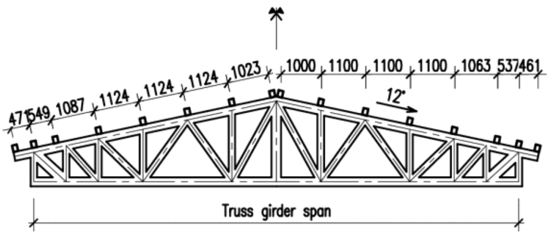
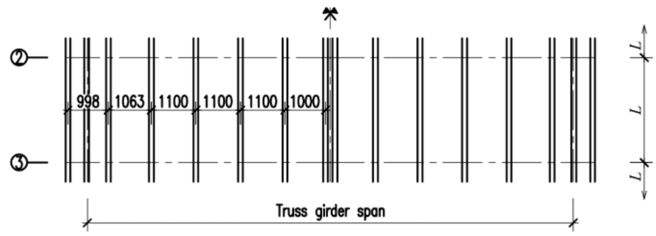
HOW TO CHECK ROOF PURLIN?

…worked examples for BO003 and BO006

WORKED EXAMPLE 1: ROOF PURLIN AS A SIMPLE BEAM

(in this example service class no 2 is considered)





Roof geometry

L = 3200 mm	roof purlin span = roof truss girder spacing
a = 1124 mm	roof purlin spacing parallel to the roof plane
a_0 = 1100 mm	roof purlin spacing in plan view
α = 12°	pitch of the roof (slope)

Beam geometry and material

h = 160 mm	beam section depth
b = 100 mm	beam section width

$$I_y = \frac{1}{12} b \cdot h^3 = \frac{1}{12} 0{,}10 \cdot 0{,}16^3 = 34{,}1.10^6 \text{ mm}^4$$

$$I_z = \frac{1}{12} b^3 \cdot h = \frac{1}{12} 0{,}10^3 \cdot 0{,}16 = 13{,}3.10^6 \text{ mm}^4$$

$$W_y = \frac{1}{6} b \cdot h^2 = \frac{1}{6} 0{,}10 \cdot 0{,}16^2 = 427.10^3 \text{ mm}^3$$

$$W_z = \frac{1}{6} b^2 \cdot h = \frac{1}{6} 0{,}10^2 \cdot 0{,}16 = 267.10^3 \text{ mm}^3$$

Solid timber C20 according to the EN 338

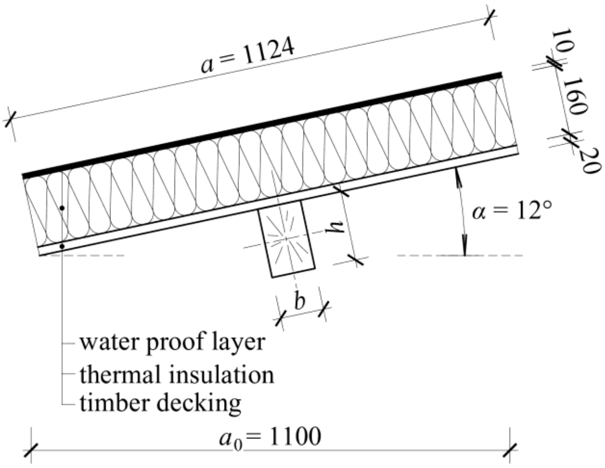
$f_{m,k}$ = 20 MPa	characteristic bending strength
$f_{v,k}$ = 2,2 MPa	characteristic shear strength
$E_{0,mean}$ = 9,5 GPa	mean value of modulus of elasticity parallel to the grain
γ_M = 1,3	partial factor for a material properties
k_{mod}	modification factor for duration of load and moisture content

Load states

Permanent loads

Self-weight of the beam $g_{0,k} = b \cdot h \cdot 5 = 0{,}100 \cdot 0{,}160 \cdot 5 = 0{,}08$ kN/m

Roof cladding



	Volume weight [kN/m³]	Thickness [m]	[kN/m²]
Water proof layer	-	-	0,10
Thermal insulation	0,25	0,16	0,04
Timber decking	5,00	0,02	0,10
Suma			0,24

