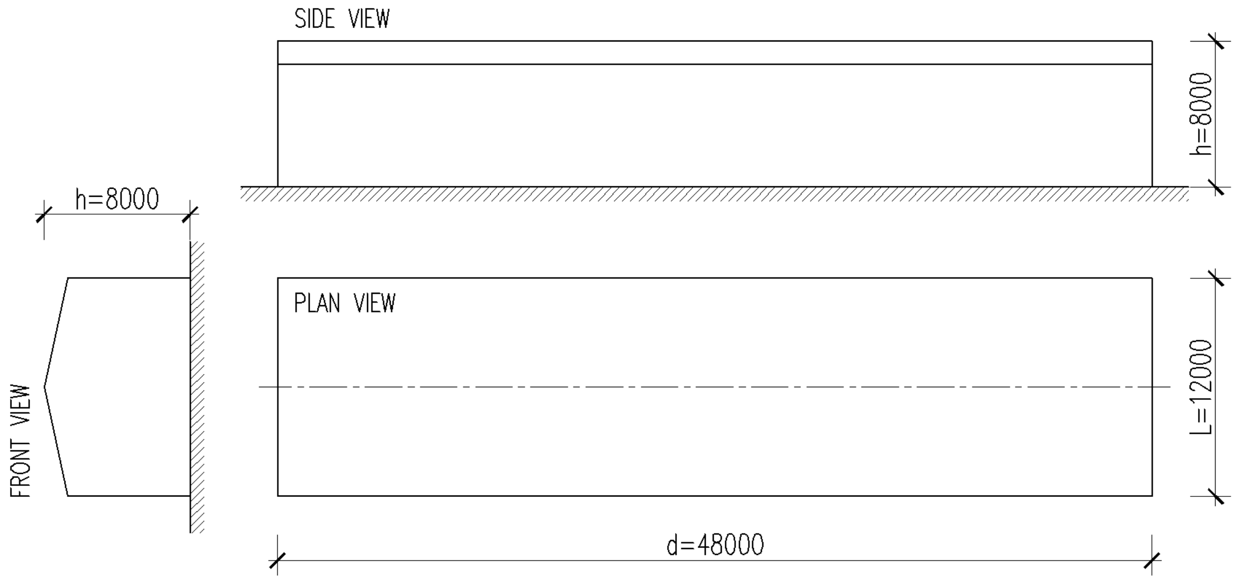
# HOW TO DESIGN TRANSVERSAL BRACING?

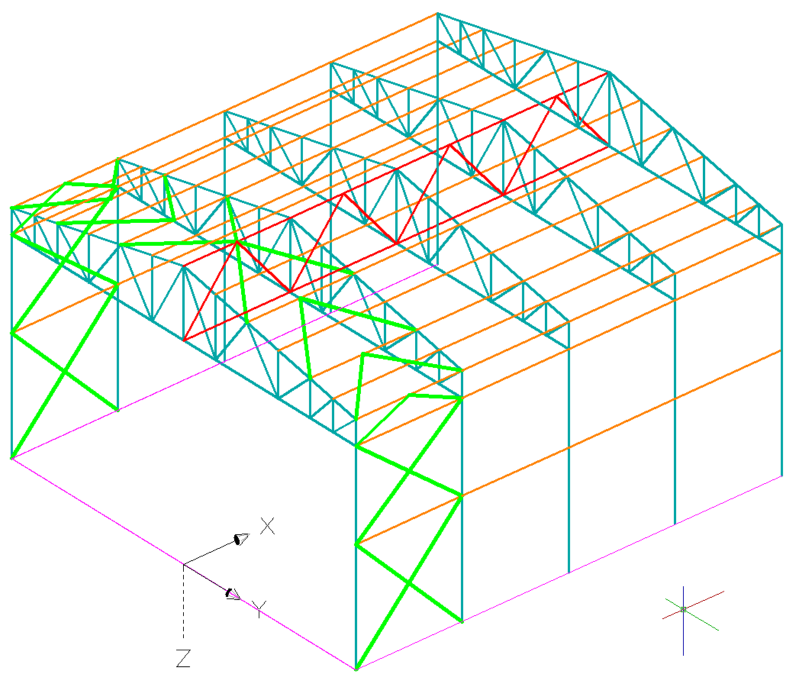
## …worked examples for BO003 and BO006

### WORKED EXAMPLE 5: TRANSVERSAL BRACING OF TRUSS GIRDER ROOF STRUCTURE

Make assessment of transversal bracing of truss girder roof structure. Bracing members are made of solid timber C20. Diagonals of K-shape bracing are designed as solid sections. Service class no 2 is considered.




