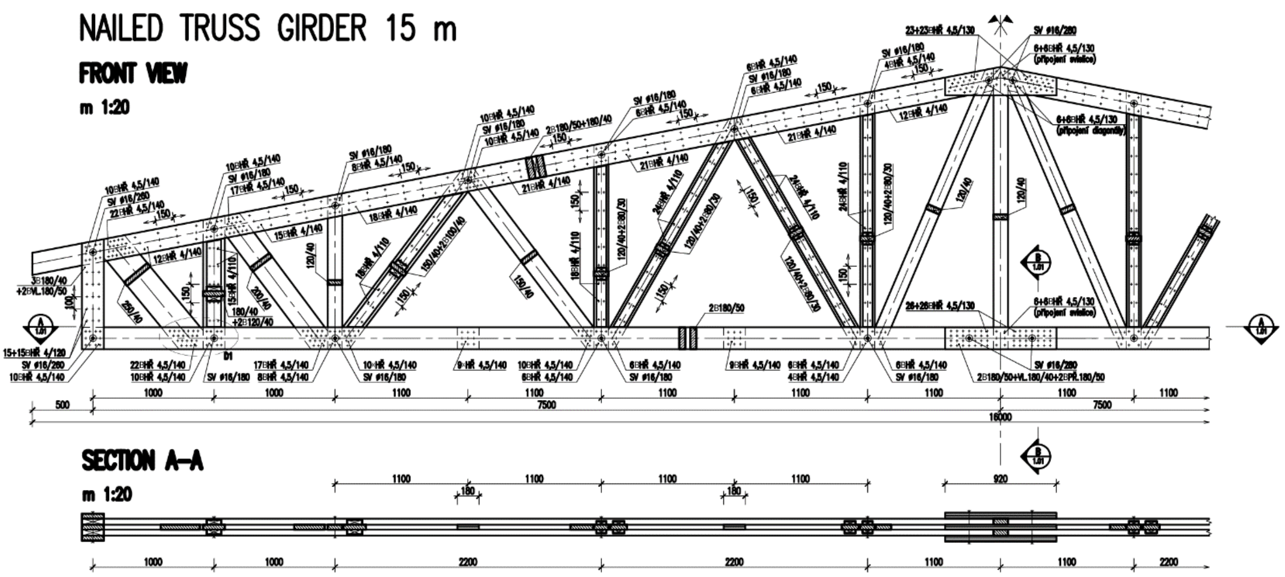
# HOW TO CALCULATE DEFLECTION OF TIMBER TRUSS GIRDER?

## …worked examples for BO003 and BO006

## WORKED EXAMPLE 2: NAILED TRUSS GIRDER

Calculate deflection of timber truss girder in the mid-span. Girder is made of solid timber C20. Nailed joints are made of nail with diameter 4,5 mm without pre-drilling.




**Roof geometry**

$L$ = 15000 mm truss girder span

$l$ = 3200 mm roof purlin span = roof truss girder spacing

$\alpha$ = 12° pitch of the roof (slope)

**Beam geometry and material**

Solid timber C20 according to the EN 338

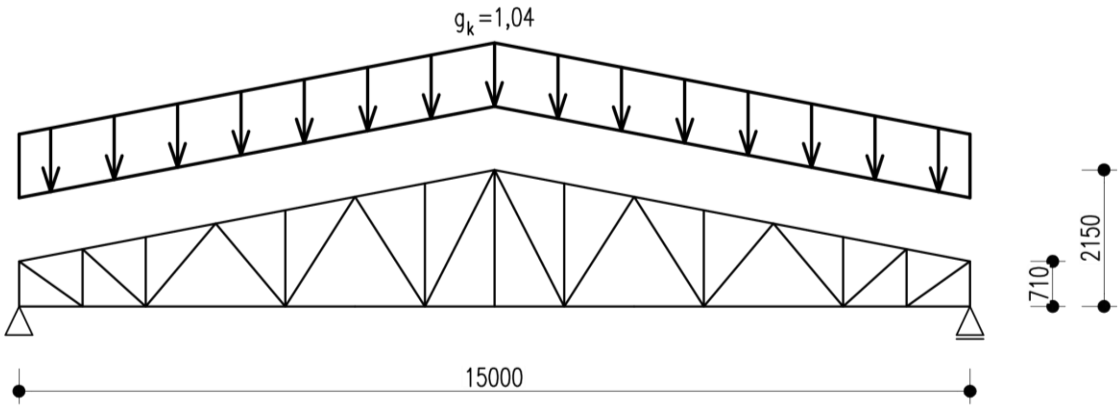
$E_{0,mean}$ = 9,5 GPa mean value of modulus of elasticity parallel to the grain