$g_1 = 0{,}24$ kN/m^2

$g_{1,k} = g_1 \cdot a = 0{,}24 \cdot 1{,}124 = 0{,}27$ kN/m

LS1 Overall dead load $g_k = g_{0,k} + g_{1,k} = 0{,}08 + 0{,}27 = 0{,}35$ kN/m

Imposed loads

LS2 Snow $q_s = 0{,}8$ kN/m^2

$q_{s,k} = q_s \cdot a_0 = 0{,}8 \cdot 1{,}100 = 0{,}88$ kN/m

LS3 Service load - planar $f = 0{,}75$ kN/m^2

$f_k = f \cdot a = 0{,}75 \cdot 1{,}124 = 0{,}84$ kN/m

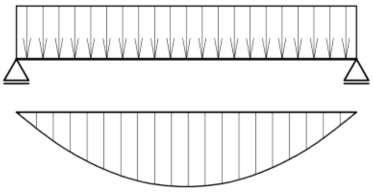
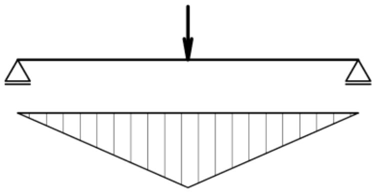
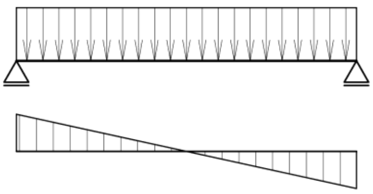
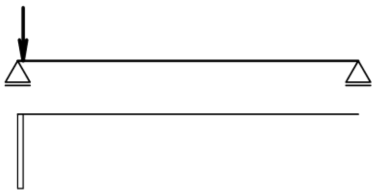
LS4 Service load - force $F_k = 1{,}0$ kN

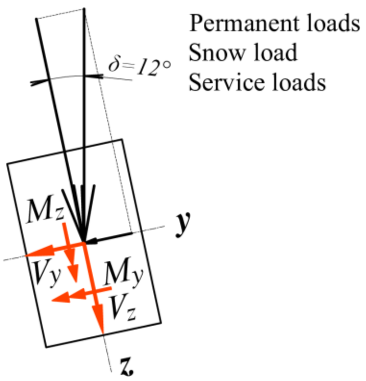
LS5 Wind - pressure $q_w = 0{,}1$ kN/m^2

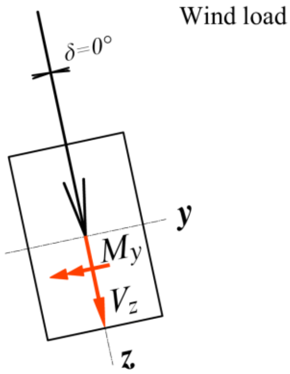
$q_{w,k} = q_w \cdot a = 0{,}1 \cdot 1{,}124 = 0{,}11$ kN/m

Internal forces

Internal forces should be calculated according to the equations listed in table:

Load	Uniformly distributed load	Force
Bending moment	$M_{Ek,y} = \frac{1}{8} \cdot f_k \cdot \cos\delta \cdot L^2$ $M_{Ek,z} = \frac{1}{8} \cdot f_k \cdot \sin\delta \cdot L^2$	$M_{Ek,y} = \frac{1}{4} \cdot F_k \cdot \cos\delta \cdot L$ $M_{Ek,z} = \frac{1}{4} \cdot F_k \cdot \sin\delta \cdot L$
Diagram		
Shear force	$V_{Ek,y} = \frac{1}{2} \cdot f_k \cdot \sin\delta \cdot L$ $V_{Ek,z} = \frac{1}{2} \cdot f_k \cdot \cos\delta \cdot L$	$V_{Ek,y} = F_k \cdot \sin\delta$ $V_{Ek,z} = F_k \cdot \cos\delta$
Diagram		





Internal forces of all load states and load combinations:

	LS1	LS2	LS3	LS4	LS5	LC1	LC2	LC3
δ	12°	12°	12°	12°	0°	-	-	-
ψ_0	-	0,5	0,7	0,7	0,6	-	-	-
$M_{Ek,y}$; $M_{Ed,y}$	0,44	1,10	1,05	0,78	0,14	3,47	3,12	2,73
$M_{Ek,z}$; $M_{Ed,z}$	0,09	0,23	0,22	0,17	0	0,71	0,64	0,53
$V_{Ek,y}$; $V_{Ed,y}$	0,12	0,29	0,28	0,21	0	0,89	0,79	0,67
$V_{Ek,z}$; $V_{Ed,z}$	0,55	1,37	1,31	0,98	0,18	4,34	3,90	3,41

Load combinations are created according next rules (according to the rule 6.10 in EN 1990). Load states LS3 and LS4 have not to be combined together!!!:

$$\mathrm{LC1} = 1{,}35 \cdot \mathrm{LS1} + 1{,}5 \cdot \left(\mathrm{LS2} + \psi_0 \cdot \mathrm{LS3} + \psi_0 \cdot \mathrm{LS5}\right)$$

$$\mathrm{LC2} = 1{,}35 \cdot \mathrm{LS1} + 1{,}5 \cdot \left(\psi_0 \cdot \mathrm{LS2} + \mathrm{LS3} + \psi_0 \cdot \mathrm{LS5}\right)$$

$$\mathrm{LC3} = 1{,}35 \cdot \mathrm{LS1} + 1{,}5 \cdot \left(\psi_0 \cdot \mathrm{LS2} + \psi_0 \cdot \mathrm{LS3} + \mathrm{LS5}\right)$$

where ψ_0 is factor for combination value of a imposed load (see EN 1990)