Axonometry




Spatial structure members:

blue – truss girder + columns

orange – purlins

red – longitudinal vertical bracing

green – transversal bracing (roof part + wall part)

## Building geometry

$L$ = 12 m truss girder span = building width

$d$ = 48 m building length

$h$ = 8 m building high

$\alpha$ = 12° pitch of the roof (slope)

## Material

Solid timber C20 according to the EN 338

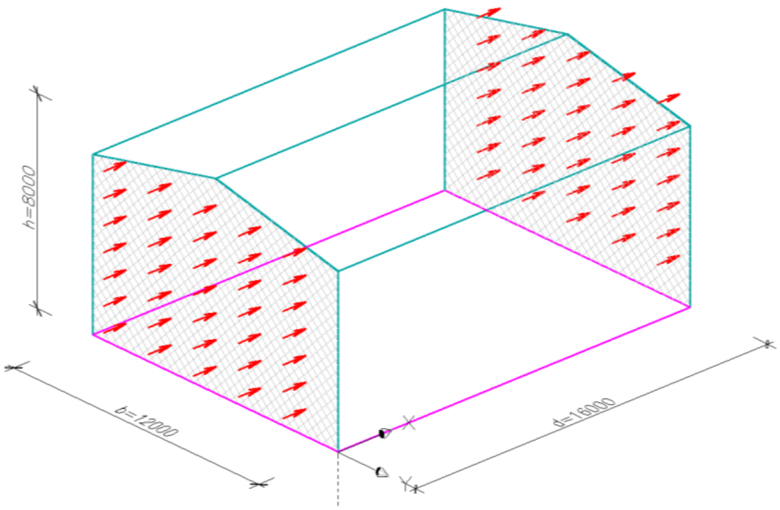
$f_{c,0,k}$ = 19 MPa characteristic compressive strength along the grain

$$f_{c,0,d} = k_{mod} \cdot \frac{f_{c,0,k}}{\gamma_M} = 0{,}9 \cdot \frac{19}{1{,}3} = 13{,}15 \text{ MPa}$$ design compressive strength along the grain

## Load states

LS1 Wind pressure and wind suction on gable walls

$q_p(z) = 0{,}864\ \mathrm{kN/m^2}$ (for CZ wind area II, terrain category II and building high 8 m above terrain)




pressure coefficients for external pressure for areas D and E for geometrical ratio $h/d = 8/48 = 0{,}166$

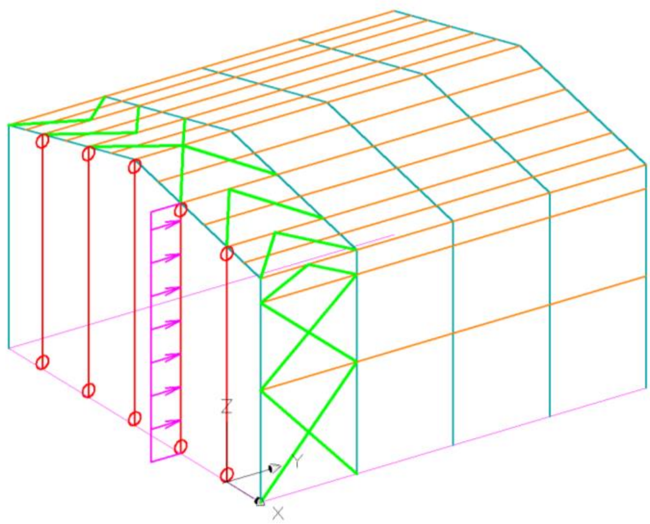
$c_{pe,D}$ = +0,7 $c_{pe,E}$ = -0,3

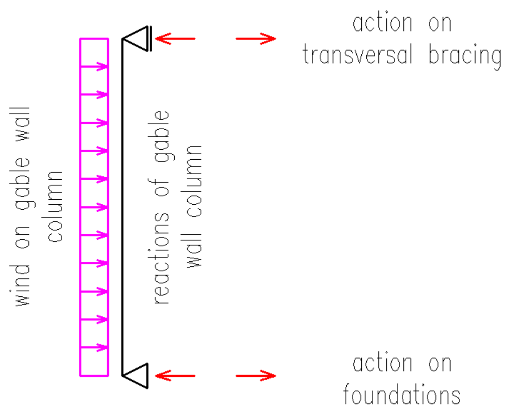
wind pressure on windward wall and wind suction on leeward wall

$$w_{e,D} = q_p(z) \cdot c_{pe,D} = 0{,}864 \cdot 0{,}7 = 0{,}60\ \mathrm{kN/m^2} \qquad w_{e,E} = q_p(z) \cdot c_{pe,E} = 0{,}864 \cdot -0{,}3 = -0{,}26\ \mathrm{kN/m^2}$$

actions on transversal roof bracing

$$f_{w,D} = w_{e,D} \cdot \frac{h}{2} = 0{,}60 \cdot \frac{8}{2} = 2{,}40\ \mathrm{kN/m} \qquad f_{w,E} = w_{e,E} \cdot \frac{h}{2} = 0{,}26 \cdot \frac{8}{2} = 1{,}04\ \mathrm{kN/m}$$





