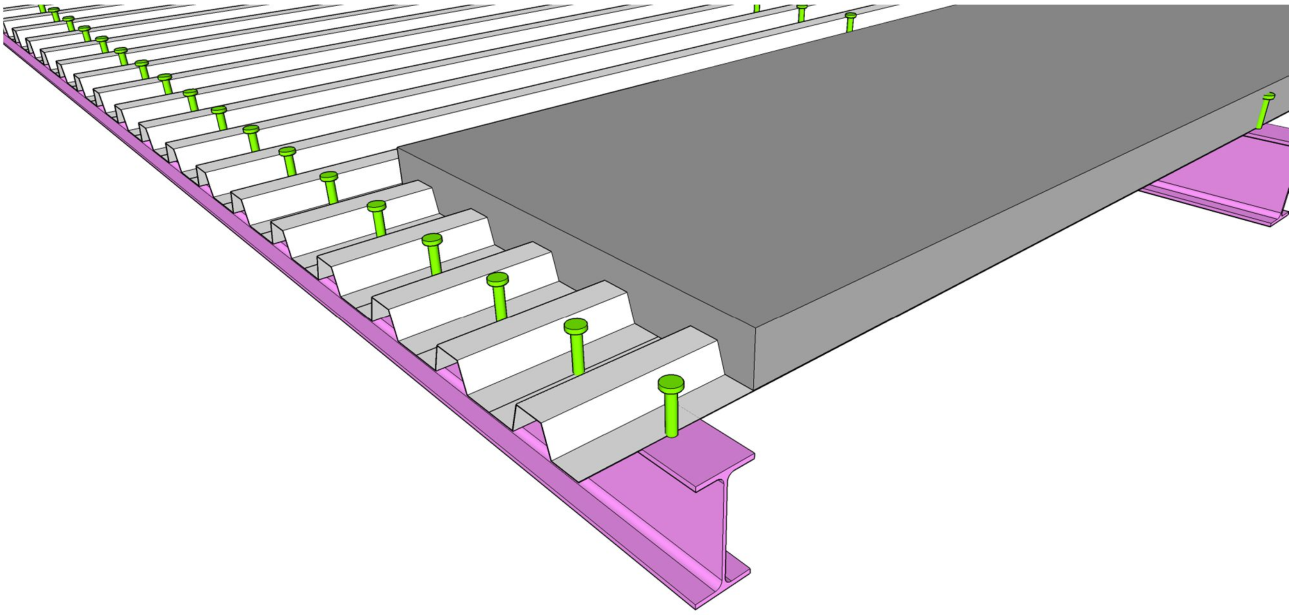
HOW TO CHECK STEEL-CONCRETE COMPOSITE BEAM?

…worked examples for CO001

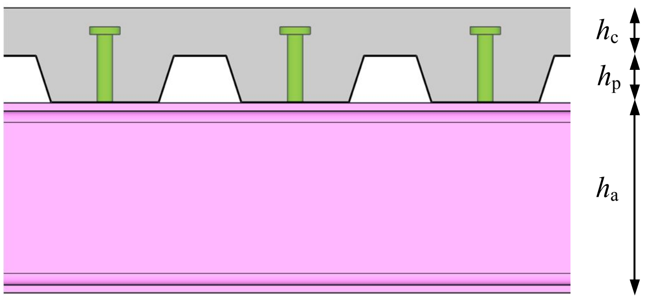
WORKED EXAMPLE 1: BEAM WITH TRAPEZOIDAL SHEET

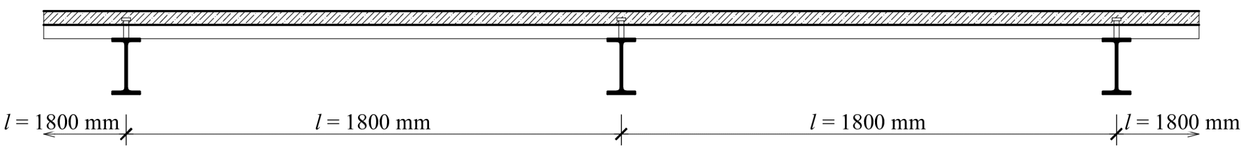
Secondary beam is composed of IPE section (S235) and concrete (C25/30) slab of 50 mm high on trapezoidal sheet VSŽ 11002 which is oriented perpendicular to the IPE beam. Shear force is transferred using shear studs Köco 19/75.