Ultimate limit state

Shear resistance

$$\tau_\mathrm{y} = \frac{V_\mathrm{Ed,y} \cdot S_\mathrm{y}}{I_\mathrm{y} \cdot b_\mathrm{ef}} = \frac{3}{2} \cdot \frac{V_\mathrm{Ed,y}}{h \cdot b_\mathrm{ef}} = \frac{3}{2} \cdot \frac{0{,}89.10^3}{160 \cdot 67} = 0{,}12 \text{ MPa}$$

$$\tau_\mathrm{z} = \frac{V_\mathrm{Ed,z} \cdot S_\mathrm{z}}{I_\mathrm{z} \cdot h_\mathrm{ef}} = \frac{3}{2} \cdot \frac{V_\mathrm{Ed,z}}{b \cdot h_\mathrm{ef}} = \frac{3}{2} \cdot \frac{4{,}34.10^3}{100 \cdot 107} = 0{,}61 \text{ MPa}$$

where

$$h_\mathrm{ef} = k_\mathrm{cr} \cdot h = 0{,}67 \cdot 160 = 107 \text{ mm}$$

$$b_\mathrm{ef} = k_\mathrm{cr} \cdot b = 0{,}67 \cdot 100 = 67 \text{ mm}$$

$k_\mathrm{cr} = 0{,}67$ reduction factor taking into account cracks (value is valid for solid timber)

Design shear strength

$$f_\mathrm{v,d} = k_\mathrm{mod} \cdot \frac{f_\mathrm{v,k}}{\gamma_\mathrm{M}} = 0{,}9 \cdot \frac{2{,}2}{1{,}3} = 1{,}52 \text{ MPa}$$

Reliability criterion (on the safe side)

$$\frac{\tau_\mathrm{y}}{f_\mathrm{v,d}} + \frac{\tau_\mathrm{z}}{f_\mathrm{v,d}} = \frac{0{,}12}{1{,}52} + \frac{0{,}61}{1{,}52} = 0{,}48 \quad \leq \quad 1{,}0$$

=> ***condition is satisfied***

Bending resistance

$$\sigma_\mathrm{m,y} = \frac{M_\mathrm{Ed,y}}{W_\mathrm{y}} = \frac{3{,}47.10^6}{427.10^3} = 8{,}13 \text{ MPa}$$

$$\sigma_\mathrm{m,z} = \frac{M_\mathrm{Ed,z}}{W_\mathrm{z}} = \frac{0{,}71.10^6}{267.10^3} = 2{,}66 \text{ MPa}$$

Design bending strength

$$f_\mathrm{m,y,d} = f_\mathrm{m,z,d} = k_\mathrm{mod} \cdot \frac{f_\mathrm{m,k}}{\gamma_\mathrm{M}} = 0{,}9 \cdot \frac{20}{1{,}3} = 13{,}8 \text{ MPa}$$

Reliability criterions

$$\frac{\sigma_{m,y}}{f_{m,y,d}} + k_m \cdot \frac{\sigma_{m,z}}{f_{m,z,d}} = \frac{8,13}{13,8} + 0,7 \cdot \frac{2,66}{13,8} = 0,72 \quad \leq \quad 1,0$$

=> *condition is satisfied*

$$k_m \cdot \frac{\sigma_{m,y}}{f_{m,y,d}} + \frac{\sigma_{m,z}}{f_{m,z,d}} = 0,7 \cdot \frac{8,13}{13,8} + \frac{2,66}{13,8} = 0,60 \quad \leq \quad 1,0$$

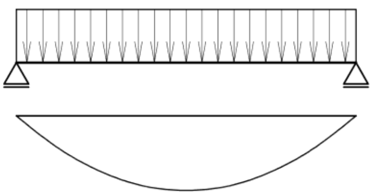
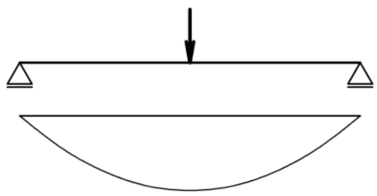
=> *condition is satisfied*

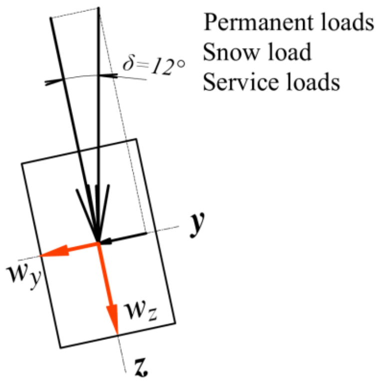
$k_m = 0,7$ for rectangular sections made of solid timber (factor makes allowance for re-distribution of stresses and the effect of inhomogeneities of the material in a cross-section)