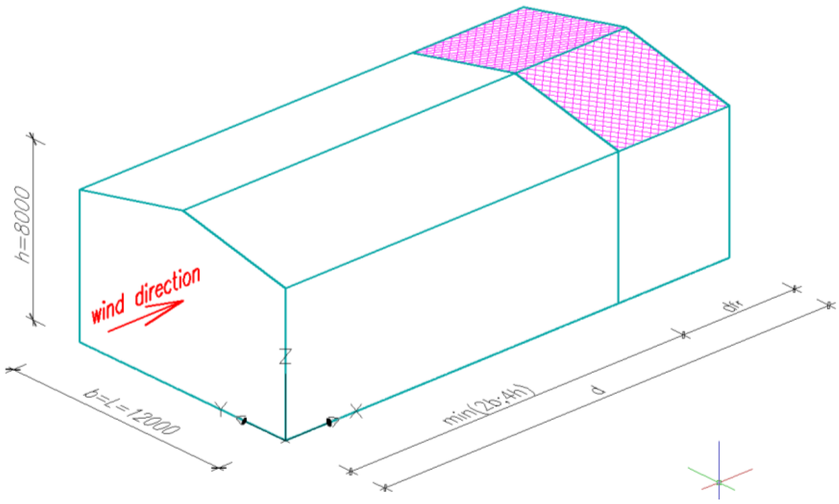
LS2 Wind friction on the roof cladding

Friction length

$$d_{\mathrm{fr}} = d - \min\{2 \cdot b; 4 \cdot h\} = 48 - \min\{2 \cdot 12; 4 \cdot 8\} = 48 - \min\{24; 32\} = 24 \text{ m}$$

Friction area

$$A_{\mathrm{fr}} = d_{\mathrm{fr}} \cdot \frac{L}{\cos\alpha} = 24 \cdot \frac{12}{\cos 12°} = 294 \text{ m}^2$$




Friction force

$$F_{\mathrm{fr}} = c_{\mathrm{fr}} \cdot A_{\mathrm{fr}} \cdot q_{\mathrm{p}}(z) = 0{,}02 \cdot 294 \cdot 0{,}864 = 5{,}08 \text{ kN}$$

where $c_{\mathrm{fr}}$ = 0,02 is friction coefficient for asphalt shingle

Transfer to uniformly distributed load

$$f_{\mathrm{fr}} = \frac{F_{\mathrm{fr}}}{L/\cos\alpha} = \frac{5{,}08}{12/\cos 12°} = 0{,}41 \text{ kN/m}$$

LS3 Stabilizing load (forces)

$$f_{\mathrm{st}} = k_L \cdot \frac{n \cdot N_{\mathrm{c,d}}}{k_{\mathrm{f,3}} \cdot L/\cos\alpha} = 1 \cdot \frac{4{,}0 \cdot 48{,}23}{30 \cdot 12/\cos 12°} = 0{,}52 \text{ kN/m}$$

where

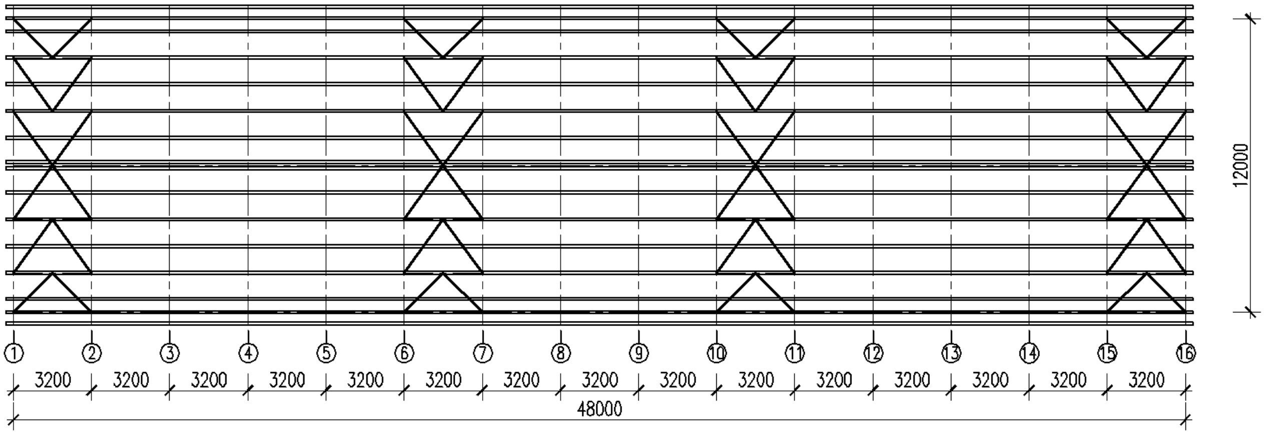
length coefficient

$$k_L = \min\left\{\begin{matrix} 1 \\ \sqrt{\dfrac{15}{L/\cos\alpha}} \end{matrix}\right\} = \min\left\{\begin{matrix} 1 \\ \sqrt{\dfrac{15}{12/\cos 12°}} \end{matrix}\right\} = \min\left\{\begin{matrix} 1 \\ 1{,}11 \end{matrix}\right\} = 1$$