Geometry

L = 6000 mm	secondary beam span
l = 1800 mm	secondary beam spacing
h_c = 50 mm	concrete slab high
h_p = 50 mm	trapezoidal sheet high
h_a = 200 mm	IPE beam high





Loads

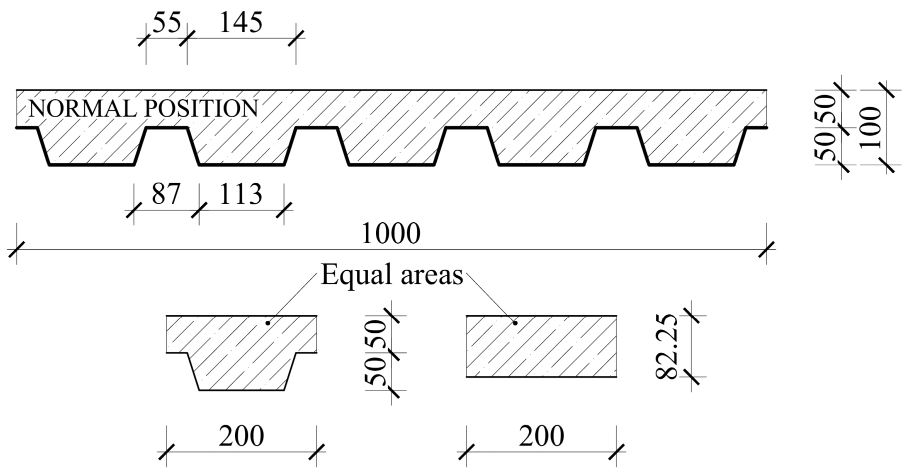
Dead loads

IPE weight $g_a = A_a \cdot 78{,}50 = 2848.10^{-6} \cdot 78{,}50 = 0{,}22 \text{ kN/m}$

Trapezoidal sheet weight $g_{trap} = 0{,}22 \text{ kN/m}$

Concrete slab weight $g_c = h_{eq} \cdot 25 \cdot l = 0{,}08225 \cdot 25 \cdot 1{,}8 = 3{,}70 \text{ kN/m}$

where equivalent concrete slab high h_{eq} is constant high of concrete slab of cross section area equal to cross section area of concrete slab concreted into trapezoidal sheet according to the figure below.



Overall dead load $g_k = g_a + g_{trap} + g_c = 0{,}22 + 0{,}22 + 3{,}7 = 4{,}14 \text{ kN/m}$

Imposed loads

Service load on ceiling $q_{serv} = 3{,}0 \text{ kN/m}^2$

$q_k = q_{serv} \cdot l = 3{,}0 \cdot 1{,}8 = 5{,}40 \text{ kN/m}$

Loads combinations

For SLS $f_k = 1{,}0 \cdot g_k + 1{,}0 \cdot q_k = 4{,}14 + 5{,}4 = 9{,}54 \text{ kN/m}$

For ULS $f_d = 1{,}35 \cdot g_k + 1{,}5 \cdot q_k = 1{,}35 \cdot 4{,}14 + 1{,}5 \cdot 5{,}4 = 13{,}69 \text{ kN/m}$

Internal forces

$$M_{Ed} = \frac{1}{8} \cdot f_d \cdot L^2 = \frac{1}{8} \cdot 13{,}69 \cdot 6^2 = 61{,}61 \text{ kNm}$$ bending moment

$$V_{Ed} = \frac{1}{2} \cdot f_d \cdot L = \frac{1}{2} \cdot 13{,}69 \cdot 6 = 41{,}08 \text{ kN}$$ shear force