$\rho_{mean}$ = 390 kg/m$^3$ mean value of timber density

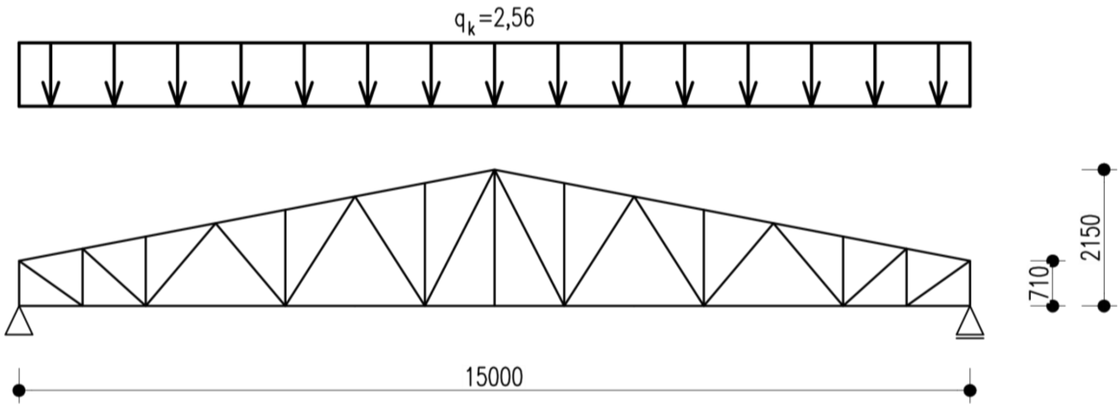
$d$ = 4,5 mm nail diameter (without pre-drilling)

**Loads**

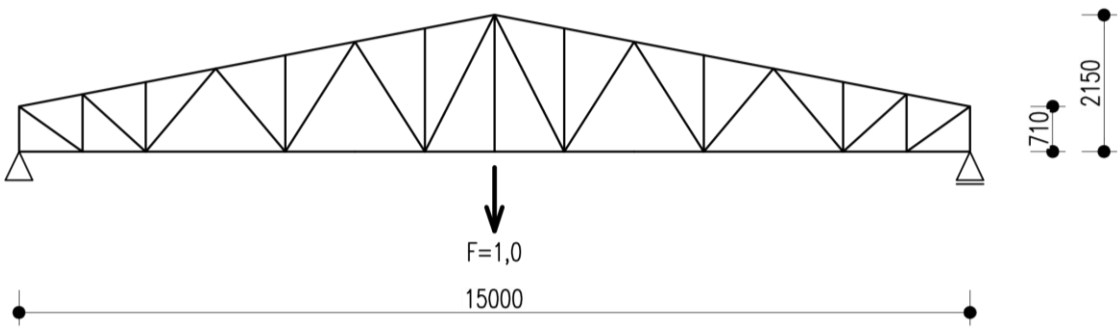
LS1 Permanent load $g_k = 1{,}04$ kN/m




LS2 Snow load $q_k = 2{,}56$ kN/m




LS3 Virtual unit load $F = 1{,}00$



## a) Virtual force method

Virtual force method is based on virtual work principle. There is a new load state – virtual load. In the point where deflection is calculated there is a unit force and direction of that force is equal to the direction of calculate deflection.

Calculation using Maxwell-Mohr equation (1), where integral could be solved using Vereščagin law (2). Using this method, the deflections due to slip in the joints it is possible to take into account (3).

$$u = \int_0^l \frac{N \cdot N_1}{E \cdot A}\,\mathrm{d}l \tag{1}$$

$$u = \sum_{i=1}^{n} \frac{N_i \cdot N_{1,i} \cdot l_i}{E_i \cdot A_i} \tag{2}$$

$$u = \sum_{i=1}^{n} \left( \frac{N_i \cdot N_{1,i} \cdot l_i}{E_i \cdot A_i} + \Delta_i \cdot N_{1,i} \right) \tag{3}$$

kde

$N_i$ is normal force in *i*-member due to external loads [N],

$N_{1,i}$ normal force in *i*-member due to virtual unit load [-],

$l_i$ length of *i*-member [mm],

$A_i$ cross sectional area of *i*-member [mm$^2$],

$E_i$ Young's modulus of *i*-member [MPa] and

$\Delta_i$ slip in the joint of *i*-member [mm].