factor of manufacturing quality

$k_{\mathrm{f,3}} = 30$ (for bad manufacturing quality) number of roof girders (upper chords) stabilized by one transversal bracing

$$n = \frac{number\ of\ roof\ girders}{number\ of\ transversal\ bracing} = \frac{16}{4} = 4$$

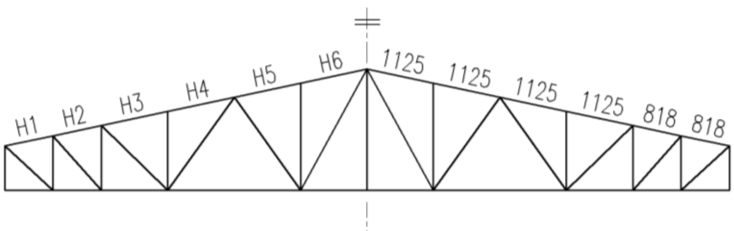



weighted average of design normal forces in upper chords where lengths of upper chord $l_i$ is weight

$$N_{\mathrm{c,d}} = \frac{\Sigma N_i \cdot l_i}{\Sigma l_i}$$

| Upper chord | Load state | | Load combination |
|---|---|---|---|
| | LS1 permanent | LS2 snow | LC1=1,35×LS1+1,5×LS2 |
| | [kN] | | [kN] |
| H1 | -5,50 | -13,25 | -27,30 |
| H2 | -8,51 | -20,48 | -42,20 |
| H3 | -10,60 | -25,53 | -52,60 |
| H4 | -10,60 | -25,53 | -52,60 |
| H5 | -10,82 | -26,05 | -53,67 |
| H6 | -10,82 | -26,05 | -53,67 |

upper chords notation and lengths




$$N_{\mathrm{c,d}} = \frac{27{,}30 \cdot 0{,}818 + 42{,}20 \cdot 0{,}818 + 52{,}60 \cdot 1{,}124 + 52{,}60 \cdot 1{,}124 + 53{,}67 \cdot 1{,}124 + 53{,}67 \cdot 1{,}124}{0{,}818 + 0{,}818 + 1{,}124 + 1{,}124 + 1{,}124 + 1{,}124} = 48{,}23\ \mathrm{kN}$$

## **Load combinations**

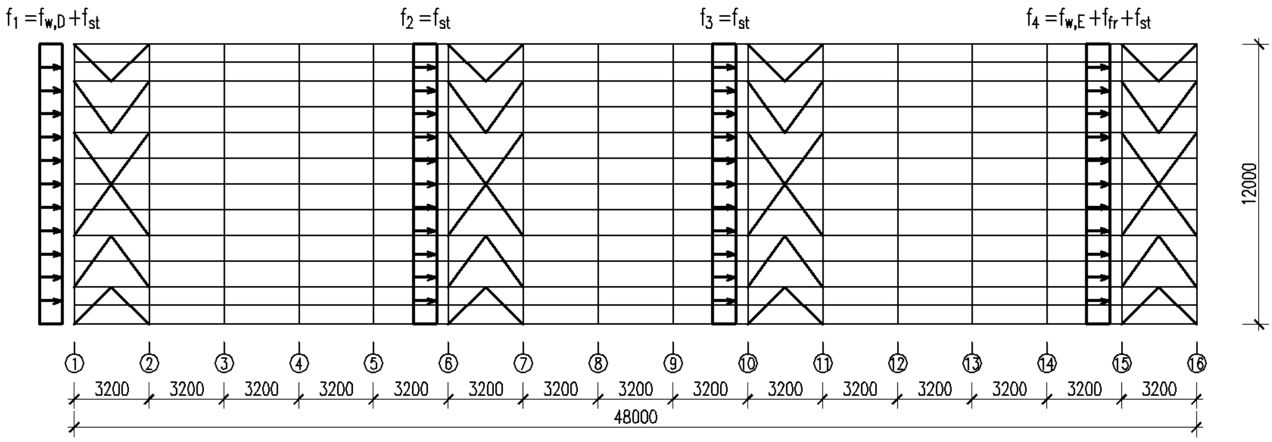
There are special load combination for each transversal bracing (on the safe side)

$$f_1 = f_{\mathrm{w,D}} \cdot \gamma_{\mathrm{Q}} + f_{\mathrm{st}} = 2{,}40 \cdot 1{,}5 + 0{,}52 = 4{,}12 \text{ kN/m}$$