Alternative a: Elastic solution

Shear resistance

Shear resistance of composite beam is the same as for steel IPE section – shear resistance of concrete slab is neglected. Shear stress in IPE web should not exceed shear strength of steel.

$$\tau_{\mathrm{Ed}} = \frac{V_{\mathrm{Ed}} \cdot S_{\mathrm{y}}}{I_{\mathrm{y}} \cdot t_{\mathrm{w}}} = \frac{41{,}08.10^{3} \cdot 104{,}8.10^{3}}{19{,}43.10^{6} \cdot 5{,}6} = 39{,}57 \text{ MPa}$$

where

t_{w} is web thickness

I_{y} is second moment of area

S_{y} is first moment of area for horizontal cut in the centroidal axis

Reliability criterion

$$\frac{\tau_{\mathrm{Ed}}}{\frac{f_{\mathrm{y}}/\sqrt{3}}{\gamma_{\mathrm{M0}}}} = \frac{39{,}57}{\frac{235/\sqrt{3}}{1{,}0}} = 0{,}29 \quad \leq \quad 1{,}0$$ => ***condition is satisfied***

Bending resistance

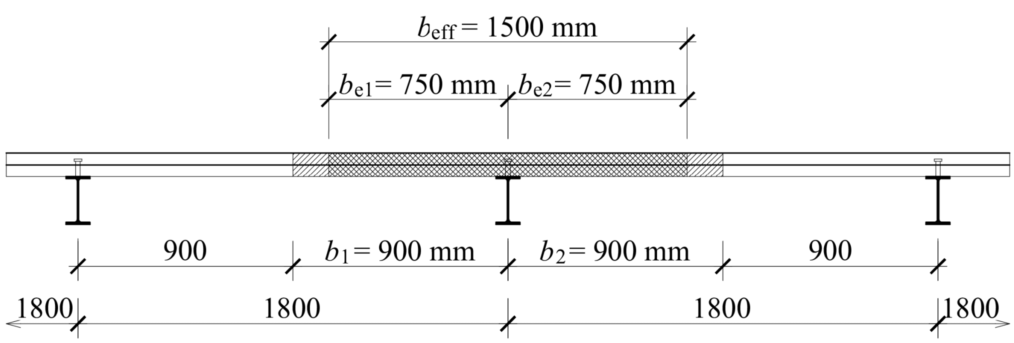
Normal stress in IPE flanges should not exceed yield strength of steel and normal stress in concrete slab should not exceed compressive strength of concrete. The calculation is carried out with effective width of concrete slab b_{eff} due to shear lug.

$$b_{\mathrm{eff}} = b_0 + \sum b_{\mathrm{ei}} = 0 + 750 + 750 = 1500 \text{ mm}$$

where

$$b_{\mathrm{e1}} = b_{\mathrm{e2}} = l_0/8 = 6000/8 = 750 \text{ mm} \leq b_1 = b_2 = 900 \text{ mm}$$

where $l_0 = L$ for simple beam



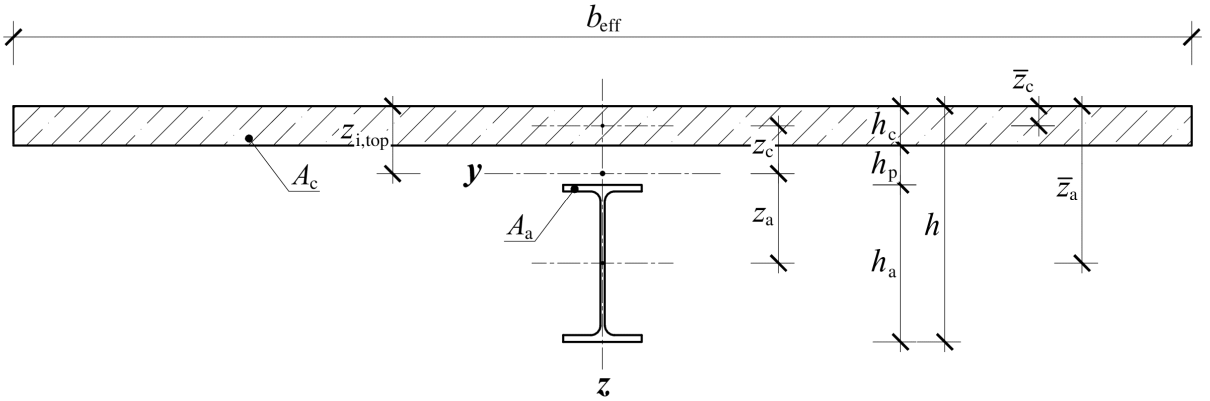
The composite section is substituted by all-steel section with reduced slab width using n ratio.