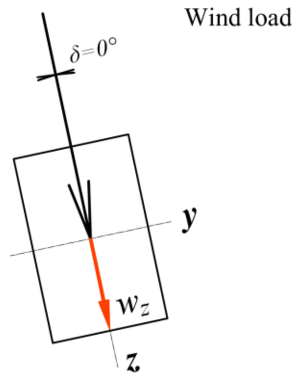
<u>Serviceability limit state</u>

Instantaneous deflection

Deflections should be calculated according to the equations listed in table:

Load	Uniformly distributed load	Force
Instantaneous deflection (in the time of load installation)	$w_{inst,y} = \frac{5}{384} \cdot \frac{f_k \cdot \sin\delta \cdot L^4}{E_{0,mean} \cdot I_z}$ $w_{inst,z} = \frac{5}{384} \cdot \frac{f_k \cdot \cos\delta \cdot L^4}{E_{0,mean} \cdot I_y}$	$w_{inst,y} = \frac{1}{48} \cdot \frac{F_k \cdot \sin\delta \cdot L^3}{E_{0,mean} \cdot I_z}$ $w_{inst,z} = \frac{1}{48} \cdot \frac{F_k \cdot \cos\delta \cdot L^3}{E_{0,mean} \cdot I_y}$
Diagram		





Instantaneous deflections of all load states and load combinations:

	LS1	LS2	LS3	LS4	LS5	LC1	LC2	LC3
δ	12°	12°	12°	12°	0°	-	-	-
$w_{inst,y}$	0,78	1,96	1,88	1,12	0	-	-	-
$w_{inst,z}$	1,44	3,61	3,46	0,21	0,47	-	-	-
w_{inst}	1,64	4,11	3,94	1,14	0,47	8,78	7,91	6,92

where

$$w_{inst} = \sqrt{w_{inst,y}{}^2 + w_{inst,z}{}^2}$$ this simplification could be used for small angles

Load combinations are created according next rules (according to the rule 6.14b in EN 1990). Load states LS3 and LS4 have not to be combined together!!!:

$$LC1 = 1{,}0 \cdot LS1 + 1{,}0 \cdot (LS2 + \psi_0 \cdot LS3 + \psi_0 \cdot LS5)$$

$$LC2 = 1{,}0 \cdot LS1 + 1{,}0 \cdot (\psi_0 \cdot LS2 + LS3 + \psi_0 \cdot LS5)$$

$$LC3 = 1{,}0 \cdot LS1 + 1{,}0 \cdot (\psi_0 \cdot LS2 + \psi_0 \cdot LS3 + LS5)$$

Action	ψ_0	ψ_1	ψ_2
Snow load except FIN, NOR, SWE, ISL up to 1000 meters above sea level	0,5	0,2	0,0
Service load on roofs category H	0,7	0,2	0,0
Wind load	0,6	0,2	0,0

Limit value of instantaneous deflection

$$w_{inst,lim} = \frac{L}{300} = \frac{3200}{300} = 10{,}7 \text{ mm}$$

Reliability criterion

$$\frac{w_{inst}}{w_{inst,lim}} = \frac{8{,}78}{10{,}7} = 0{,}82 \quad \leq \quad 1{,}0$$ => ***condition is satisfied***

Final deflection

Final deflections should be calculated according to the equations listed below:

$$w_{fin} = w_{inst} \cdot (1 + k_{def})$$ for permanent actions

$$w_{fin} = w_{inst} \cdot (1 + \psi_2 \cdot k_{def})$$ for the leading variable action

$$w_{fin} = w_{inst} \cdot (\psi_0 + \psi_2 \cdot k_{def})$$ for accompanying variable action

$k_{def} = 0{,}8$ for service class no 2 and solid timber (factor for the evaluation of creep deformation taking into account the relevant service class)

ψ_2 factor for quasi-permanent value of a imposed load (see EN 1990)

Final deflections of all load states and load combinations:

	LS1	LS2	LS3	LS5	LC
LS2 is leading imposed load	2,95	4,11	2,76	0,28	10,10
LS3 is leading imposed load	2,95	2,06	3,94	0,28	9,23
LS5 is leading imposed load	2,95	2,06	2,76	0,47	8,24

Limit value of final deflection

$$w_{\mathrm{fin,lim}} = \frac{L}{200} = \frac{3200}{200} = 16{,}0 \text{ mm}$$

Reliability criterion

$$\frac{w_{\mathrm{inst}}}{w_{\mathrm{inst,lim}}} = \frac{10{,}10}{16{,}0} = 0{,}63 \quad \leq \quad 1{,}0$$

=> ***condition is satisfied***