$$f_2 = f_{\mathrm{st}} = 0{,}52 \text{ kN/m}$$

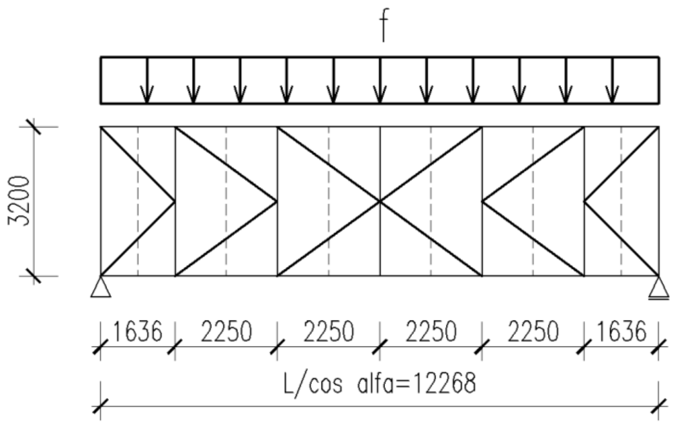
$$f_3 = f_{\mathrm{st}} = 0{,}52 \text{ kN/m}$$

$$f_4 = f_{\mathrm{w,E}} \cdot \gamma_{\mathrm{Q}} + f_{\mathrm{fr}} \cdot \gamma_{\mathrm{Q}} + f_{\mathrm{st}} = 1{,}04 \cdot 1{,}5 + 0{,}41 \cdot 1{,}5 + 0{,}52 = 2{,}70 \text{ kN/m}$$




All of the transversal bracing are the same geometry and members cross sections. The most loaded of them will be analysed with load *f*

$$f = \max\{f_1; f_2; f_3; f_4\} = \max\{4{,}12; 0{,}52; 0{,}52; 2{,}70\} = 4{,}12 \text{ kN/m}$$




## Internal forces

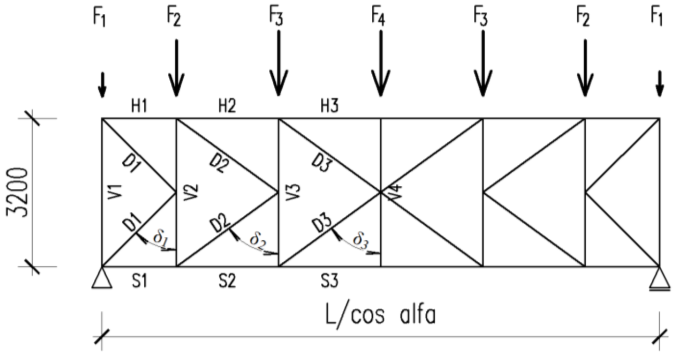
Uniformly distributed load *f* is transformed on forces $F_i$ acting in joints. The purlins which are not joint to the bracing (dashed verticals) are not considered in the calculation of internal forces.

$$F_1 = f \cdot \frac{l_1}{2} = 4{,}12 \cdot \frac{1636}{2} = 3{,}37 \text{ kN}$$

$$F_2 = f \cdot \frac{l_1 + l_2}{2} = 4{,}12 \cdot \frac{1{,}636 + 2{,}250}{2} = 8{,}00 \text{ kN}$$

$$F_3 = f \cdot \frac{l_2 + l_3}{2} = 4{,}12 \cdot \frac{2{,}250 + 2{,}250}{2} = 9{,}27 \text{ kN}$$

$$F_4 = f \cdot \frac{l_3 + l_4}{2} = 4{,}12 \cdot \frac{2{,}250 + 2{,}250}{2} = 9{,}27 \text{ kN}$$




Reactions

$$R_a = R_b = \frac{\Sigma F_i}{2} = \frac{2 \cdot (3{,}30 + 7{,}83 + 9{,}07) + 9{,}07}{2} = 24{,}74 \text{ kN}$$

Normal force in diagonals D1

$$N_{D1} = \pm \frac{R_a - F_1}{2 \cdot \cos \delta_1} = \pm \frac{24{,}74 - 3{,}30}{2 \cdot \cos 45°} = \pm 15{,}16 \text{ kN}$$

Normal force in diagonals D2

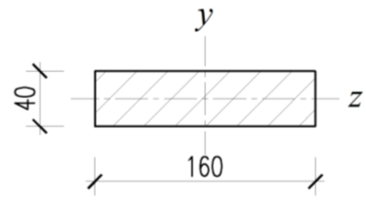
$$N_{D2} = \pm \frac{R_a - F_1 - F_2}{2 \cdot \cos \delta_2} = \pm \frac{24{,}74 - 3{,}30 - 7{,}83}{2 \cdot \cos 54°} = \pm 11{,}58 \text{ kN}$$

Normal force in diagonals D3

$$N_{D3} = \pm \frac{R_a - F_1 - F_2 - F_3}{2 \cdot \cos \delta_3} = \pm \frac{24{,}74 - 3{,}30 - 7{,}83 - 9{,}07}{2 \cdot \cos 54°} = \pm 3{,}86 \text{ kN}$$

In one diagonal is tension force and in the second there is compression force of the same value. The compression is more unfavourable situation thus the diagonals will be check on flexural buckling.

