nonlinear fracture mechanics in the sense of extreme value statistics has been recently applied for two crack initiation problems which exhibits Weibull-type the statistical size effect: Dog-bone shaped concrete specimens in uniaxial tension, (Lehký & Novák 2002 and size effect of four point bending plane concrete beams due to bending span, (Novák et al. 2003). In both cases a good agreement of numerically simulated mean size effect curve and an experimental curve have been achieved.

5 CONCLUSIONS

The paper briefly describes the small-sample simulation technique suitable for statistical, sensitivity and reliability analyses of computationally intensive problems. The FREET software development and its recent applications are documented. The software is designed in the form suitable for a relatively easy assessment of any user-defined computational problem. It can be applied without serious difficulties even to realistic computationally demanding reliability problems.

ACKNOWLEDGEMENTS

The support from the Grant Agency of the Czech Republic under the grants No. 103/02/1030, No. 103/03/1035 and under project reg. No. CEZ: J22/98: 261100007 is acknowledged.

REFERENCES

Bažant, Z.P., Novák, D. & Vořechovský, M. 2003. Statistical size effect prediction in quasibrittle materials. *Proc., ICASP 9*, San Francisco, USA.

Bucher, C.G. & Bourgund, U. 1987. Efficient use of response surface methods. Ins. of Enginnering Mechanics, Innsbruck University, Report No. 9-87.

Červenka, V. & Pukl, R. 2002. *ATENA - Computer Program for Nonlinear Finite Element Analysis of Reinforced Concrete Structures*. Program documentation. Prague, Czech Republic: Červenka Consulting.

Grigoriu, M. 1983. Methods for approximate reliability analysis. *J. Structural Safety*, Vol. 1, 155-165.

Hasofer, A. M. & Lind, N. C. 1974. Exact and invariant second-moment code format. *Journal of Eng. Mech. Division, ASCE*, Vol. 100 (EM1), 111-121.

Huntington, D. E. & Lyrintzis, C. S. 1998. Improvements to and limitations of Latin hypercube sampling. *Probabilistic Engineering Mechanics*, Vol. 13, No. 4, 245-253.

Lehký D. & Novák, D. 2002. Statistical size effect of concrete in uniaxial tension. *4th PhD Symposium in Civil Engineering*, Munich, Germany.

Li, K. S. & Lumb, P. 1985. Reliability analysis by numerical integration and curve fitting. *J. Structural Safety*, Vol. 3, 29–36.

McKay, M. D., Conover, W. J. & Beckman, R. J. 1979. A comparison of three methods for selecting values of input variables in the analysis of output from a computer code. *T*echnometrics 21, 239–245.

Novák, D. & Kijawatworawet, W. 1990. *A comparison of accurate advanced simulation methods and Latin Hypercube Sampling method with approximate curve fitting to solve reliability problems*. Internal Working Report No. 34 - 90, Institute of Eng. Mechanics, University of Innsbruck, Austria.

Novák, D. et al. 2002. *FREET - Feasible Reliability Engineering Tool*. Program documentation, Brno University of Technology, Faculty of Civ. Engineering / Červenka Consulting, Prague, Czech Republic.

Novák, D., Teplý, B. & Shiraishi, N. 1993. Sensitivity analysis of structures: A Review. *Int Conf. CIVIL COMP"93*, Edinburgh, Scotland, August: 201-207.

Novák, D., Teplý, B. & Keršner, Z. 1998. The role of Latin Hypercube Sampling method in reliability engineering. *Structural Safety and Reliability*, Proc. of 7th Int. Conference ICOSSAR97, Kyoto, Japan, Balkema, Rotterdam.

Novák, D., Vořechovský, M., Pukl, R. & Červenka, V. 2000. Statistical nonlinear analysis - size effect of concrete beams. *Fracture Mechanics of Concrete and Concrete Structures*, Proc. of 4th Int. Conference FraMCoS, Cachan, France, Balkema.

Schuëller, G. I. 1997. Structural reliability — recent advances. (Freudenthal lecture) *Proc., ICOSSAR-97, Kyoto, Japan*, Balkema, Rotterdam, Vol. I, 3–35.

Schuëller, G. I., Bucher, C. G., Bourgund, U., Ouypornprasert, W. 1989. On efficient computational schemes to calculate structural failure probabilities. *Probabilistic Eng. Mechanics*, Vol. 4, No. 1, 10–18.

Pukl R., Strauss A., Novák, D.,Červenka V., & Bergmeister K. 2003. Probabilistic-based assessment of concrete structures using non-linear fracture mechanics. *Proc., ICASP 9*, San Francisco, USA.

Vořechovský & M., Novák, D. 2002. Correlated random variables in probabilistic simulation. *4th PhD Symposium in Civil Engineering*, Munich, Germany.

Vořechovský & Novák, D. 2003. Statistical correlation in stratified sampling. *Proc., ICASP 9*, San Francisco, USA.