$$n = \frac{E_{\mathrm{a}}}{E_{\mathrm{c}}} = \frac{210}{15} = 14{,}0$$

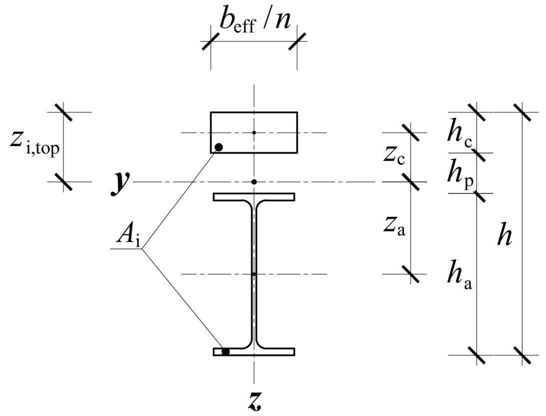
where

$E_a = 210 \text{ GPa}$ Young's modulus of steel

$E_c = \frac{E_{cm}}{2} = \frac{30}{2} = 15 \text{ GPa}$ commonly used simplification to take into account long term effects



Ideal (equivalent) all-steel section:



Cross section area of equivalent (ideal) all-steel section

$$A_i = A_a + \frac{A_c}{n} = 2848 + \frac{75000}{14} = 8205 \text{ mm}^2$$

where

$A_c = b_{eff} \cdot h_c = 1500 \cdot 50 = 75000 \text{ mm}^2$ is cross section area of concrete slab

$A_a = 2848 \text{ mm}^2$ is cross section area of IPE beam

First moment of area of equivalent all-steel section

$$S_i = S_a + \frac{S_c}{n} = 569{,}6.10^3 + \frac{1875.10^3}{14} = 703{,}5.10^3 \text{ mm}^3$$

where

$S_a = A_a \cdot \bar{z}_a = 2848 \cdot 200 = 569{,}6.10^3 \text{ mm}^3$ is first moment of steel part to the upper fibres of slab

$S_c = A_c \cdot \bar{z}_c = 75000 \cdot 25 = 1875.10^3 \text{ mm}^3$ first moment of concrete part to the upper fibres of slab

Position of centroid axis of equivalent all-steel section

$$z_{i,top} = \frac{S_i}{A_i} = \frac{703{,}5.10^3}{8205} = 85{,}7 \text{ mm}$$

Second moment of area (moment of inertia) of equivalent all-steel section

$$I_i = I_{a,i} + \frac{I_{c,i}}{n} = 56{,}61.10^6 + \frac{292{,}4.10^6}{14} = 77{,}50.10^6 \text{ mm}^4$$

where

$I_{a,i} = I_a + A_a \cdot z_a^2 = 19{,}43.10^6 + 2848 \cdot 114{,}3^2 = 56{,}61.10^6 \text{ mm}^4$ is moment of inertia of steel part

$I_{c,i} = I_c + A_c \cdot z_c^2 = 15{,}63.10^6 + 75000 \cdot 60{,}7^2 = 292{,}4.10^6 \text{ mm}^4$ is moment of inertia of concrete part

$I_a = 19{,}43.10^6 \text{ mm}^4$ moment of inertia of IPE to the own centroid axis (taken from static tables)