**Diagonal D1**




$b$ = 40 mm section depth

$h$ = 160 mm section high

$N_{Ed}$ = -15,16 kN

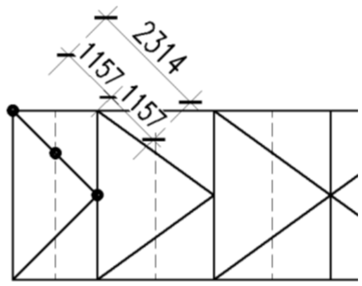
Buckling to the $z$-axis (in $y$ direction) is decisive.

Design compression stress along the grain

$$\sigma_{c,0,d} = \frac{N_{Ed}}{A} = \frac{15{,}16.10^3}{6400} = 2{,}37 \text{ MPa}$$

Critical length (half of the actual length of diagonal due to joining to the neglected verticals = purlins)

$$L_{cr,z} = 1157 \text{ mm}$$




Moment of inertia

$$I_z = \frac{1}{12} b^3 \cdot h = \frac{1}{12} 40^3 \cdot 160 = 0{,}853.10^6 \text{ mm}^4$$

Radius of gyration

$$i_z = \sqrt{\frac{I_z}{A}} = \sqrt{\frac{0{,}853.10^6}{6400}} = 11{,}5 \text{ mm}$$

Slenderness

$$\lambda_z = \frac{L_{cr,z}}{i_z} = \frac{1157}{11{,}5} = 100$$

Relative slenderness

$$\lambda_{rel,z} = \frac{\lambda_z}{\pi} \cdot \sqrt{\frac{f_{c,0,k}}{E_{0,05}}} = \frac{100}{\pi} \cdot \sqrt{\frac{19}{6400}} = 1{,}74$$

Factor for reduction factor calculation

$$k_z = 0{,}5 \cdot \left(1 + \beta_c \cdot (\lambda_{rel,z} - 0{,}3) + \lambda_{rel,z}^2\right) = 0{,}5 \cdot \left(1 + 0{,}2 \cdot (1{,}74 - 0{,}3) + 1{,}74^2\right) = 2{,}16$$

Reduction factor

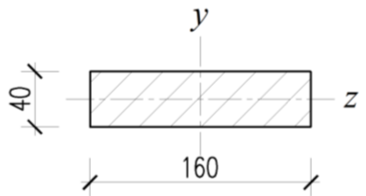
$$k_{c,z} = \frac{1}{k_z + \sqrt{k_z^2 - \lambda_{rel,z}^2}} = \frac{1}{2{,}16 + \sqrt{2{,}16^2 - 1{,}74^2}} = 0{,}29$$

Reliability criterion

$$\frac{\sigma_{c,0,d}}{k_{c,z} \cdot f_{c,0,d}} = \frac{2{,}37}{0{,}29 \cdot 13{,}15} = 0{,}62 \quad \leq \quad 1{,}0$$

*=> condition is satisfied*

**Diagonal D2**




$b$ = 40 mm section depth

$h$ = 160 mm section high

$N_{Ed}$ = -11,58 kN

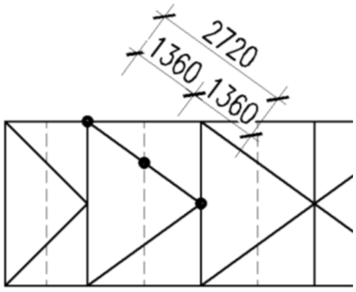
Buckling to the $z$-axis (in $y$ direction) is decisive.

Design compression stress along the grain

$$\sigma_{c,0,d} = \frac{N_{Ed}}{A} = \frac{11{,}58.10^3}{6400} = 1{,}81\ \mathrm{MPa}$$

Critical length (half of the actual length of diagonal due to joining to the neglected verticals = purlins)

$$L_{cr,z} = 1360\ \mathrm{mm}$$




Moment of inertia

$$I_z = \frac{1}{12} b^3 \cdot h = \frac{1}{12} 40^3 \cdot 160 = 0{,}853.10^6\ \mathrm{mm}^4$$

Radius of gyration

$$i_z = \sqrt{\frac{I_z}{A}} = \sqrt{\frac{0{,}853.10^6}{6400}} = 11{,}5\ \mathrm{mm}$$