Slip in the joint could be calculated from slip modulus $K$ in a connection:

$$\Delta_i = \frac{N_{i,\text{pernail}}}{K_{\text{ser},i}}$$

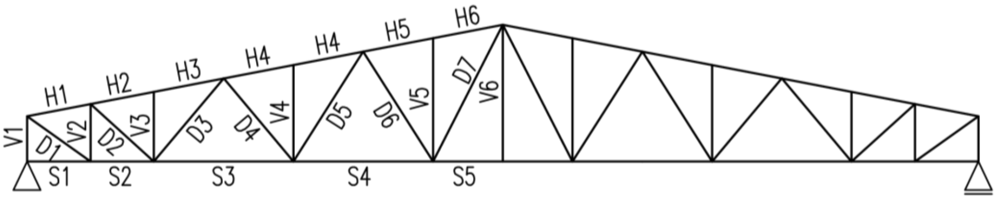
where

$K_{\text{ser}} = \rho_{\text{mean}}^{1,5} \cdot \dfrac{d^{\,0,8}}{30} = 390^{1,5} \cdot \dfrac{4,5^{0,8}}{30} = 855\ \text{N/mm}$ is slip modulus for nails without pre-drilling

$N_{i,\text{pernail}}$ is normal force per one nail in the joint

The table is carried out for half of the girder because problem is symmetrical (geometry and loads). Thus, the real deflection is two times sum of numbers in the columns. Upper and bottom chord are made of one piece of timber, thus the deflection due to slip in the jaints is caused only on the ends of diagonals and verticals.

Truss girder members notation

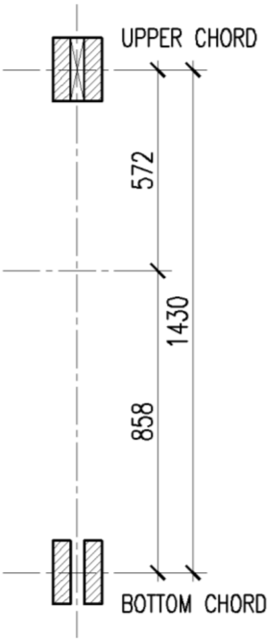



Deflections according to the equation (3)

| Prut | $N_i$ | | $N_{1,i}$ | $l_i$ | $A_i$ | $w_i$ | |
|---|---|---|---|---|---|---|---|
| | ZS1 | ZS2 | ZS3 | | | ZS1 | ZS2 |
| | [kN] | [kN] | [-] | [mm] | [mm$^2$] | [mm] | [mm] |
| H1 | -8,47 | -20,47 | -0,56 | 1018 | 27000 | 0,02 | 0,05 |
| H2 | -12,92 | -31,24 | -0,93 | 1018 | 27000 | 0,05 | 0,12 |
| H3 | -12,92 | -31,24 | -0,93 | 1120 | 27000 | 0,05 | 0,13 |
| H4 | -16,24 | -39,25 | -1,41 | 1120 | 27000 | 0,10 | 0,24 |
| H5 | -16,24 | -39,25 | -1,41 | 1120 | 27000 | 0,10 | 0,24 |
| H6 | -15,42 | -37,27 | -1,68 | 1120 | 27000 | 0,11 | 0,27 |
| H7 | -15,42 | -37,27 | -1,68 | 1120 | 27000 | 0,11 | 0,27 |
| D1 | 10,08 | 24,37 | 0,68 | 1227 | 10000 | 0,09 | 0,21 |
| D2 | 5,88 | 14,21 | 0,48 | 1347 | 8000 | 0,05 | 0,12 |
| D3 | -3,70 | -8,93 | -0,42 | 1707 | 14000 | 0,02 | 0,05 |
| D4 | 1,36 | 3,28 | 0,31 | 1707 | 6000 | 0,01 | 0,03 |
| D5 | 0,15 | 0,37 | -0,28 | 2048 | 9600 | 0,00 | 0,00 |
| D6 | -1,35 | -3,26 | 0,22 | 2048 | 9600 | -0,01 | -0,02 |
| D7 | 2,59 | 6,26 | -0,21 | 2414 | 4800 | -0,03 | -0,07 |
| V1 | -7,94 | -19,20 | -0,50 | 710 | 39600 | 0,01 | 0,02 |
| V2 | -5,84 | -14,11 | -0,39 | 902 | 16800 | 0,01 | 0,03 |
| V3 | -1,11 | -2,69 | 0,00 | 1094 | 4800 | 0,00 | 0,00 |
| V4 | -1,17 | -2,82 | 0,00 | 1516 | 9600 | 0,00 | 0,00 |
| V5 | -1,17 | -2,82 | 0,00 | 1938 | 9600 | 0,00 | 0,00 |
| V6 | 0,00 | 0,00 | 1,00 | 2150 | 9600 | 0,00 | 0,00 |
| S1 | 0,00 | 0,00 | 0,00 | 1000 | 18000 | 0,00 | 0,00 |
| S2 | 8,22 | 19,87 | 0,55 | 1000 | 18000 | 0,03 | 0,06 |
| S3 | 14,97 | 36,18 | 1,19 | 2200 | 18000 | 0,23 | 0,55 |
| S4 | 15,76 | 38,09 | 1,53 | 200 | 18000 | 0,03 | 0,07 |
| S5 | 13,85 | 33,49 | 1,74 | 1100 | 18000 | 0,16 | 0,38 |
| 2×SUM | | | | | | **2,28** | **5,52** |