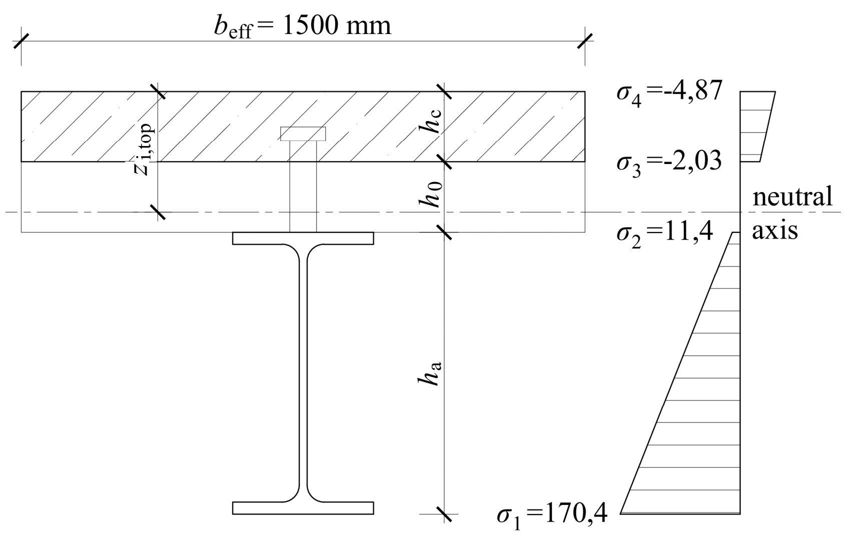
$I_c = \frac{1}{12} \cdot b_{eff} \cdot h_c^3 = \frac{1}{12} \cdot 1500 \cdot 50^3 = 15{,}63.10^6 \text{ mm}^4$ moment of inertia of concrete slab to the own centroid axis

$$\sigma_1 = \frac{M_{Ed}}{I_i} \cdot z_1 = \frac{61{,}61.10^6}{77{,}50.10^6} \cdot 214{,}3 = 170{,}4 \text{ MPa}$$

$$\sigma_2 = \frac{M_{Ed}}{I_i} \cdot z_2 = \frac{61{,}61.10^6}{77{,}50.10^6} \cdot 14{,}3 = 11{,}4 \text{ MPa}$$

$$\sigma_3 = \frac{1}{n} \cdot \frac{M_{Ed}}{I_i} \cdot z_3 = \frac{1}{14} \cdot \frac{61{,}61.10^6}{77{,}50.10^6} \cdot (-35{,}7) = -2{,}03 \text{ MPa}$$

$$\sigma_4 = \frac{1}{n} \cdot \frac{M_{Ed}}{I_i} \cdot z_4 = \frac{1}{14} \cdot \frac{61{,}61.10^6}{77{,}50.10^6} \cdot (-85{,}7) = -4{,}87 \text{ MPa}$$



Criterion for steel part

$$\frac{\sigma_1}{f_y/\gamma_{M0}} = \frac{170{,}4}{235/1{,}0} = 0{,}73 \quad \leq \quad 1{,}0$$ => *condition is satisfied*

Criterion for concrete part

$$\frac{\sigma_4}{f_{ck}/\gamma_c} = \frac{4{,}87}{20/1{,}5} = 0{,}37 \quad \leq \quad 1{,}0$$ => *condition is satisfied*

Shear studs

Will be added

Deflection in serviceability limit state

Overall deflection of composite beam depends on the way of erectionning. In the case of erectionning without intermediate supports some load cases act on steel IPE section with no composite action and other load cases act on composite section.