Slenderness

$$\lambda_z = \frac{L_{cr,z}}{i_z} = \frac{1360}{11{,}5} = 118$$

Relative slenderness

$$\lambda_{rel,z} = \frac{\lambda_z}{\pi} \cdot \sqrt{\frac{f_{c,0,k}}{E_{0,05}}} = \frac{118}{\pi} \cdot \sqrt{\frac{19}{6400}} = 2{,}05$$

Factor for reduction factor calculation

$$k_z = 0{,}5 \cdot \left(1 + \beta_c \cdot (\lambda_{rel,z} - 0{,}3) + \lambda_{rel,z}^2\right) = 0{,}5 \cdot \left(1 + 0{,}2 \cdot (2{,}05 - 0{,}3) + 2{,}05^2\right) = 2{,}78$$

Reduction factor

$$k_{c,z} = \frac{1}{k_z + \sqrt{k_z^2 - \lambda_{rel,z}^2}} = \frac{1}{2{,}78 + \sqrt{2{,}78^2 - 2{,}05^2}} = 0{,}21$$

Reliability criterion

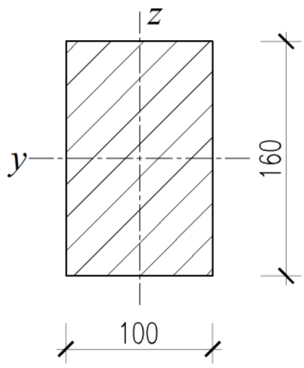
$$\frac{\sigma_{c,0,d}}{k_{c,z} \cdot f_{c,0,d}} = \frac{1{,}81}{0{,}21 \cdot 13{,}15} = 0{,}66 \quad \leq \quad 1{,}0$$

*=> condition is satisfied*

## Diagonal D3

Has the same geometry as diagonal D3 bur smaller normal force.

## Vertical V1




Vertical V1 is actually roof purlin

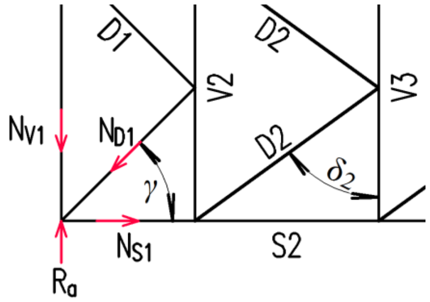
$b$ = 100 mm section depth

$h$ = 160 mm section high

Normal force in vertical V1 can be calculate from static condition of vertical forces

$$\sum F_y = 0 \quad => \quad R_a - N_{D1} \cdot \sin\gamma - N_{V1} = 0$$

$$N_{V1} = R_a - N_{D1} \cdot \sin\gamma = 24{,}74 - 15{,}16 \cdot \sin 45° = 14{,}02 \text{ kN}$$




Design compression stress along the grain

$$\sigma_{c,0,d} = \frac{N_{Ed}}{A} = \frac{14{,}02.10^3}{16000} = 0{,}876 \text{ MPa}$$

Because of reliability criterions of member stressed by normal force and biaxial bending reduction factors for both axis have to be calculated

Critical length

$$L_{cr,y} = L_{cr,z} = 3200 \text{ mm}$$

Moment of inertia

$$I_y = \frac{1}{12} b \cdot h^3 = \frac{1}{12} 0{,}10 \cdot 0{,}16^3 = 34{,}1.10^6 \text{ mm}^4$$

$$I_z = \frac{1}{12} b^3 \cdot h = \frac{1}{12} 0{,}10^3 \cdot 0{,}16 = 13{,}3.10^6 \text{ mm}^4$$

Radius of gyration

$$i_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{34{,}1.10^6}{16000}} = 46{,}2 \text{ mm}$$

$$i_z = \sqrt{\frac{I_z}{A}} = \sqrt{\frac{13{,}3.10^6}{16000}} = 28{,}9 \text{ mm}$$

Slenderness

$$\lambda_y = \frac{L_{cr,y}}{i_y} = \frac{3200}{46{,}2} = 69{,}3$$

$$\lambda_z = \frac{L_{cr,z}}{i_z} = \frac{3200}{28{,}9} = 111$$

Relative slenderness

$$\lambda_{rel,y} = \frac{\lambda_y}{\pi} \cdot \sqrt{\frac{f_{c,0,k}}{E_{0,05}}} = \frac{69{,}3}{\pi} \cdot \sqrt{\frac{19}{6400}} = 1{,}20$$

$$\lambda_{rel,z} = \frac{\lambda_z}{\pi} \cdot \sqrt{\frac{f_{c,0,k}}{E_{0,05}}} = \frac{111}{\pi} \cdot \sqrt{\frac{19}{6400}} = 1{,}92$$