| $n_{bolt,i}$ | $N_{i,pernail}$ | | $\Delta_i$ | | $w_{slip,i}$ | |
|---|---|---|---|---|---|---|
| | ZS1 | ZS2 | ZS1 | ZS2 | ZS1 | ZS2 |
| | [kN] | [kN] | [mm] | [mm] | [mm] | [mm] |
| - | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 |
| - | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 |
| - | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 |
| - | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 |
| - | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 |
| - | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 |
| - | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 |
| 22 | 0,46 | 1,11 | 0,54 | 1,30 | 0,36 | 0,88 |
| 17 | 0,35 | 0,84 | 0,40 | 0,98 | 0,20 | 0,47 |
| 10 | -0,37 | -0,89 | -0,43 | -1,04 | 0,18 | 0,44 |
| 10 | 0,14 | 0,33 | 0,16 | 0,38 | 0,05 | 0,12 |
| 6 | 0,03 | 0,06 | 0,03 | 0,07 | -0,01 | -0,02 |
| 6 | -0,23 | -0,54 | -0,26 | -0,64 | -0,06 | -0,14 |
| 6 | 0,43 | 1,04 | 0,50 | 1,22 | -0,10 | -0,25 |
| 10 | -0,79 | -1,92 | -0,93 | -2,25 | 0,46 | 1,12 |
| 10 | -0,58 | -1,41 | -0,68 | -1,65 | 0,27 | 0,65 |
| 8 | -0,14 | -0,34 | -0,16 | -0,39 | 0,00 | 0,00 |
| 6 | -0,19 | -0,47 | -0,23 | -0,55 | 0,00 | 0,00 |
| 4 | -0,29 | -0,70 | -0,34 | -0,82 | 0,00 | 0,00 |
| 6 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 |
| - | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 |
| - | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 |
| - | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 |
| - | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 |
| - | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 |
| | | | | | **2,71** | **6,55** |

**b) Simplified method (approximate solution)**




Truss girder is substituted by solid beam with equivalent moment of inertia. In the case of girder with variable depth, the equivalent depth is considered as average depth:

$$h = \left(h_{\text{sup}} + h_{\text{apex}}\right)/2 = \left(710 + 2150\right)/2 = 1430 \text{ mm}$$

Centre of gravity position:

$$z_{\text{up}} = \frac{A_{\text{bot}} \cdot h}{A_{\text{up}} + A_{\text{bot}}} = \frac{18000 \cdot 1430}{27000 + 18000} = 572 \text{ mm}$$

$$z_{\text{bot}} = \frac{A_{\text{up}} \cdot h}{A_{\text{up}} + A_{\text{bot}}} = \frac{27000 \cdot 1430}{27000 + 18000} = 858 \text{ mm}$$

Moments of inertia ($I_{\text{y,up}}$ and $I_{\text{y,bot}}$) to the own centroidal axis of bottom and upper chord could be neglected:

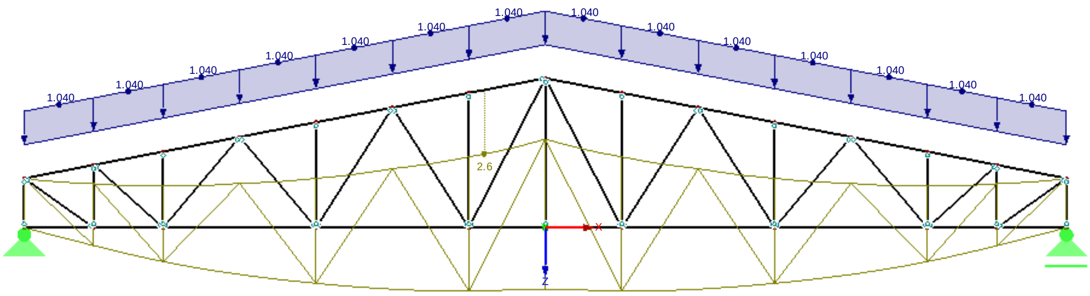
$$I_{\text{y}} = A_{\text{up}} \cdot z_{\text{up}}^2 + A_{\text{bot}} \cdot z_{\text{bot}}^2 = 27000 \cdot 572^2 + 18000 \cdot 858^2 = 2{,}21.10^{10} \text{ mm}^4$$

Deflection at mid-span of simple beam with equivalent moment of inertia is:

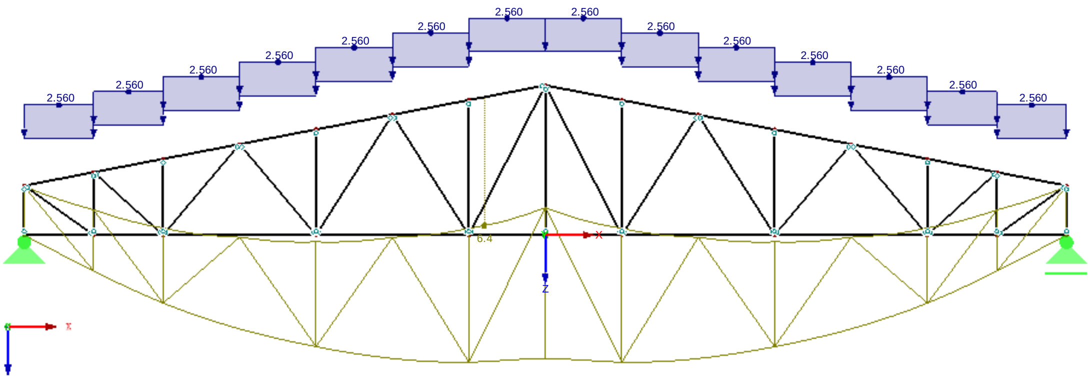
$$w_{\text{LS1}} = \frac{5}{384} \cdot \frac{g_{\text{k}} \cdot L^4}{E_{0,\text{mean}} \cdot I_y} = \frac{5}{384} \cdot \frac{1{,}04 \cdot 15\,000^4}{9\,500 \cdot 2{,}21.10^{10}} = 3{,}3 \text{ mm}$$

$$w_{\text{LS2}} = \frac{5}{384} \cdot \frac{q_{\text{k}} \cdot L^4}{E_{0,\text{mean}} \cdot I_y} = \frac{5}{384} \cdot \frac{2{,}56 \cdot 15\,000^4}{9\,500 \cdot 2{,}21.10^{10}} = 8{,}0 \text{ mm}$$

## c) Computer solution (static software)




$w_{LS1} = 2{,}5$ mm




$w_{LS2} = 6{,}1$ mm

## Results comparison

| Load state | Force method | | Simplified method | Software |
|---|---|---|---|---|
| | with slip in joints | without slip in joints | without slip in joints | |
| ZS1 | 2,3 + 2,7 = 5,0 | 2,3 | 3,3 | 2,5 |
| ZS2 | 5,5 + 6,6 = 12,1 | 5,5 | 8,0 | 6,1 |
| LC1 = ZS1 + ZS2 | 17,1 | 7,8 | 11,3 | 8,6 |

Calculated deflections are instantaneous values, it is necessary to check also final values of deflections!

Reliability criterions:

$$\frac{w_{inst}}{w_{inst,lim}} \leq 1{,}0$$

$$\frac{w_{fin}}{w_{fin,lim}} \leq 1{,}0$$