Load cases acting on steel IPE section: weight of concrete mixture, trapezoidal sheet weight and IPE weight

$$f_{k,1} = g_a + g_{trap} + g_c = 0{,}22 + 0{,}22 + 3{,}7 = 4{,}14 \text{ kN/m}$$

$$w_1 = \frac{5}{384} \cdot \frac{f_{k,1} \cdot L^4}{E_a \cdot I_a} = \frac{5}{384} \cdot \frac{4{,}14 \cdot 6000^4}{210.10^3 \cdot 19{,}43.10^6} = 17{,}1 \text{ mm}$$

Load cases acting on composite section: imposed loads

$$f_{k,2} = q_k = 5{,}40 \text{ kN/m}$$

$$w_2 = \frac{5}{384} \cdot \frac{f_{k,2} \cdot L^4}{E_i \cdot I_i} = \frac{5}{384} \cdot \frac{5{,}40 \cdot 6000^4}{210.10^3 \cdot 77{,}50.10^6} = 4{,}3 \text{ mm}$$

Overall deflection:

$$w = w_1 + w_2 = 17{,}1 + 4{,}3 = 21{,}4 \text{ mm}$$

Limit value of deflection:

$$w_{lim} = \frac{L}{250} = \frac{6000}{250} = 24 \text{ mm}$$

Criterion for deflection

$$\frac{w}{w_{lim}} = \frac{21{,}4}{24} = 0{,}89 \quad \leq \quad 1{,}0$$ => *condition is satisfied*

Alternative b: Plastic solution

Shear resistance

Shear resistance of composite beam is the same as for steel IPE section – shear resistance of concrete slab is neglected.

$$V_{\mathrm{pl,Rd}} = \frac{A_{\mathrm{v}} \cdot f_{\mathrm{y}} / \sqrt{3}}{\gamma_{\mathrm{M0}}} = \frac{1025 \cdot 235 / \sqrt{3}}{1,0} = 139,07 \text{ kN}$$

where

A_{v} is shear affected area of IPE = web area

Reliability criterion

$$\frac{V_{\mathrm{Ed}}}{V_{\mathrm{pl,Rd}}} = \frac{41,08}{139,07} = 0,30 \quad \leq \quad 1,0$$ => ***condition is satisfied***

Bending resistance

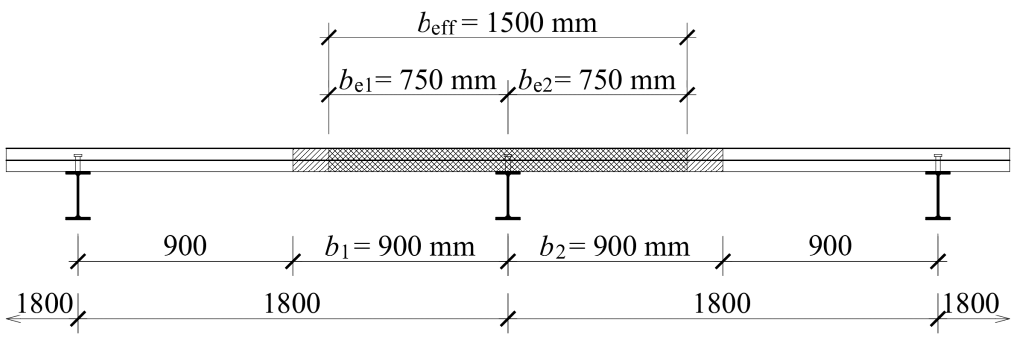
Effective width of concrete slab due to shear lug

$$b_{\mathrm{eff}} = b_0 + \sum b_{\mathrm{ei}} = 0 + 750 + 750 = 1500 \text{ mm}$$

where

$$b_{\mathrm{e1}} = b_{\mathrm{e2}} = l_0/8 = 6000/8 = 750 \text{ mm} \leq b_1 = b_2 = 900 \text{ mm}$$

where $l_0 = L$ for simple beam



High of compressed part of concrete slab is calculated from equation where normal force in IPE section is equal to normal force in concrete slab for assumption that neutral axis passes through concrete slab.