Factor for reduction factor calculation

$$k_y = 0{,}5 \cdot \left(1 + \beta_c \cdot (\lambda_{rel,y} - 0{,}3) + \lambda_{rel,y}^2\right) = 0{,}5 \cdot \left(1 + 0{,}2 \cdot (1{,}20 - 0{,}3) + 1{,}20^2\right) = 1{,}31$$

$$k_z = 0{,}5 \cdot \left(1 + \beta_c \cdot (\lambda_{rel,z} - 0{,}3) + \lambda_{rel,z}^2\right) = 0{,}5 \cdot \left(1 + 0{,}2 \cdot (1{,}92 - 0{,}3) + 1{,}92^2\right) = 2{,}51$$

Reduction factor

$$k_{c,y} = \frac{1}{k_y + \sqrt{k_y^2 - \lambda_{rel,y}^2}} = \frac{1}{1{,}31 + \sqrt{1{,}31^2 - 1{,}20^2}} = 0{,}54$$

$$k_{c,z} = \frac{1}{k_z + \sqrt{k_z^2 - \lambda_{rel,z}^2}} = \frac{1}{2{,}51 + \sqrt{2{,}51^2 - 1{,}92^2}} = 0{,}24$$

Reliability criterions

$$\frac{\sigma_{c,0,d}}{k_{c,y} \cdot f_{c,0,d}} + \boxed{\frac{\sigma_{m,y}}{f_{m,y,d}} + k_m \cdot \frac{\sigma_{m,z}}{f_{m,z,d}}} = \frac{0{,}876}{0{,}54 \cdot 13{,}15} + \boxed{\frac{8{,}13}{13{,}8} + 0{,}7 \cdot \frac{2{,}66}{13{,}8}} = 0{,}84 \quad \leq \quad 1{,}0$$

=> *condition is satisfied*

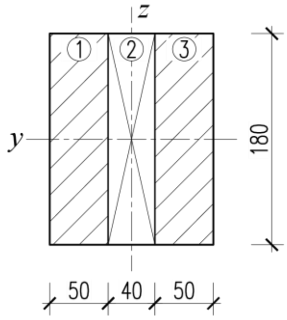
$$\frac{\sigma_{c,0,d}}{k_{c,z} \cdot f_{c,0,d}} + \boxed{k_m \cdot \frac{\sigma_{m,y}}{f_{m,y,d}} + \frac{\sigma_{m,z}}{f_{m,z,d}}} = \frac{0{,}876}{0{,}24 \cdot 13{,}15} + \boxed{0{,}7 \cdot \frac{8{,}13}{13{,}8} + \frac{2{,}66}{13{,}8}} = 0{,}87 \quad \leq \quad 1{,}0$$

=> *condition is satisfied*

it is known from roof purlin assessment

## Upper and bottom chord H3 and S3

Transversal bracing chords (both upper and bottom) are actually upper chord of truss girder



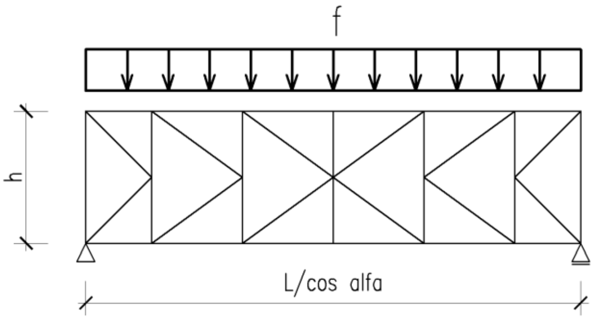

$b$ = 50+40+50=140 mm section depth

$h$ = 180 mm section high

Normal force in chords H3 and S3 can be simplified calculate from bending moment on simple beam

$$M = \frac{1}{8} \cdot f \cdot (L/\cos\alpha)^2 = \frac{1}{8} \cdot 4{,}12 \cdot (12/\cos 12°)^2 = 77{,}51 \text{ kNm}$$

$$N_{S3} = N_{H3} = \pm \frac{M}{h} = \pm \frac{77{,}51}{3{,}2} = \pm 24{,}22 \text{ kN}$$




Design compression stress along the grain

$$\sigma_{c,0,d} = \frac{N_{Ed}}{A_{tot}} = \frac{(53{,}67 + 24{,}22).10^3}{23400} = 3{,}33 \text{ MPa}$$

Reliability criterions

$$\frac{\sigma_{c,0,d}}{k_{c,z} \cdot f_{c,0,d}} = \frac{3{,}33}{\boxed{0{,}986 \cdot 13{,}15}} = 0{,}26 \quad \leq \quad 1{,}0$$

*=> condition is satisfied*

$$\frac{\sigma_{c,0,d}}{k_{c,z} \cdot f_{c,0,d}} = \frac{3{,}33}{\boxed{0{,}178 \cdot 13{,}15}} = 1{,}42 \quad \geq \quad 1{,}0$$

***=> condition is not satisfied***

it is known from truss girder assessment