$F_{\mathrm{a}} = F_{\mathrm{c}}$ $\qquad A_{\mathrm{a}} \cdot f_{\mathrm{yd}} = A_{\mathrm{c}} \cdot 0,85 \cdot f_{\mathrm{cd}}$ $\qquad$ where $A_{\mathrm{c}} = b_{\mathrm{eff}} \cdot x$

from there

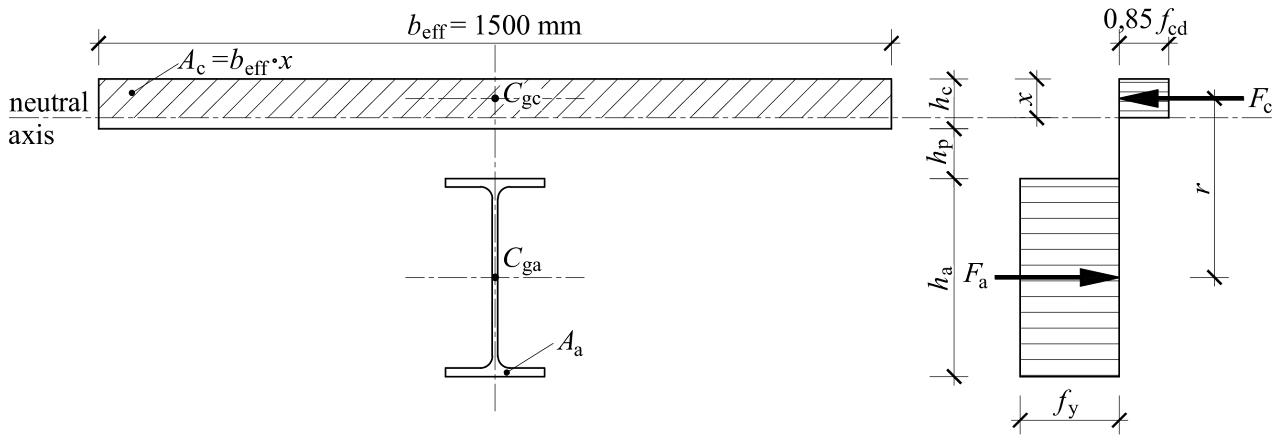
$$x = \frac{A_{\mathrm{a}} \cdot f_{\mathrm{yd}}}{0,85 \cdot b_{\mathrm{eff}} \cdot f_{\mathrm{cd}}} = \frac{2848 \cdot 235}{0,85 \cdot 1500 \cdot 13,3} = 39 \text{ mm}$$

where

$$f_{cd} = \frac{f_{ck}}{\gamma_C} = \frac{20}{1{,}5} = 13{,}3 \text{ MPa}$$

$$f_{yd} = \frac{f_y}{\gamma_M} = \frac{235}{1{,}0} = 235 \text{ MPa}$$

$x = 39$ mm $\leq$ $h_c = 50$ mm => *assumption is satisfied*



Plastic bending resistance

$$M_{pl,Rd} = F_a \cdot r = F_c \cdot r = 669{,}28 \cdot 0{,}161 = 107{,}75 \text{ kNm}$$

where

$$r = h_a/2 + h_c + h_p - x = 100 + 50 + 50 - 39 = 161 \text{ mm} = 0{,}161 \text{ m}$$

$$F_a = A_a \cdot f_{yd} = 2848 \cdot 235 = 669{,}28.10^3 \text{ N} = 669{,}28 \text{ kN}$$

$$F_c = A_a \cdot 0{,}85 \cdot f_{cd} = 150 \cdot 39 \cdot 0{,}85 \cdot 13{,}3 = 669{,}28.10^3 \text{ N} = 669{,}28 \text{ kN}$$

Reliability criterion

$$\frac{M_{Ed}}{M_{pl,Rd}} = \frac{61{,}61}{107{,}75} = 0{,}'57 \leq 1{,}0$$ => *condition is satisfied*

Shear studs

$d = 19$ mm diameter of shear stud

$h_{sc} = 75$ mm length of shear stud

Characteristic bearing resistance of one shear stud with diameter from 16 mm to 25 mm

$$P_{rk} = \min\left\{\begin{matrix} 0{,}8 \cdot f_u \cdot \frac{\pi \cdot d^2}{4} \\ 0{,}29 \cdot \alpha \cdot d^2 \sqrt{f_{ck} \cdot E_{cm}} \end{matrix}\right\} = \min\left\{\begin{matrix} 0{,}8 \cdot 400 \cdot \frac{\pi \cdot 19^2}{4} \\ 0{,}29 \cdot 0{,}99 \cdot 19^2 \sqrt{20 \cdot 30000} \end{matrix}\right\} = \min\left\{\begin{matrix} 90{,}73 \\ 80{,}24 \end{matrix}\right\} = 80{,}24 \text{ kN}$$

where

$E_{cm} = 30$ GPa concrete Young's modulus for short load duration

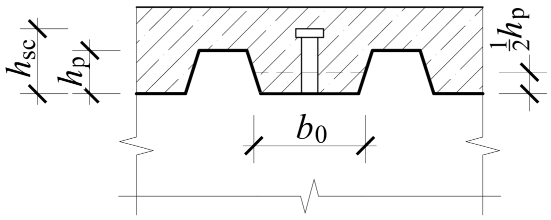
$$\alpha = 0{,}2 \cdot \left(\frac{h_{sc}}{d} + 1 \right) = 0{,}2 \cdot \left(\frac{75}{19} + 1 \right) = 0{,}99 \qquad \text{for} \qquad \frac{h_{sc}}{d} = \frac{75}{19} = 3{,}94 \quad \leq \quad 4$$

Design bearing resistance for trapezoidal sheet oriented perpendicular to the IPE beam

$$P_{rd} = \frac{P_{rk} \cdot k_t}{\gamma_V} = \frac{80{,}24 \cdot 0{,}85}{1{,}25} = 54{,}56 \text{ kN}$$

where k_t is factor for concrete slabs concreted into trapezoidal sheet oriented perpendicular to the IPE beam

$$k_t = \frac{0{,}7}{\sqrt{n_r}} \cdot \frac{b_0}{h_p} \cdot \left(\frac{h_{sc}}{h_p} - 1 \right) = \frac{0{,}7}{\sqrt{1}} \cdot \frac{129}{50} \cdot \left(\frac{75}{50} - 1 \right) = 0{,}903 \quad \geq \quad k_{t,\max} = 0{,}85 \quad => \quad k_t = 0{,}85$$



Overall shear force between IPE section and concrete slab

$$N_{cf} = \min \begin{Bmatrix} A_a \cdot f_{yd} \\ A_c \cdot 0{,}85 \cdot f_{cd} + A_s \cdot f_{sd} \end{Bmatrix} = \min \begin{Bmatrix} 2848 \cdot 235 \\ 39 \cdot 1500 \cdot 0{,}85 \cdot 13{,}3 + 0 \end{Bmatrix} = \min \begin{Bmatrix} 669{,}28 \\ 669{,}28 \end{Bmatrix} = 669{,}28 \text{ kN}$$

Minimum number of shear studs for full composite action

$$n_f = \frac{N_{cf}}{P_{Rd}} = \frac{669{,}28}{54{,}56} = 12{,}3$$

Number of shear studs which is possible (maximal realizable number) to put onto beam is influenced by shape of trapezoidal sheet. The wave length of VSŽ 11002 is 200 mm, thus shear studs spacing is 200 mm too.

$$n_{max} = \frac{L/2}{l_{wave}} = \frac{6000/2}{200} = 15$$

=> *full composite action*

Other way

Maximal spacing of shear studs for full composite action

$$b_{max} = \frac{L/2}{n_f} = \frac{6000/2}{12{,}3} = 243{,}9 \text{ mm} \quad \geq \quad l_{wave} = 200 \text{ mm}$$

=> *full composite action*

Deflection in serviceability limit state

The same as for elastic solution

$$\frac{w}{w_{lim}} = \frac{21{,}4}{24} = 0{,}89 \quad \leq \quad 1{,}0$$

=> *condition is satisfied*